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Higher Order Radiative Corrections to Electron Scattering*

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The higher order radiative corrections are examined in the infrared region, and Schwinger's conjecture regarding the functional dependence of these corrections on the energy resolution is proved.

RECENT high-energy electron scattering experiments[1] have been carried out under conditions for which the radiative corrections are quite large in magnitude, although their effect on the relative angular distribution is not too important. Schwinger's[2] original calculation of the radiative corrections, correct to lowest order in both the fine-structure constant and the external potential, resulted in a fractional correction to the elastic cross section σ_{el} computed without regard for radiation

$$\sigma(\theta,E,\Delta E)\cong(1-\delta)\sigma_{el}(\theta), \qquad (1)$$

where, in the high-energy limit, δ is given by

$$\delta\cong\frac{4\alpha}{\pi}\ln\left(\frac{2E\sin\theta/2}{m}\right)\ln\left(\frac{E}{\Delta E}\right). \qquad (2)$$

The various quantities occurring in Eq. (2) have their usual meanings; in particular, ΔE is the upper limit of the energy that can be radiated by the electron if it is to be recorded by the detection system. The occurrence of ΔE in δ is a consequence of the well-known infrared divergence associated with the bremsstrahlung and the fact that real and virtual photon processes must be considered together.

The restriction of the validity of Eq. (1) to the lowest order of the scattering potential was removed by Suura,[3] and explicit calculations by Newton[4] and Chrétien[5] confirmed Eq. (1) for the special case of the second Born approximation to scattering in a Coulomb potential. Schwinger noted that if the energy resolution of the detector were improved (i.e., ΔE decreased), δ would become large and Eq. (1) would lose validity. He pointed out that under these conditions higher order radiative corrections would become important; and on the basis of Bloch-Nordsieck[6] type arguments, he conjectured that the proper form for Eq. (1) would be obtained by the substitution

$$1-\delta\rightarrow e^{-\delta}. \qquad (3)$$

In this note we shall re-examine the question of the infrared divergence and show that Schwinger's conjecture is asymptotically true for small ΔE ($\Delta E\ll E$).

Starting with Bloch and Nordsieck,[6] the infrared divergence problem has been considered by many authors. For references and discussion of earlier literature, we refer the reader to Jauch and Rohrlich,[7] who have recently given a comprehensive quantum-electrodynamical treatment of soft-quantum processes. In this paper we shall examine in somewhat more detail how the statistical independence of the soft quanta arises. The complicated overlapping of the real and virtual soft quanta will be disentangled by symmetrizing over the order of their emission and absorption. The importance of this symmetrization procedure for emission of real photons was pointed out by Gupta[8] in a discussion of multiple bremsstrahlung. These detailed considerations will be presented later in two lemmas, and the main features of the proof of Schwinger's conjecture (assuming the statistical independence of the soft photons) will be given first.

For a given electron energy E and scattering angle θ, we seek the asymptotic dependence of the cross section $\sigma(\theta,E,\Delta E)$ on ΔE in the limit $\Delta E\ll E$. As a device for handling the infrared divergence, we introduce a minimum photon momentum K_m.[9] As shown in Lemma I, a Poisson distribution is obtained for the emitted photons and the desired cross section is given by[10]

$$\sigma(\theta,E,\Delta E)=\lim_{K_m\rightarrow 0}\left[\sum_{n=0}^{\infty}\frac{(\alpha A)^n}{n!}\int_{K_m}^{\Sigma k_i=\Delta E}\cdots\int\frac{dk_1}{k_1}\frac{dk_2}{k_2}\cdots\frac{dk_n}{k_n}\right]\times\sigma'(\theta,E,K_m), \qquad (4)$$

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1 Experimental and theoretical results on high-energy electron scattering are summarized in R. Hofstadter, Revs. Modern Phys. **28**, 214 (1956).

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3 H. Suura, Phys. Rev. **99**, 1020 (1955).

4 R. G. Newton, Phys. Rev. **97**, 1162 (1955); **98**, 1514 (1955).

5 M. Chrétien, Phys. Rev. **98**, 1515 (1955).

6 F. Bloch and A. Nordsieck, Phys. Rev. **52**, 54 (1937).

7 J. M. Jauch and F. Rohrlich, Helv. Phys. Acta **27**, 613 (1954); *Theory of Photons and Electrons* (Addison-Wesley Press, Cambridge, 1955), pp. 390–405.

