Columbia discussions immediately preceding this experiment.

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† Also at International Business Machines, Watson Scientific Laboratories, New York, New York.

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[11] *Note added in proof.*—We have now observed an energy dependence of a in the $1+a\cos\theta$ distribution which is somewhat less steep but in rough qualitative agreement with that predicted by the two-component neutrino theory $(\mu\rightarrow e+\nu+\bar{\nu})$ without derivative coupling. The peak-to-valley ratios for electrons traversing 9.3 g/cm², 15.6 g/cm², and 19.8 g/cm² of graphite are observed to be 1.80±0.07, 1.84±0.11, and 2.20±0.10, respectively.

Results from an Enriched Negative *K*-Meson Beam*

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WE have recently obtained a K^--meson beam from the Bevatron in which the intensity was greatly enhanced by selection of particles emitted in the forward direction. We further improved the usefulness of the beam incident on our emulsion stacks by causing the magnetically analyzed particles of 435 Mev/c to traverse a polystyrene degrader of 18.36 g/cm² and undergo a second bending of 180°, thus discarding the pion component of the beam. The remaining background tracks are chiefly muons and electrons. A small emulsion stack exposed in order to evaluate the beam has already yielded useful information. Although much more work is planned on this and a larger stack, some of the data now in hand are of sufficient interest and reliability for a preliminary report. In order to make quantitative measurements the emulsion density was carefully determined, and we employed our new range-energy curve.[1] The numbers in Table I were derived from along-the-track scanning of 1224 K mesons. Of these, 21 decayed in flight, 182 interacted inelastically in flight with emulsion nuclei, 6 scattered elastically from free protons in the emulsion, 2 interacted in flight with free protons to produce negative hyperons, and only 2 interacted at rest with free protons to produce charged hyperons (the two had opposite signs). The K-meson energy interval for which the interaction cross sections were calculated was 30 to 90 Mev. Analysis of hyperfragments and their parent stars was carried out on an IBM 650 digital computer using a program kindly supplied by Dr. C. Violet. We are greatly indebted to Ernestine Beleal, Anna-Mary Bush, Thoma Davis, John Dyer, Renée Feldman, Hester Lowe, Lynn Reynolds, and Toni Woodford for their conscientious scanning work.

TABLE I. Measurements obtained from the interaction and decay of negative K mesons in emulsion.

K^- mean life	$(1.46_{-0.31}^{+0.38})\times10^{-8}$ sec
K^--proton elastic scattering cross section	$(52_{-21}^{+31})\times10^{-27}$ cm²
K^- free path for inelastic collisions in emulsion	$27.2\pm\ 2.3$ cm
Σ^+ mass (from $\Sigma^+\rightarrow$proton decay)	$(2327.8\pm\ 0.7)\ m_e$
Σ^- mass (from $\Sigma^--\Sigma^+$ mass difference)	$(2341.5\pm\ 2.3)\ m_e$
K^- mass (from $K^-+p\rightarrow\Sigma^++\pi^-$ at rest)	$(965.3\pm\ 1.5)\ m_e$
K^- mass (from $K^-+p\rightarrow\Sigma^-+\pi^+$ in flight)	$(961.4\pm\ 3.3)\ m_e$
K^- mass (from K^-+p elastic collisions)	$(978\ \ \pm25\ \ )\ m_e$
Binding of Λ^0 in ${}_\Lambda He^5$	$3.0\pm\ 0.6$ Mev
Binding of Λ^0 in ${}_\Lambda He^4$	$1.2\pm\ 1.0$ Mev
Binding of Λ^0 in ${}_\Lambda Li^9$	$3.8\pm\ 3.0$ Mev
Decay branching ratio $(\Sigma^+\rightarrow p+\pi^0)/(\Sigma^+\rightarrow n+\pi^+)$	13/13
Frequency distribution of prongs from K^- stars at rest	
Prongs 0: 1: 2: 3: 4: 5:6:7:8:9	
Distrib. 36:43:63:30:28:20:9:2:2:1	
Frequency distribution of prongs from K^- stars in flight	
Prongs 0: 1: 2: 3: 4: 5:6:7:8:9:10:11:12	
Distrib. 16:22:46:34:31:14:8:4:0:1: 1: 0: 1	
Frequency distribution of prongs from K^- stars that emit hyperfragments	
Prongs 1:2: 3: 4:5:6:7:8	
Distrib. 1:3:11:12:6:3:2:3:1	
Frequency distribution of hyperfragment prongs	
Prongs 1: 2: 3:4:5:6:7	
Distrib. 4:11:15:5:3:1:1	
Frequency of hyperfragment emission from K^- stars	28/1152
Ratio of mesonic to nonmesonic decay of hyperfragments	9/42

