Photoproduction of Pion Pairs in Hydrogen*†

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The photoproduction of pion pairs ($\pi^-+\pi^+$) has been measured by observing a π^- signal from liquid hydrogen bombarded with bremsstrahlung of energies up to 600 Mev. The excitation function and π^- energy spectrum measured at 60° in the laboratory system agree qualitatively with the calculations of Cutkowsky and Zachariasen. The major part of the process corresponds to the emission of the π^+ in the resonant P state and the π^- in an S state. The net increase in the π^+ yield due to ($\pi^-+\pi^+$) and ($\pi^0+\pi^+$) pairs is also observed.

I. INTRODUCTION

THE results of experiments on both pion-nucleon scattering and photoproduction of single pions from protons have indicated a strong resonance in the pion-nucleon state corresponding to the total isotopic spin $I=\frac{3}{2}$ and the total angular momentum $J=\frac{3}{2}$.[1] To obtain further evidence relating to the pion-nucleon interaction, one can investigate the state with two mesons and a nucleon: for example, experiments on photoproduction of pion pairs from hydrogen or inelastic pion-nucleon scattering. Such studies may furnish information concerning a meson-meson interaction.

Inelastic pion-scattering experiments have been done at Brookhaven using the π^- beam produced in the Cosmotron. Collisions of these mesons with protons have been observed in a hydrogen diffusion cloud chamber by Eisberg *et al.*[2] A number of multiple pion processes have been observed. Also, Walker and Crussard[3] exposed nuclear emulsions in the π^- beam. The results, primarily the angular distributions, are said to be indicative of decay of an excited pion-nucleon system in the resonant (3,3) state. A detailed theoretical study of this work has not appeared.[4]

Negative pions can be photoproduced from protons only in ($\pi^++\pi^-$) pairs by the reaction

$$\gamma+p\rightarrow p+\pi^++\pi^-. \quad (1)$$

The first pion-pair photoproduction results were reported by two groups[5,6] at the California Institute of Technology who observed the yield of π^- mesons with 500-Mev bremsstrahlung incident upon a high-pressure hydrogen-filled target. Peterson and Henry,[5] using nuclear emulsions as detectors, reported preliminary results of $(11\pm2)\%$ at a lab angle of 73° for the ratio of π^- to π^+ yields from the hydrogen gas. This measurement included a range of pion energies with the minimum detectable meson energy 11 Mev.

Sands *et al.*[6] looked for negative pions from the same target and at the same maximum bremsstrahlung energy with a magnetic spectrometer and counters. Their results at 73° gave a π^-/π^+ ratio of $(1.5\pm0.3)\%$ for pion energies of 47 ± 10 Mev. The measurements were repeated with 375-Mev bremsstrahlung (below threshold), and most but not all of the negative signal disappeared. Sands *et al.* look upon the result as an upper limit for pair production.

The measurements of the two California Institute of Technology groups seemed to be in poor agreement, and some doubt remained concerning the existence of the pion-pair process. When preliminary results[7] of the work reported here were obtained, an explanation was indicated in terms of a rapid variation of the pair-production cross section with the π^- energy.

Peterson[8] has subsequently published new results based on continued scanning of plates from exposures taken with 500-Mev bremsstrahlung. He reports the following values for the hydrogen π^-/π^+ ratio: at a lab angle of 140°, $(26\pm8)\%$ and $(0\pm2)\%$ for mean pion energies of 20 and 65 Mev, respectively; and at 73°, $(16\pm6)\%$ and $(4\pm2)\%$ for mean pion energies of 25 and 54 Mev. These results are compatible with ours and serve as an independent verification of the peaking of the cross section at low π^- energies.

Measurements of the π^- yield from hydrogen with 800-Mev bremsstrahlung are being made by Luckey and Wilson[9] at Cornell University; no results have been reported.

Several experimenters have looked for negative pions from hydrogen below threshold: Jenkins *et al.*[10] at Cornell obtained a π^-/π^+ ratio of $(1\pm4)\%$ for 34-Mev pions at 90° in the lab with 310-Mev bremsstrahlung;

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‡ Now with the Lockheed Aircraft Corporation, Missile Systems Division, Palo Alto, California.

[1] See H. A. Bethe and F. de Hoffmann, *Mesons and Fields* (Row, Peterson and Company, Evanston, 1955), Vol. 2, for a summary of much of the literature.
[2] Eisberg, Fowler, Lea, Shephard, Shutt, Thorndike, and Whittemore, Phys. Rev. **97**, 797 (1955).
[3] W. D. Walker and J. Crussard, Phys. Rev. **98**, 1416 (1955).
[4] See, however, the discussion of S. Barshay, Phys. Rev. **103**, 1102 (1956).
[5] V. Z. Peterson and I. G. Henry, Phys. Rev. **96**, 850 (1954).
[6] Sands, Bloch, Teasdale, and Walker, Phys. Rev. **99**, 652 (1955).
[7] R. M. Friedman and K. M. Crowe, Phys. Rev. **100**, 1799(A) (1955).
[8] V. Z. Peterson, Bull. Am. Phys. Soc. Ser. II, **1**, 173 (1956).
[9] D. Luckey and R. R. Wilson, Bull. Am. Phys. Soc. Ser. II, **1**, 172 (1956).
[10] Jenkins, Luckey, Palfrey, and Wilson, Phys. Rev. **95**, 652 (1955).