8 S. N. Gupta, Phys. Rev. **98**, 1507 (1955); **99**, 1015 (1955).

9 This is the usual quantum electrodynamics with photon operators for momentum k less than K_m omitted from the theory. This procedure is not invariant, but for the present purpose it is simpler than introducing a photon mass.

10 We use units in which $\hbar=c=1$.

where $\sigma'(\theta,E,K_m)$ is the renormalized cross section for scattering without emission of real photons, and where

$$A=\frac{1}{2\pi^2}k^2\int d\Omega_k\left(\frac{p_\mu}{p\cdot k}-\frac{p_\mu'}{p'\cdot k}\right). \tag{5}$$

Equation (4) expresses the usual $1/k$ dependence for the emission of soft photons, and the factors A are the result of the integration over the angles of the emitted photons. According to the definition of $\sigma(\theta,E,\Delta E)$, the total energy of all the emitted photons must be less than ΔE.[11]

If the photon energies occurring in Eq. (4) could individually range between K_m and ΔE, a very simple result would be obtained since the series would reduce to exponential form. The actual limits of integration can be handled easily by a method used previously by Jauch and Rohrlich[7] in which the upper limit of integration is expressed by means of an integral

$$\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\exp[i\lambda(1-\sum k_i/\Delta E)]}{\lambda-i\epsilon}d\lambda=\begin{cases}1 \text{ if } \sum k_i<\Delta E\\ 0 \text{ if } \sum k_i>\Delta E,\end{cases} \tag{6}$$

where ϵ is a small positive number. When one uses this integral representation, the series can easily be summed to give

$$\sigma(\theta,E,\Delta E)=\lim_{K_m\to 0}\left[\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{d\lambda}{\lambda-i\epsilon}e^{i\lambda}\exp\left(\alpha A\int_{K_m/\Delta E}^{1}e^{-i\lambda x}\frac{dx}{x}\right)\right]\times\sigma'(\theta,E,K_m). \tag{7}$$

The asymptotic value of the expression in square brackets (for $K_m\ll\Delta E$) can now be extracted and $\sigma(\theta,E,\Delta E)$ takes the form

$$\sigma(\theta,E,\Delta E)=F(\alpha A)\lim_{K_m\to 0}\exp\left(\alpha A\ln\frac{\Delta E}{K_m}\right)\sigma'(\theta,E,K_m), \tag{8}$$

where

$$F(\alpha A)=\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{d\lambda}{\lambda-i\epsilon}e^{i\lambda}\exp\left[\alpha A\int_0^1\frac{dx}{x}(e^{-i\lambda x}-1)\right] \tag{9}$$
$$=1-\tfrac{1}{12}\pi^2(\alpha A)^2+\cdots.$$

It is shown in Lemma II that, in the limit of small K_m, σ' satisfies the differential equation

$$\partial\sigma'/\partial K_m=(\alpha A/K_m)\sigma'. \tag{10}$$

The limit in Eq. (8) therefore exists, and we can infer that

$$\sigma(\theta,E,\Delta E)\propto e^{\alpha A\ \ln\Delta E}. \tag{11}$$

[11] The series in Eq. (4) terminates after a finite number of terms ($nK_m\leq\Delta E$). While this would seem to introduce difficulties, it actually causes no trouble as K_m becomes small since the series "converges" before the last term is reached. This may be seen from the fact that the last term is less than $(\alpha A\ \ln N)^N/N!$, where N is the largest integer less than $(\Delta E/K_m)$.

Because of the nature of Lemma II, the dependence of the constant of proportionality on E and θ cannot be determined by this type of analysis; only the functional dependence on ΔE is fixed. However, it is convenient to express the cross section in terms of the elastic scattering cross section and an unknown integration constant $E(\theta)$

$$\sigma(\theta,E,\Delta E)=e^{-\alpha A\ \ln[E(\theta)/\Delta E]}\sigma_{\text{el}}(\theta). \tag{12}$$

Some of the higher order radiative corrections appear explicitly in the α of Eq. (12); other radiative corrections, such as $F(\alpha A)$, are contained implicitly in $E(\theta)$.