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Energy of Interacting Fermi Systems

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THE purpose of this note is to make known a number of investigations concerning the energy of interacting Fermi systems. All of these investigations are based on an approximation which may be crudely characterized as an expansion in the ratio of the distance between particles to the distance traversed between successive virtual scatterings. This parameter can be small either because the interaction is weak, or because, although strong and repulsive, the interaction has a range short compared to the distance between particles. The similarity between these two alternatives is mathematically formulated and utilized by systematically rearranging perturbation theory according to the pseudopotential description of Fermi.[1] When the potential is weak, this rearrangement reduces essentially to ordinary perturbation theory; when it is strong, but of short range, the modified procedure may still be employed. In the present note we report on calculations dealing with each of these possibilities.

Our interest in the results of perturbation theory for weak interactions lies in the applicability of this method to nuclear physics. For a gas of nucleons with a density near the observed one, and interacting by conventional attractive nucleon-nucleon potentials, the perturbation expression for the energy[2,3] converges quite rapidly.[4] We have examined this perturbation expression to determine how large a repulsion is required in odd states in order to obtain conditional saturation from exchange forces alone. Such a repulsion does not seem to be compatible with the scattering data. We have also, preliminary to studying saturation with more realistic forces, examined the first-order energies, as a function of density, of states with given relative angular momentum. In the neighborhood of the observed density, s, p, and d states are all significant.

The second-order perturbation energy, which gives a first approximation to the correlational energy between each pair of particles, has also been calculated in the earlier discussions.[2,3] We have calculated this energy taking the exclusion principle into account (as in these references) and also neglecting it. At the observed density, with conventional forces, the latter result is about four times as large as the former, and consequently the Pauli principle is quite important. When it is taken into account, the second-order perturbation energy is about one-tenth of the first order and convergence seems quite good.[4]

The third-order perturbation expression contains two terms in addition to another contribution to the correlational energy of each pair of particles. The first of these represents corrections to the energy arising from correlations in the motion of more than two particles; the second describes the effect on the correlations of two of the average velocity-dependent field of the remainder. The former has recently been estimated by both Brueckner[5] and Bethe.[4] A slightly more detailed examination of it suggests a somewhat larger estimate (about one-eighth of the second-order energy at the observed density). The latter term is one to which great weight has been given in recent discussions.[1,6] Attempts have been made to take it into account more accurately by making the equation for the pseudopotential approximately "self-consistent." A careful calculation of this term yields a result not significantly larger than that obtained for the former one. With conventional forces, a quadratic approximation to the velocity dependence of the potential overestimates the latter correction by a factor of two; an effective-mass approximation which also neglects the exclusion principle yields a value eight times too large. The essential point appears to be that because of the exclusion principle, and either the short range or weakness of the interaction, collisions take place rarely. When they do take place, however, the average momentum change is quite large. Consequently the quadratic approximation to the potential, although valid at low energies, considerably overestimates the difference in potential between typical initial and intermediate states. This overestimate is even greater when strong and more singular forces, with their characteristically higher momenta, are employed. After a smaller initial reduction due to the exclusion principle, the second correction term is still overestimated by the reduced-mass approximation to it, by a factor of three in the case of a tensor force, and a factor of five for a force with a hard core. In all cases the actual value of this correction term appears to be sufficiently small to permit its determination by modified perturbation theory. The more complicated self-consistent procedures of Brueckner and Levinson seem unnecessary.[7]