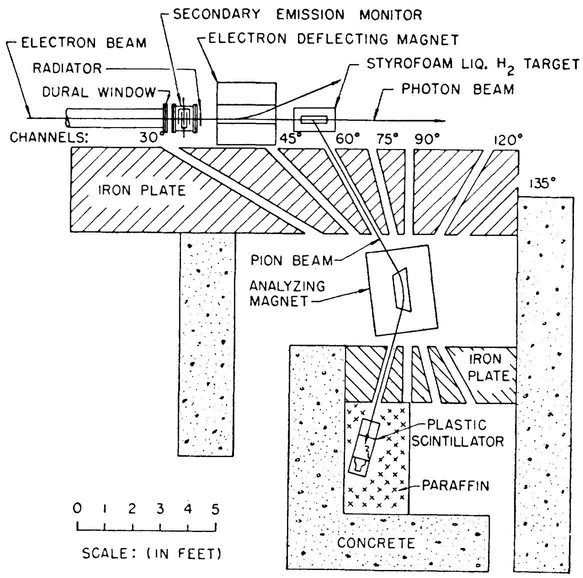

FIG. 1. Experimental arrangement shown in Setup II (60° lab angle with 45° deflection of pion beam).

Littauer and Walker[11] reported a ratio of $(4\pm5)\%$ for 65 ± 15 Mev pions at 135° with 310-Mev bremsstrahlung; White *et al.*[12] at Berkeley obtained an upper limit of 2% for 322-Mev bremsstrahlung[13] which they believe is due to π^- production in the target walls and collimators.

Osborne[14] at the Massachusetts Institute of Technology measured the low-energy π^+ yield at bremsstrahlung energies extending 20 Mev beyond the kinematical pion-pair threshold of 325 Mev for reaction (1). Note that the π^+ yield is due to single-pion production, production of charged pion pairs [reaction (1)], and pair production of the type

$$\gamma+p\rightarrow n+\pi^+ +\pi^0. \tag{2}$$

No increase in yield which would have been attributed to pair production was seen within the statistical errors of $\sim5\%$.

II. EXPERIMENTAL PROCEDURE

A. Method

The pion-detection system used in this measurement has also been employed in several other photomeson experiments. We will describe the system briefly here and refer the reader to a more detailed description.[15]

The first part of this experiment was concerned with measuring the yield of π^- mesons from hydrogen relative to the π^+ yield. Using the average value for the single π^+ cross section reported by Walker *et al.*[16] and Tollestrup, Keck, and Worlock,[17] absolute values for the pair cross section could be obtained.

The counting yield of positive pions from hydrogen for bremsstrahlung of upper energy $k_{\max}$ is

$$Y_{\pi^+}=\eta^+ N_H\left(\frac{d\sigma_{\pi^+}}{d\Omega}\right)_{\text{single}}\Delta\Omega N_k\Delta k +\eta^+ N_H\left(\frac{d^2\sigma_{\pi^+}}{d\Omega dE_{\pi^+}Q}\right)_{\text{pair}}\Delta\Omega\Delta E_\pi Q. \tag{3}$$

The first term is the contribution from single π^+ production, and the second from production of both $(\pi^+ +\pi^-)$ and $(\pi^+ +\pi^0)$ pairs, reactions (1) and (2), respectively. Here η^+ is the π^+ counting efficiency; $\Delta\Omega$ and ΔE_π are the solid angle and spread in energy accepted by the spectrometer; $N_k\Delta k$ is the number of photons of mean energy k within the interval Δk corresponding to ΔE_π, where k is the unique photon energy required for photoproduction of a meson of given energy and lab angle; $(d\sigma_{\pi^+}/d\Omega)_{\text{single}}$ is the differential π^+ laboratory cross section; N_H is the number of hydrogen atoms per cm²; $(d^2\sigma_{\pi^+}/d\Omega dE_{\pi^+}Q)_{\text{pair}}$ is the laboratory cross section for π^+ mesons from pair production; Q is the number of effective quanta defined as

$$k_{\max}Q=\int_0^{k_{\max}} kN_k dk. \tag{4}$$

The π^- yield is

$$Y_{\pi^-}=\eta^- N_H\left(\frac{d^2\sigma_{\pi^-}}{d\Omega dE_{\pi^-}Q}\right)_{\text{pair}}\Delta\Omega\Delta E_\pi Q, \tag{5}$$

where η^- is the π^- counting efficiency, and the cross section is that for π^- mesons from charged pair production, reaction (1). Equation (3) can be put into more convenient form for comparison with (5) by expressing the single π^+ cross section as follows:

$$\left(\frac{d^2\sigma_{\pi^+}}{d\Omega dE_{\pi^+}Q}\right)_{\text{single}} =\left(\frac{d\sigma_{\pi^+}}{d\Omega}\right)_{\text{single}}\left(\frac{1}{k}\frac{dk}{dE_\pi}\right)\frac{kN_k(k,k_{\max})}{Q(k_{\max})}. \tag{6}$$

Let R be the ratio of the pair π^+ cross section to the single π^+ cross section defined in (6). Now Eq. (3) can be written as

$$Y_{\pi^+}=\eta^+ N_H\left(\frac{d^2\sigma_{\pi^+}}{d\Omega dE_{\pi^+}Q}\right)_{\text{single}}\Delta\Omega\Delta E_\pi Q(1+R). \tag{7}$$

[11] R. M. Littauer and D. Walker, Phys. Rev. **86**, 838 (1952).
[12] Jakobson, Schulz, and White, Phys. Rev. **91**, 695 (1953).
[13] Recent measurements show the maximum energy might have been as high as 340 Mev instead of the 322 Mev used for calculations of that paper [R. S. White (private communication)].
[14] L. S. Osborne, Massachusetts Institute of Technology Laboratory for Nuclear Science Progress Report, August 31, 1954 (unpublished).
[15] Motz, Crowe, and Friedman, "Photopions from nuclei: minus-plus ratios" (to be published).
[16] Walker, Teasdale, Peterson, and Vette, Phys. Rev. **99**, 210 (1955).
[17] Tollestrup, Keck, and Worlock, Phys. Rev. **99**, 220 (1955).