If we ignore the higher-order radiative corrections contained in $E(\theta)$, we can expand Eq. (12) to the first order in α and compare the result with previous calculations; this will fix $E(\theta)$ to zeroth order in α. Comparing with Schwinger's result, Eqs. (1) and (2), which treated the external potential to lowest order, we find

$$\ln E(\theta)\cong\ln E. \tag{13}$$

Schwinger's conjecture follows directly from this. In case the external potential is too strong for the Born approximation to be valid, $E(\theta)$ might be expected to depend on the potential. However, the radiative corrections are associated primarily with the soft photons which are emitted and absorbed far from the scattering center and $E(\theta)$ is in the nature of an upper limit to the energy of the virtual photons which make an important contribution. Since the final result is insensitive to the precise value of $E(\theta)$, we expect that even when the Born approximation is not valid, a good approximation to $E(\theta)$ is given by

$$E(\theta)\cong E, \tag{14}$$

This assumption is strengthened by the explicit calculation of references 3, 4, and 5. This completes the main part of the proof.

Lemma I.—We shall show that the matrix element for the emission of n soft photons of momenta k_1, k_2, $\cdots k_n$ and polarizations e_1, e_2, $\cdots e_n$ is given asymptotically ($\sum k_i\ll E$) by

$$M_n(p',p;k_1,k_2,\cdots k_n)=\left\{\prod_i\left[-\frac{e}{(2k_i)^{\frac{1}{2}}}\right]\left(\frac{e_i\cdot p}{k_i\cdot p}-\frac{e_i\cdot p'}{k_i\cdot p'}\right)\right\}M(p,p';K_m), \tag{15}$$

where $M(p',p;K_m)$ is the matrix element for an electron to scatter from a state of momentum p to one of momentum p', including all the virtual processes. The remarkable fact about Eq. (15) is that all the real photons are dynamically independent of each other and of the virtual photons. This leads to a Poisson distribution for the emitted photons, and Eqs. (4) and (5) follow directly upon squaring the matrix element and averaging over the polarizations. Equation (15) was derived for the case of one-photon emission by

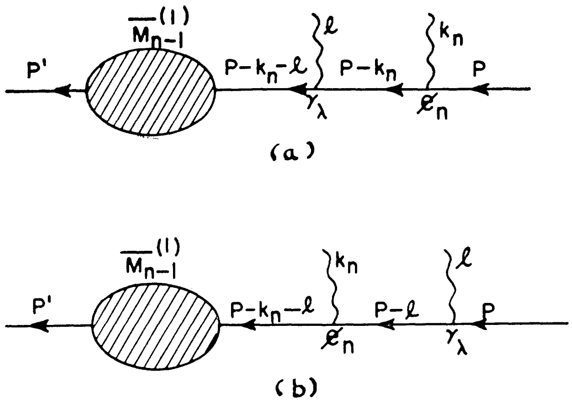

FIG. 1. Diagrams representing the emission of n real photons obtained by inserting the nth photon (momentum k_n and polarization e_n) before (a) and after (b) the first vertex of the diagram for $(n-1)$ photon emission. The photon (momentum l and polarization λ) can be either real or virtual. $\bar{M}_{n-1}$ represents the remaining part of the diagram for $(n-1)$ photon emission after its second vertex.

Newton[12] and Jauch and Rohrlich,[7] who showed that the infrared divergence arose from the emission of the photon from an external electron line. Neglecting virtual photons, Gupta[8] derived the expression for an arbitrary number of photons. The essential step in deriving Eq. (15) is the symmetrization of the order of emission of all the real and virtual soft photons, as was done by Gupta for the real photons, the infrared divergence arising from those emissions which are external to the main scattering process.

Because of the difficulties associated with the overlapping of the emission of real and virtual photons, we shall examine the derivation of Eq. (15) in some detail. The proof will be by induction on n. In the various diagrams contributing to M_n, we assign momenta so that the initial electron momentum p appears in every electron line before the first potential scattering, and the final momentum p' in every electron line after the last potential scattering. We first consider a particular diagram contributing to M_{n-1}, say $\bar{M}_{n-1}$, and insert the emission operator for the nth photon in all possible ways. The matrix element for the emission of a photon (k_n,e_n) before the first vertex γ_λ of $\bar{M}_{n-1}$ (see Fig. 1) may be written