Our first application of this perturbation theory to a system in which there is a strong short-range interaction has been to the case of a pure hard-core gas. Such a gas is characterized by one parameter, the product of the hard-core radius, a, and the Fermi momentum, k_F, and the energy may be expressed as an expansion in it. To third order, the energy of such a gas, with the spin degrees of freedom of a neutron-proton system, is[8,9]

$$E=\frac{k_F^2}{2m}\left[\frac{3}{5}+\frac{2}{\pi}(k_Fa)+\frac{12}{35\pi^2}(11-2\log 2)(k_Fa)^2+0.78(k_Fa)^3\right].$$

Approximations used in evaluating the third-order term may be in error by ten percent. To third order, effects of a velocity dependence of the average potential do not contribute, but effects of three- and four-body correlations do. The terms they give rise to would be significant in the nucleus if the hard-core interaction were present in all states and if its effect were not diminished by the surrounding nuclear attraction.

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Saturation of Nuclear Forces

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In this note we wish to report on a calculation[1] of the energy of nuclear matter interacting by two-body forces which qualitatively describe the observed low-energy nuclear scattering data. In the S-dominant triplet $J=1$ states, these forces contain a tensor component which reproduces the deuteron properties. In the singlet S state, they have a hard core (0.5×10^{-13} cm), which leads to the prediction of a repulsive phase shift consistent with the high-energy proton-proton scattering.[2] In the remaining states the forces have been chosen to reproduce the phase shifts of Feshbach and Lomon[3] in the region below 150 Mev. Comparatively manageable calculations with these forces have been made possible by taking them to be separable.[4] These calculations have been performed and found to predict saturation at a density and energy not very different from the ones determined empirically. This saturation results primarily from repulsive interactions in states with nonzero angular momentum.

Probably the most significant results of these calculations are their quite reasonable qualitative features. The first of these is that the important contributions to the nuclear energy depend primarily on the properties of nuclear forces at the relative kinetic energies (below 125 Mev) present in the nucleus. A tensor force affects the scattering data at these energies and therefore can alter the energy; a hard core of the usual dimensions has a much smaller effect. In particular, a hard core in the singlet S state alters the energy by only one Mev near the normal density, and also at 1.4 times that density. At the normal density, the hard core actually *decreases* the nuclear energy since at this density, the attraction, which must also be increased to maintain the correct scattering properties, overcompensates in the energy the repulsion of the hard core. At the higher density the results differ by one Mev in the opposite direction as the hard core starts to become really significant.

Even when the potential contains a hard core and deep attraction it does not seem possible to neglect the statistical effects of other particles on the scattering of two. They appear to be important whenever the scattering length of the two-body interaction is large compared to the distance between particles. The dynamical effects of other particles, on the other hand, have a small effect on the scattering of two.[1]

More important than the changes produced by hard cores are the effects of the low-energy interactions in states of higher relative angular momenta. For lack of better information, we have used for each of these a potential which duplicates the corresponding Feshbach-Lomon phase shift. These phase shifts (unlike any determined from meson-theoretic potentials) describe the scattering data below 100 Mev at least qualitatively. They are characterized by two properties: one is the prediction of very little P-wave scattering when averaged over spins, but of fairly substantial individual P-wave phase shifts; the other is the presence of a net repulsion in relative D states due to a sizable repulsion in the D-dominant triplet $J=1$ state. The latter feature is one which cannot be reconciled with simple central forces whatever their exchange. However, it seems to agree with the prediction of a singular tensor force in the triplet $J=1$ state.[5] Since the scattering is weaker and the intermediate momenta higher, the repulsive effects of the exclusion principle are not so important in most of these states of higher angular momenta. In all but one, the $^3P_0$ state, the energy is closely related to the corresponding phase shift.

Using the forces described above, we have determined the nuclear energy at the observed nuclear density and at 1.4 times that density. At the observed density we obtain a volume energy per nucleon of 18 Mev,[6] and at the higher density we obtain a smaller binding energy. The calculated minimum lies between these densities and probably fairly close to the observed one.

While we place little weight on the numerical results, they do not appear to depend sensitively on variations in potential consistent with the same scattering phase shifts. They do appear to make it plausible, although far from definite, that the gross properties of nuclear matter may be qualitatively understood on the basis of two-body forces alone. In particular, it seems that the "real" forces present in relatively low-energy scattering may give rise to saturation at a density near the