Then, the observed π^-/π^+ ratio is

$$\frac{Y_{\pi^-}}{Y_{\pi^+}}=\frac{\eta^-}{\eta^+}\left(\frac{d^2\sigma_{\pi^-}}{d\Omega dE_{\pi^-}Q}\right)_{\text{pair}}\Bigg/\left(\frac{d^2\sigma_{\pi^+}}{d\Omega dE_{\pi^+}Q}\right)_{\text{single}}(1+R). \quad (8)$$

The second part of this experiment was to measure the increase in π^+ yield due to pairs to obtain values for R.

B. Apparatus

The experimental arrangement is shown in Fig. 1. The electron beam from the Stanford Mark III linear accelerator[18] is doubly-deflected to eliminate neutron and gamma-ray impurities, and energy-analyzed.[19] The primary electron energy was defined by the magnetic field in the first deflecting magnet. This magnet has been calibrated[20] using the floating-wire technique to an accuracy of better than $\pm 0.5\%$.

The analyzed beam is steered down an evacuated pipe and passes into the experimental area through a 0.005-in. Dural window. A photon beam is produced by placing a radiator before a magnet which deflects out the electrons before they can reach the target. Usually an 0.010-in. Cu radiator was used (0.018 radiation length), which resulted in a photon beam diameter at the target of the order of 1 in. For those runs where no deflection was made, the radiator could be placed in front of the target; this made possible the use of a larger radiator while keeping the beam size small at the target.

Monitoring of the beam intensity was accomplished by putting a secondary electron monitor[21] after the vacuum pipe, as shown in Fig. 1. For primary beam energies from 300 to 600 Mev, the response of the monitor was found to be independent of beam intensity and energy by calibrating it against a Faraday cup integrator.[22] The variation in response measured between the two energy extremes was $(-1.4\pm 1.0)\%$.

The Styrofoam-jacketed liquid hydrogen target used is shown in Fig. 2. The inner cup was 2.5-in. wide, 10.5-in. long, and 6.5-in. high. At equilibrium the loss rate was 2.0 liters/hr. The liquid hydrogen used was obtained from the Department of Chemistry, University of California, Berkeley. The deuterium concentration is the same as in distilled water, about one part in 6000; there were no other known impurities.[23]

Pions were observed in an earlier arrangement, referred to as "Setup I," at 75° with a 30° deflection for the pion beam. Channels were constructed, as shown in Fig. 1, so that meson production could be studied for several pion lab angles. The aperture size used was

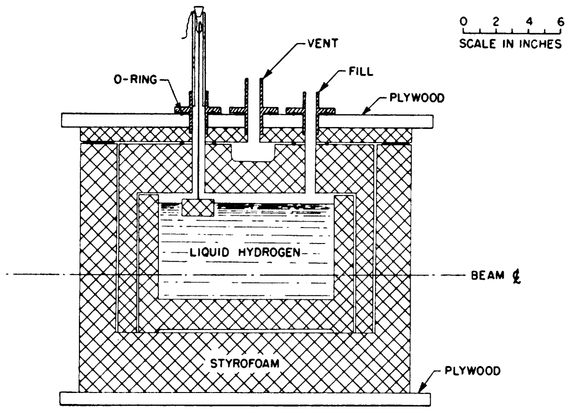

FIG. 2. The Styrofoam-jacketed liquid hydrogen target.

4×4 in., and this could be decreased to any desired size by insertion of steel shims. Pair production was studied in this arrangement, "Setup II," at 60° with a 45° pion beam deflection.

The analyzing magnet has poles 18-in. long, 6-in. wide, with a 3-in. gap. Both the entrance and exit angles were set for normal incidence for all runs. The mean momentum of the pion channel was determined by the floating-wire method. In addition, in Setup II an electron-scattering calibration was made. The location of the lithium elastic-scattering peak gave an energy calibration relative to the primary electron beam energy calibration. The elastic peak gave a value of 15% for the resolution full width at half-maximum $(\Delta p/p)$ for the pion-analyzing system. In Setup II, pions of kinetic energies up to 100 Mev can be deflected. Absorbers were placed before the magnet to measure pions of higher energies.

The plastic scintillator in which the mesons stop and decay is 10 in. long and 7 in. in diameter. The scintillator is viewed with a 5-in. DuMont 6364 photomultiplier tube. The counter response was tested with a radioactive source, and the pulse height found to be uniform within 5% over its length. This technique would be unreliable for detecting end effects which could, for example, give rise to an increase in pulse height for particles stopping near the axis of the cylinder.

An independent and sensitive check of the energy calibration of both the accelerator beam-analyzing system and the pion-detecting system was made by observing single pion production in hydrogen near threshold. The result of this calibration for the 69-Mev magnet setting differs by an amount somewhat greater than the estimated errors. The discrepancy is equal to 8% in pion momentum, or 8 Mev.