$$\frac{e}{(2k_n)^{\frac{1}{2}}}\bar{M}_{n-1}^{(1)}(p',p-k_n)\frac{1}{\boldsymbol{p}-\boldsymbol{l}-\boldsymbol{k}_n-im}\times\gamma_\lambda\frac{1}{\boldsymbol{p}-\boldsymbol{k}_n-im}\boldsymbol{e}_nG_l, \quad (16)$$

where $\bar{M}_{n-1}^{(1)}$ represents the part of $\bar{M}_{n-1}$ after the second vertex, l is the momentum of either a real or virtual photon, $\boldsymbol{p}\equiv\sum_\mu p_\mu\gamma_\mu$, and G_l is either $(1/l^2)$ or $[e/(2l)^{\frac{1}{2}}]$. Rationalizing the last denominator and putting $k_n=0$ in the first denominator, we obtain

$$\frac{e}{(2k_n)^{\frac{1}{2}}}\bar{M}_{n-1}^{(1)}(p',p-k_n)\frac{1}{\boldsymbol{p}-\boldsymbol{l}-im}\gamma_\lambda\left(-\frac{e_n\cdot p}{k_n\cdot p}\right)G_l. \quad (17)$$

The difference between Eqs. (16) and (17) is

$$\frac{e}{(2k_n)^{\frac{1}{2}}}\bar{M}_{n-1}^{(1)}(p',p-k_n)\frac{1}{\boldsymbol{p}-\boldsymbol{l}-\boldsymbol{k}_n-im}\times\boldsymbol{k}_n\frac{1}{\boldsymbol{p}-\boldsymbol{l}-im}\gamma_\lambda\left(-\frac{e_n\cdot p}{k_n\cdot p}\right)G_l. \quad (18)$$

We have omitted a term $\boldsymbol{k}_n\boldsymbol{e}_n$ from the last numerator in both (17) and (18) since it would give a contribution of order ΔE to the cross section. Expression (17) with k_n set equal to 0 in $\bar{M}_{n-1}^{(1)}$ together with a similar expression obtained by considering the emission of the nth photon after the last vertex of $\bar{M}_{n-1}$ would give the desired result [Eq. (22) below]. However, even though expression (18) would not lead to an infrared divergence for the nth photon, it cannot be neglected because the infrared divergence associated with the photon of momentum l has been increased (it would in fact lead to a $1/K_m$ rather than a $\log K_m$ term in the cross section). We compare (18) with the matrix element for the emission just after the first vertex (Fig. 1):

$$\frac{e}{(2k_n)^{\frac{1}{2}}}\bar{M}_{n-1}^{(1)}(p',p-k_n)\frac{1}{\boldsymbol{p}-\boldsymbol{l}-\boldsymbol{k}_n-im}\times\boldsymbol{e}_n\frac{1}{\boldsymbol{p}-\boldsymbol{l}-im}\gamma_\lambda G_l. \quad (19)$$

Now, moving $\boldsymbol{k}_n$ to the left of $(\boldsymbol{p}-\boldsymbol{l}-\boldsymbol{k}_n-im)^{-1}$ in expression (18), we obtain

$$\frac{1}{(p-l-k_n)^2+m^2}\{-\boldsymbol{k}_n[\boldsymbol{p}-\boldsymbol{l}-im]+2k_n\cdot p-2k_n\cdot l\}.$$

The first term in the numerator cancels the second denominator of (18), so that it does not lead to a higher infrared divergence with respect to an l integration. The last term of the numerator is proportional to l; hence it cancels the low-momentum singularity of the denominator $(\boldsymbol{p}-\boldsymbol{l}-im)$. Thus as the singular term of (18), we have

$$\frac{e}{(2k_n)^{\frac{1}{2}}}\bar{M}_{n-1}^{(1)}(p',p-k_n)\frac{-2e_n\cdot p}{(p-l-k_n)^2+m^2}\times\frac{1}{\boldsymbol{p}-\boldsymbol{l}-im}\gamma_\lambda G_l. \quad (20)$$

In the same way, moving $\boldsymbol{e}_n$ to the left in Eq. (19), we

[12] R. G. Newton, Phys. Rev. 94, 1773 (1954).

obtain the singular term

$$\frac{e}{(2k_n)^{\frac{1}{2}}}\bar{M}_{n-1}{}^{(1)}(p',p-k_n)\frac{2e_n\cdot p}{(p-l-k_n)^2+m^2}\times\frac{1}{\not{p}-\not{l}-im}\gamma_\lambda G_l. \quad (21)$$

(The term in $\boldsymbol{k}_n\boldsymbol{e}_n$ is actually not singular.) Hence expressions (20) and (21) cancel each other completely.