At the 69-Mev setting, pions are widely distributed along the axis of the plastic scintillator. Nonuniformity of light collection causing pions which stop nearest the phototube to be counted with higher efficiency would produce the observed effect. The existence of such a condition cannot be excluded by our uniformity meas-

[18] M. Chodorow *et al.*, Rev. Sci. Instr. **26**, 134 (1955).
[19] W. K. H. Panofsky and J. A. McIntyre, Rev. Sci. Instr. **25**, 287 (1954).
[20] The calibration was performed by Professor W. M. Woodward.
[21] H. R. Fechter and G. W. Tautfest, Rev. Sci. Instr. **26**, 229 (1955).
[22] K. L. Brown and G. W. Tautfest, Rev. Sci. Instr. **27**, 696 (1956).
[23] D. N. Lyon (private communication).

urements, and we believe this is the most likely explanation of the discrepancy.

We have adjusted the mean pion energy by $+4\pm4$ Mev, the uncertainty of which is small compared to the resolution width of ±11 Mev. The other settings have been similarly adjusted.

C. Pion Counting

The linear accelerator beam pulse was reduced to 0.1-μsec duration. The positrons or electrons from the π-μ-β decay were detected in delayed coincidence with the beam pulse.

A block diagram of the electronics is shown in Fig. 3. The negative signal from the anode of the phototube was clipped and amplified with a gain of ≈1000 and rise time of 2×10^{-8} sec. The pulses were fed into three paralleled fast discriminators, whose outputs were placed in coincidence with several delayed gating pulses. With the gate timings shown in Fig. 3, the ratio of the counts in Gate 1 to those in Gate 2 should be 2:1 if the signal was entirely due to muon decay. The delayed neutron background has a much longer period (≥20 μsec) and is scaled in Gate 3.

For those cases where high counting rates were measured—the π^+ yield from hydrogen—a fast scaler was used, as shown in the lower part of Fig. 3. Its resolving time ($\sim$30 mμsec) made counting-rate corrections negligible.

Pions were counted as follows: Positive pions stopped in the scintillator and decayed into positive muons. These muons had a range of only 1.5 mm and decayed into positrons with energies ranging up to 52.7 Mev. The signal produced by the positron was then placed in delayed coincidence in the manner indicated. To count negative pions, the analyzing magnet current was reversed. Those negative pions that stopped in the scintillator were captured by the carbon nuclei producing stars, and no delayed pulses resulted. The target-to-counter distance is 12 ft in Setup II, so that

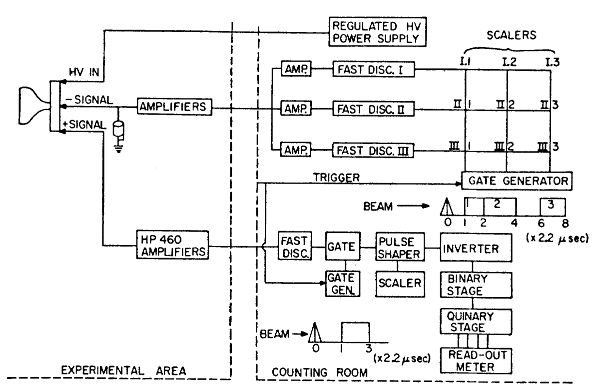

FIG. 3. Block diagram of the electronics. The upper circuit was used for all runs. The faster circuit shown below was used in some of the later runs. The gate timings generally used are shown below the respective gate generators.

a large fraction of the pions decayed in flight: 50% for 34-Mev pions, 38% for 69-Mev pions. Only the π^- decays in flight in which the μ^- reached the scintillator were counted. The π^- counting efficiency was calculated to be $\sim\frac{1}{6}$ that of the π^+ for the 69-Mev setting of the spectrometer. The negative efficiency is also reduced by $\sim$9% since the observed μ^- mean lifetime is less than that for the μ^+ due to absorption by the carbon nuclei in the scintillator.

Figure 4 shows the calculated π^+ energy resolution for 69-Mev pions. The ordinate represents the relative fraction of pions at a given energy which produce positrons in the counter. The small high-energy tail is due to the decay-in-flight contribution. The calculated width of the resolution agrees with that observed in the electron-scattering calibration.

The π^- resolution arises from decays in flight. The calculated spectrum[24] for the 69-Mev setting is shown in Fig. 5. The calculation can be separated into decays

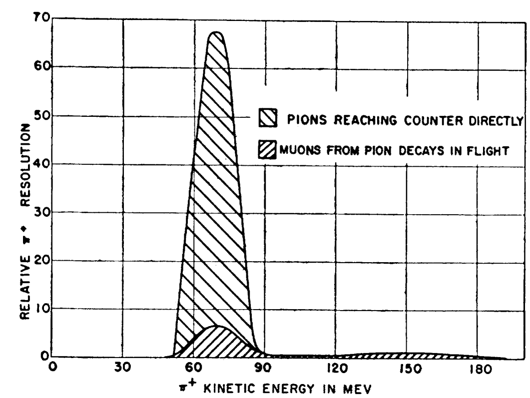

FIG. 4. The relative positive pion resolution for the 69-Mev magnet setting. The high-energy tail arises from the decay-in-flight contribution.

before the magnet, and decays after the magnet: (1) Decay muons at a given lab angle come off with two energies corresponding to decay in the forward and backward directions in the pion's rest frame. Thus high-energy pions can produce muons of low enough energy to be deflected by the magnet into the counter. (2) The counting efficiency is greatest in the region directly in front of the counter. The pion resolution from decays after the magnet will thus be proportional to the direct part of the π^+ resolution.

It can be seen from Figs. 4 and 5 that the largest part of the resolution arising from decays in flight is centered about the same mean energy and has the same width as the direct π^+ spectrum. As far as the observed yields are concerned, the effect of the second peak is almost negligible for positives, while for negatives corrections can be applied if the pion spectrum from the target is known.

[24] Similar calculations were performed by Professor R. L. Walker, California Institute of Technology.

D. Determination of Efficiency

The relative π^-/π^+ counting efficiency η^-/η^+ can be calculated to within $\sim\pm 5\%$. This efficiency has been measured by observing the π^-/π^+ ratio from a carbon target and comparing the results with other measurements.[9,11,25,26] The measured carbon π^-/π^+ ratios are consistent with a constant value for the ratio independent of pion energy, angle, and the upper limit of the bremsstrahlung. This is plausible since carbon contains an equal number of neutrons and protons and the calculated production thresholds are within 3 Mev. For the angles and energies concerned here the π^-/π^+ ratio in deuterium is nearly constant.[27] We adopt a constant value of 1.17 ± 0.05 for the carbon π^-/π^+ ratio, independent of pion energy.

From our measured carbon ratios we can obtain values for η^-/η^+ at the desired mean pion energies by correcting for the effects of the second peak on the observed yields. To determine this correction, the calculated resolution curves were folded into carbon pion spectra measured for 300- and 500-Mev bremsstrahlung.[28] Using the adopted value for the carbon ratio, the calculated ratios obtained agree to within 5% with those observed over the range of pion and bremsstrahlung energies.

The carbon π^-/π^+ ratio was measured during each hydrogen run and used in reducing that specific run.

TABLE I. Hydrogen π^-/π^+ ratios measured in Setup I (75° lab angle). See text for a discussion of the upper and lower limits.

Run	k_{max} (Mev)	T_π (Mev)	Measured ratios Upper limit (%)	Lower limit (%)
1	285	75	$+1.3\ \pm 1.2$	$-1.9\ \pm 2.5$
	520	120	...	$+5.8\ \pm 3.4$
2	520	75	$+0.25\pm 1.3$	-1.54 ± 0.84
	520	45	...	-1.18 ± 2.00
3	475	75	$+2.45\pm 0.12$	-0.10 ± 0.15
	475	45	$+4.60\pm 0.56$	-0.28 ± 0.16
4	555	75	$+3.54\pm 0.45$	$+2.06\pm 0.48$
	555	45	$+5.5\ \pm 1.9$	$+0.36\pm 0.71$
5	565	125	$+1.42\pm 0.38$	-1.66 ± 0.35
	565	75	$+3.46\pm 0.35$	$+0.38\pm 0.45$

[25] Peterson, Gilbert, and White, Phys. Rev. **81**, 1003 (1951); Camac, Corson, Littauer, Shapiro, Silverman, Wilson, and Woodward, Phys. Rev. **82**, 745 (1951); Feld, Frisch, Lebow, Osborne, and Clark, Phys. Rev. **85**, 680 (1952); Palfrey, Luckey, and Wilson, Phys. Rev. **91**, 468 (1953); J. Carothers, Phys. Rev. **92**, 538 (1953); D. Luckey, Phys. Rev. **97**, 469 (1955).

[26] The details of our measurements will be given in reference 15.

[27] Sands, Teasdale, and Walker, Phys. Rev. **95**, 592 (1954).

[28] K. M. Crowe and R. M. Friedman (to be published).

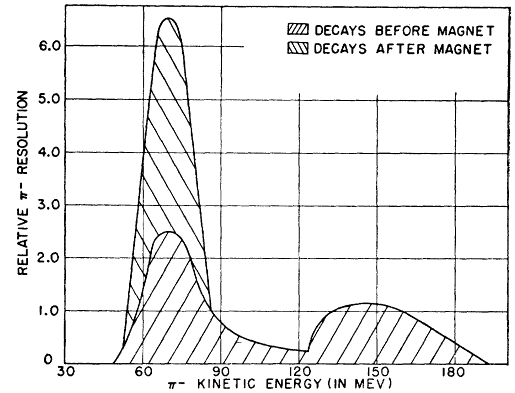

FIG. 5. Negative pion resolution arising from decays in flight at the 69-Mev magnet setting. The ordinate scale can be compared with that in Fig. 4.

III. RESULTS

A. Hydrogen Ratios in Setup I

The hydrogen π^-/π^+ ratios obtained in Setup I (75° lab angle) are listed in Table I.[29] The hydrogen data designated "measured ratios" represent the observed hydrogen ratios divided by the observed carbon ratios.

To obtain the upper limit, we took the difference of the delayed signals with liquid and gaseous hydrogen in the target and with the analyzing magnet set to count negatives. This should eliminate charged and neutral delayed backgrounds originating in the target walls, shielding, etc. However, any neutral background coming from the liquid hydrogen itself would still be included in this difference.

The lower limit is the difference in liquid hydrogen signals with the analyzing magnet field negative and with the field off. It can be seen from Table I that most of the lower limits so obtained gave either negative or null results. This situation was unsatisfactory in establishing the existence of a π^- signal. It was found that these magnet-off counts were mainly due to positive pions which scattered through the 30° deflection angle from parts of the magnet. These positive pions were probably deflected out and prevented from reaching the counter when the magnet field was set for counting negatives. In retrospect, it appears that the upper limits in Table I are valid measurements of the effect.