Next we compare (17) with the matrix element for the emission after the second vertex. By the same argument as before, the singular terms of both matrix elements cancel. We proceed in this manner until we reach the first potential scattering. Assuming that the momentum transfers delivered by the external potential are large in comparison to k_n, we may set $k_n=0$ in the part of the matrix element after the first potential without introducing any further singularities. The special case of the Coulomb field may be treated by the methods of reference 3. The result is

$$\frac{e}{(2k_n)^{\frac{1}{2}}}\bar{M}_{n-1}(p',p)\left(-\frac{e_n\cdot p}{k_n\cdot p}\right)$$

as the infrared divergent contribution. In a similar way, if we start from the last electron line, we obtain

$$\frac{e}{(2k_n)^{\frac{1}{2}}}\bar{M}_{n-1}(p',p)\left(\frac{e_n\cdot p'}{k_n\cdot p'}\right).$$

Combining these and summing over all possible diagrams $\bar{M}_{n-1}$ contributing to M_{n-1}, the final result is

$$M_n(p',p;k_1,k_2,\cdots k_n)=-\frac{e}{(2k_n)^{\frac{1}{2}}}\left(\frac{e_n\cdot p}{k_n\cdot p}-\frac{e_n\cdot p'}{k_n\cdot p'}\right)\times M_{n-1}(p',p;k_1\cdots k_{n-1}), \quad (22)$$

which leads immediately to Eq. (15).

Lemma II.—We have to show that $\sigma(\theta,E,\Delta E)$ is actually independent of K_m. The result is obvious from the work of Bloch and Nordsieck,[6] but we prefer to give a more modern treatment. From Eq. (8), we see that σ' must have as a factor $\exp(\alpha A\log K_m)$; any other dependence on K_m must approach a finite limit as $K_m\rightarrow0$. The lemma to be proved may therefore be conveniently expressed as

$$\partial\sigma'/\partial K_m=(\alpha A/K_m)\sigma', \quad (23a)$$

or

$$\partial M'/\partial K_m=(\alpha A/2K_m)M'. \quad (23b)$$

We use the same type of proof as in Lemma I. The matrix element for elastic scattering with radiative

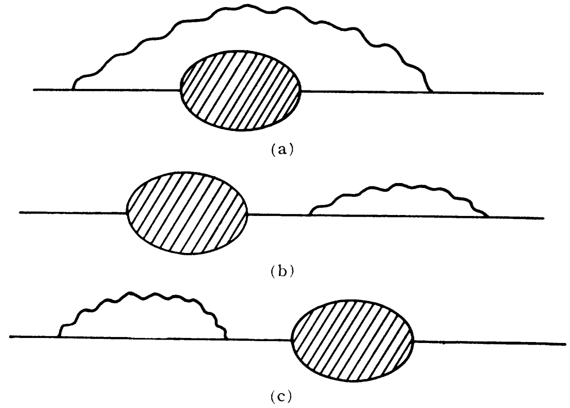

FIG. 2. Three classes of diagrams contributing to b_{-1} Eq. (25) when one of the virtual photons has its magnitude of momentum fixed equal to K_m. The single-photon line is the one whose momentum is fixed. Shaded area represents the remaining part of the elastic scattering diagrams.

corrections may be written

$$M'(\theta,E,K_m)=\sum_{n=0}^{\infty}\frac{1}{n!}\int_{K_m}^{\infty}\cdots\int\prod d^4q_i m(\theta;q_1q_2\cdots q_n), \quad (24)$$

where $m(\theta;q_i)$ is a symmetric function of the virtual photon momenta $q_1, q_2, \cdots q_n$. Differentiating Eq. (24) with respect to K_m, we find terms of the form

$$-\int d^4Km(\theta;q_1\cdots q_{n-1},K)\delta(|\mathbf{K}|-K_m)=K_m{}^{-1}b_{-1}+b_0+\cdots, \quad (25)$$