Increasing the deflection of the analyzed pion beam from 30° to 45° reduced the magnet-off background by a factor of the order of ten. All succeeding data were taken in Setup II with this larger deflection.

B. Hydrogen Ratios in Setup II

The hydrogen ratios obtained in Setup II (60° lab angle) are listed in Table II. To obtain the corrected

[29] Some of these results were reported by W. K. H. Panofsky, *Proceedings of the Fifth Annual Rochester Conference* (Interscience Publishers, Inc., New York, 1955), pp. 50–51.

TABLE II. Pair cross sections from yield of negative mesons in Setup II (60° lab angle). Results for Runs 6–11(a) are based on difference in negative signals with liquid and gaseous hydrogen in the target and thus represent upper limits. Neutral hydrogen background measured during Run 11 has been subtracted, giving the results shown as 11(b) which are lower limits.

Run	k_{max} (Mev)	T_π (Mev)	Measured ratio %	Corrected ratio %	Cross section (10^{-33} cm²/ sterad-Mev-Q)
6	560	56	3.9±1.0	6.3±1.4	4.7±1.1
7	570	76	3.3±0.3	5.4±0.7	4.5±0.6
8	570	76	3.3±0.3	5.4±0.7	4.5±0.6
	335	76	0.5±0.2	0.6±0.3	0.5±0.3
9	580	76	4.8±0.8	8.0±1.4	6.7±1.2
10	400	76	1.0±0.2	1.4±0.2	1.2±0.2
	495	76	1.0±0.2	1.4±0.2	1.2±0.2
	575	76	2.9±0.2	4.5±0.5	3.8±0.4
	575	56	6.0±0.5	9.5±1.1	7.1±0.8
	575	41	7.2±0.6	10.6±1.3	6.9±1.0
	575	28	12.5±0.9	18.2±2.2	9.9±1.4
11(a)	400	76	0.8±0.2	1.1±0.3	0.9±0.3
	500	76	1.0±0.2	1.5±0.3	1.3±0.3
	550	76	2.8±0.3	4.6±0.6	3.9±0.5
	595	76	4.4±0.3	7.2±0.8	6.0±0.7
	595	115	0.6±0.6	1.0±1.0	0.9±0.9
	595	41	9.3±0.5	15.4±1.7	10.0±1.3
	595	19	12.8±2.0	18.7±4.1	8.3±2.0
11(b)	400	76	0.2±0.3	0.3±0.4	0.3±0.3
	500	76	0.5±0.7	0.8±1.1	0.7±0.9
	550	76	2.3±0.3	3.7±0.5	3.1±0.5
	595	76	3.9±0.3	6.3±0.7	5.3±0.6
	595	115	−0.3±0.7	−0.5±1.1	−0.5±1.0
	595	41	8.7±0.6	13.5±1.6	8.8±1.1
	595	19	9.4±2.2	13.8±3.2	6.1±1.5

ratios, the measured ratios were multiplied by: (1) The adopted value for the carbon ratio, 1.17±0.05. (2) The correction $(1+R)$ for the increase in π^+ yield due to pairs based on measurements discussed in Sec. IVC. For the 76-Mev pion ratios, this correction varied from 1.08±0.03 to 1.17±0.03 for 500- and 600-Mev bremsstrahlung. To obtain the correction at the other mean pion energies measured, the 76-Mev results were scaled according to the theoretical calculations of Cutkosky and Zachariasen[30] discussed in Sec. IV. The correction decreases as the pion energy decreases: for 19-Mev pions and 600-Mev bremsstrahlung, a factor of 1.04 ±0.03 was applied. (3) The correction to the observed hydrogen ratios arising from the high-energy tail on the resolution functions (see Secs. IIC and IID). This depends principally on the carbon and hydrogen π^- spectra. However, since the hydrogen spectrum cuts off at intermediate pion energies (Fig. 9), the correction is primarily due to the carbon except at the lowest pion energy. The calculated correction factor varies from 1.20±0.04 for 76-Mev pions to 1.14±0.06 for 19-Mev pions.

The absolute values for the pair cross section were obtained as indicated in Eqs. (6) and (8), using the average π^+ cross sections reported by Walker *et al.*[16] and Tollestrup *et al.*[17] Our measurements indicate a small (<10%) contribution due to pairs in the results of Walker *et al.*[16] who used "the magnet method" with 500-Mev bremsstrahlung; Tollestrup *et al.*[17] obtained their results with the "counter-telescope method." The two sets of results disagree internally, probably in excess of the pair contributions. The magnet data were corrected for this effect by subtracting the negative signal. Since the energy spectra of pion pair fragments are not equivalent, this procedure was only partly adequate. The counter-telescope results disregarded pairs entirely. However, these comments apply only to the low-energy pion results, and these are not heavily weighted in arriving at the average single π^+ cross sections used here. At higher pion energies, the pair contribution is certainly small in comparison with the errors.[31]

For all except Run 11 in Table II, no neutral background from the liquid hydrogen was established within the statistics; the results are based on the difference in liquid-gas negative signals and represent upper limits. For Run 11, sufficient statistics were obtained to show a small neutral background from the liquid; this background has been subtracted to give the lower limits shown as 11(b). Since the two limits overlap in all cases, the exact nature of the background is of only secondary interest. There were insufficient data taken on this background to allow a meaningful estimate of the decay period. Part of the background could still be due to π^+ scattering, for example, as discussed in Sec. IIIA.

C. Increase in π^+ Yield

The excitation of 75±10 Mev positive pions from hydrogen in Setup I is shown in Fig. 6. A curve has been drawn through the experimental points to indicate

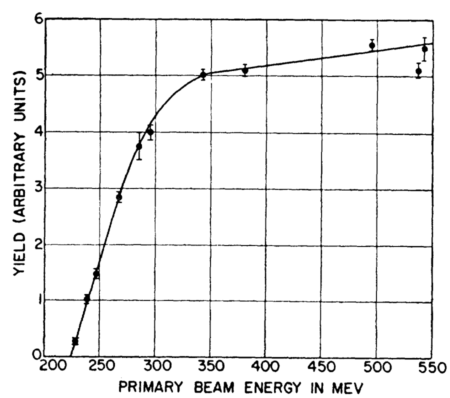

FIG. 6. Excitation function for 75±10 Mev positive pions from liquid hydrogen in Setup I. The curve shown has been drawn through the experimental points to emphasize the positive slope at high bremsstrahlung energies indicating pair production.

[30] R. E. Cutkosky and F. Zachariasen, Phys. Rev. **103**, 1108 (1956).

[31] We wish to thank Dr. Bacher, Dr. Peterson, Dr. Sands, and Dr. Tollestrup for clarifying these points.

the large slope at the high-energy end. We believe that most of this increase is attributable to pair production, since varying the upper limit of the bremsstrahlung spectrum beyond 400 Mev produces only a small change in the photon distribution in the energy range giving rise to single π^+ production.

This effect was studied in more detail in Setup II in order to obtain values for R. Two excitation measurements for 76-Mev positive pions from hydrogen were made, one using liquid hydrogen and the other a CH_2-C subtraction, with the electron beam deflected out; the results are shown in Fig. 7. The results of these measurements and of several other runs give values for R listed in Table III, based on the increase in π^+ yield as the bremsstrahlung is varied from 400 Mev (the kinematical threshold is 410 Mev). Calculated corrections for the variation in intensity of bremsstrahlung involved in single π^+ production have been applied. Using curves for copper obtained from the Bethe-Heitler formula,[32] the fractional change in the number of photons was calculated to be 4.0, 6.2, 7.3, and 8.5%, as the upper limit of the bremsstrahlung was varied from 400 to 500, 550, 575, and 600 Mev, respectively. Absolute values for the pair cross section were obtained using Eq. (8).

Table III includes several runs made with different radiator thicknesses and with the electrons in and out in order to check on multiple scattering of the electrons affecting the size of the photon beam, as well as the effect of direct electron production of pions.[33] For these runs, thick radiators were used so that the electrons contributed at most 10% to the observed yield. Panofsky estimates[34] that the electron effect would contribute a variation in the single pion yield from 400 to 600 Mev of only 1–2%.

Table III. Pair-production cross sections from increase in yield of 76±4 Mev positive pions in Setup II (60° lab angle). The column X_T is the total number of radiation lengths in the path of the electron beam and includes in addition to the radiator, air, aluminum windows, and monitor foils; if the electrons were not deflected out, the Styrofoam and liquid hydrogen contributions are also added in plus the "equivalent radiation length" for electrons, ~0.020 radiation length. R is the ratio of the cross sections for positive pions from pair and single production. The errors shown for the runs are counting statistics only (standard deviations); the errors of the Summary are computed by external consistency.

Run	Radiator	X_T (radiation lengths)	Electrons	k_{max} (Mev)	R (%)
9	0.020-in. Ta	0.177	in	575	20.7±3.5
				500	7.1±4.8
11	0.021-in. W	0.209	in	600	19.6±3.4
				550	19.4±2.4
				500	4.9±2.0
12	0.010-in. Ta plus 0.010-in. Cu	0.090	out	600	20.0±0.9
				550	14.7±0.9
				500	11.4±0.9
	0.010-in. Cu	0.024	out	600	12.2±1.3
				500	8.0±1.3
13	0.021-in. W	0.212	in	600	19.5±2.5
				500	11.8±1.8
	0.010-in. Cu	0.028	out	600	17.0±2.3

Summary

k_{max} (Mev)	R (%)	Cross section (10^{-33} cm²/sterad-Mev-Q)
600	17.7±1.7	14.9±1.4
575	20.7±3.5	17.4±3.0
550	15.2±1.4	12.8±1.2
500	9.9±1.2	8.3±1.0

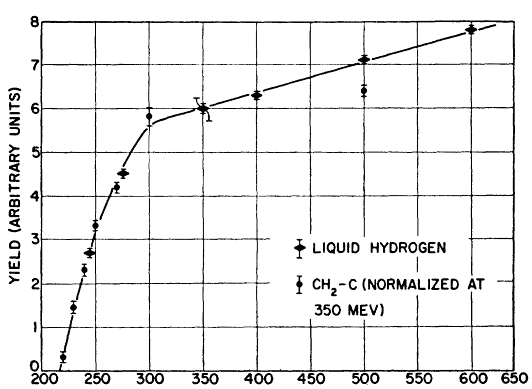

Fig. 7. Excitation function for 76±4 Mev positive pions from hydrogen in Setup II. The large slope in the experimental curve at high bremsstrahlung energies is due principally to pair production.