where the b's are functions of $q_1,\cdots q_{n-1}$; n equal terms occur in the derivative so that the factorial in Eq. (24) becomes $(n-1)!$. In order to obtain b_{-1}, we use the same type of analysis as in Lemma I. The result is effectively that the photon of momentum K terminates on the external electron lines and K is set equal to zero inside the basic diagram as in Figs. 2(a), 2(b), and 2(c). The diagrams (b) and (c) arise from the differentiation with respect to K_m of self-energy parts on the external electron lines. The result for b_{-1} is

$$b_{-1}=\tfrac{1}{2}(\alpha A)m(\theta;q_1\cdots q_{n-1}). \quad (26)$$

The symmetrization procedure discussed in Lemma I shows that the later integration over the q's will lead to no worse divergence than $(\ln K_m)^{n-1}$ in the b's of Eq. (25). The first term of Eq. (25) should dominate as $K_m\rightarrow0$ unless the sum of the remaining terms over n leads to a more singular behavior. If the latter possibility occurred, it would seem to indicate a fundamental difficulty with the theory since the limit as $K_m\rightarrow0$ would not exist. Ignoring this possibility, Eq. (23b) is obtained.

We have so far considered only the infrared divergence. However, because of Ward's identity, there is not actually any ultraviolet divergence except that which is associated with the mass renormalization and the charge renormalization due to self-energy insertions into the photon lines. Since these renormalizations are free of infrared divergencies, our argument is not affected by them.

DISCUSSION

Besides showing the cancellation of the infrared divergence, Jauch and Rohrlich[7] also considered the correction factor for electron scattering. In the latter considerations they treated the electron as a given classical current distribution. The effects of the real and virtual photons produced by such a distribution cancel automatically in the infrared region, but the virtual photons must be cut off arbitrarily in the ultraviolet region. This leads to a correction factor of the form

$$b(\Delta E)=1+\sum_n \frac{(\alpha A)^n}{n!}\left[\ln\frac{\epsilon_n}{\Delta E}\right]^n. \tag{27}$$

If the cut-off parameters ϵ_n are all set equal to $E(\theta)$, the same result as our Eq. (12) is obtained. In their book, Jauch and Rohrlich seem to make the mistake of setting $\epsilon_n=\Delta E$, thus obtaining $b(\Delta E)=1$ for $\Delta E\neq 0$; this is clearly incorrect.

It is obvious that the derivation given here could be applied to inelastic electron scattering without any difficulty; A would be changed slightly because the final electron energy would differ from the initial. Of course, if the nuclear system is excited to a continuum (e.g., electron production of pions), the extra complication of folding in the distribution functions for the two mechanisms of energy loss would be involved. Finally, as long as $E\gg\Delta E$, our results are not restricted to high-energy electron scattering. We had in mind high-energy scattering because the corrections are largest there and offer the best chance of an experimental test.[13]

ACKNOWLEDGMENTS

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[13] *Note added in proof.*—Recent experiments on the radiative corrections to high-energy electron scattering from hydrogen [R. W. McAllister, Phys. Rev. **104**, 1494 (1956) and G. W. Tautfest and W. K. H. Panofsky, Phys. Rev. **105**, (1957)] seem to confirm the lowest order correction, Eq. (1); but the higher order corrections have not yet been investigated experimentally.

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Theory of p-Wave Pion-Nucleon Interaction

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Exact relations are used to determine parameters like the cutoff and the unrenormalized coupling constant which are consistent with the low-energy scattering data. Under general assumptions about the Hamiltonian some inequalities are derived. For the Chew model those parameters can be determined with great accuracy. If one uses a Yukawa source function, the cutoff must be 4.7 and $f_0^2=0.22$ in order not to be inconsistent with those relations.

(1) INTRODUCTION

A GREAT amount of theoretical work has been done on the static model for pions (Chew model) which is characterized by

$$H'=(4\pi)^{\frac{1}{2}}f_0\int dxU(x)\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}\tau_\alpha\phi_\alpha(x). \tag{I}$$

Many approximate results[1] obtained with this model have been compared with experimental data, although it had never been shown whether those approximate solutions were anywhere near the true solutions. In particular the most promising approximation, the Tomonaga method,[2] has recently[3] been shown, in its controllable results, to deviate from the exact solution by a factor 2 to 10.

However, by using a calculation technique developed by Low[4] and by Wick,[5] relations could be obtained

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[5] G. C. Wick, Revs. Modern Phys. **27**, 339 (1955).