The contribution to the increased yield of single positive pions due to the decay-in-flight tail of the resolution function can be calculated and is negligible. Data shown in Fig. 6 made with Setup I (30° deflection) should be ten times more sensitive to the scattering of pions in the magnet than results in Fig. 7 made with Setup II (45° deflection). The increase in pions is, if anything, slightly smaller, giving a cross section of 3.3 ± 1.0 compared to $8.3\pm1.0\times10^{-33}$ cm²/sterad-Mev-Q for 500-Mev bremsstrahlung. If there is a high-energy tail to the π^+ resolution due to π^+ scattering, it can be safely ignored in the Setup-II data.

In summary, the results given in Table III should be considered an upper limit to the cross section for positive pion production from pairs, and we believe that we have corrected accurately for all the important confusable effects. Unfortunately, from our measurements alone, we cannot eliminate entirely the possible spurious effect due to multiple scattering of the primary beam or a small but long resolution tail resulting from pion scattering off the magnet. Estimates of these effects, however, indicate that they are probably

[32] H. A. Bethe and J. Ashkin, in *Experimental Nuclear Physics*, edited by E. Segrè (John Wiley and Sons, Inc., New York, 1953), Vol. 1, pp. 259 ff.

[33] Panofsky, Woodward, and Yodh, Phys. Rev. **102**, 1392 (1956); G. B. Yodh and W. K. H. Panofsky, Phys. Rev. **105**, 731 (1957).

[34] W. K. H. Panofsky (private communication); see also reference 33.

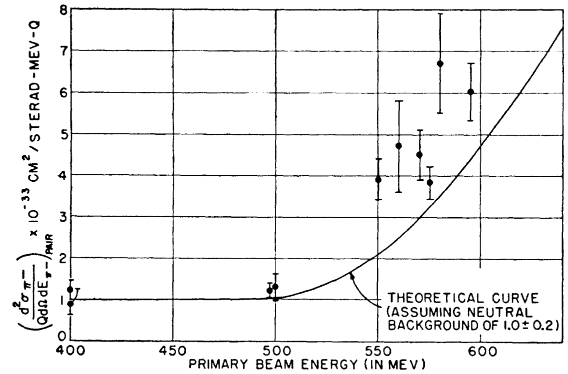

FIG. 8. Pion-pair excitation function based on the yield of 76±4 Mev negative pions in Setup II (60° lab angle). The data shown were obtained from Table II using the results of Runs 6–11(a) which are interpreted as upper limits. The graph indicates a neutral background from the hydrogen of the order of 1.0±0.2 in plotted units which should be subtracted. The theoretical curve has been drawn in accordingly.

negligible for the 500-Mev point and certainly so for the higher-energy points.

IV. DISCUSSION

Figure 8 shows the measured excitation function for 76-Mev negative pions at 60° in the lab using the data from Table II. The results of Run 11(a) have been plotted with these for the other runs so that no subtraction for neutral hydrogen background is included in the data shown. The numbers are thus interpreted as upper limits on the pair cross section. Note that the small values for the cross section obtained at threshold (400 Mev) would be reduced to null results if a subtraction of 1.0±0.2 in the plotted units were applied. The neutral hydrogen background measured in Run 11 was 0.7±0.3 units, and is clearly consistent with the value estimated from the graph. To compare with the data, the theoretical curves discussed below have accordingly been drawn starting from this background level.

The negative energy spectra from hydrogen for 595- and 575-Mev bremsstrahlung are compared with the theoretical curve for 600-Mev bremsstrahlung in Fig. 9. Here the neutral hydrogen background has been subtracted out. A peaking of the pair cross section at low π^- energies is shown by these results. Within the statistics, the high-energy π^- cross section is observed to be very small compared to the value at the peak.

The results can be compared to calculations of Cutkosky and Zachariasen[30] who have applied the static cut-off theory of Chew and Low[35] to photoproduction of a pair of pions. They assume that bombarding energies are low enough so that one of the pions is produced in an S state and the other in a P state. They obtain expressions for the process in terms of the P-wave scattering phase shifts, assuming the S-wave pion-nucleon interaction and the meson-meson interaction can be neglected. The cross section for producing both in S states has been calculated by Bincer[36] to be very small compared with these results. The processes in which both pions are in P states are presumably not yet important at these photon energies.

Cutkosky and Zachariasen consider the three possible reactions: $\gamma+p\rightarrow p+\pi^++\pi^-$; $\gamma+p\rightarrow p+\pi^0+\pi^+$; and $\gamma+p\rightarrow p+\pi^-+\pi^+$. In each case the first pion is in the P state, the second in the S state. For low energies the partial cross sections for these processes will be in the ratio 9:2:1, in complete analogy to the pion scattering theory. They have calculated cross sections in a center-of-mass system in which recoil has been neglected. These cross sections were then folded into a $1/k$ bremsstrahlung spectrum and the resulting quantity compared with our preliminary data.[7,37] The maximum energy of the bremsstrahlung was then related to the laboratory bremsstrahlung by making the usual c.m. transformation. Unfortunately, this cross section must be transformed into the laboratory cross section before a comparison can be made with our data. Although no theory treats the recoil adequately, we can approximate the purely kinematic transformation effects, which are themselves not at all negligible.

We have recalculated the theoretical curves shown in Figs. 8, 9, and 10 by transforming the Cutkowsky-Zachariasen cross sections back into the laboratory frame using the c.m. transformation and integrating over the $1/k$ photon spectrum. It should be emphasized that in the Cutkosky-Zachariasen calculations[30] the proton has been treated as being infinitely heavy and its recoil neglected. This neglect makes the reaction

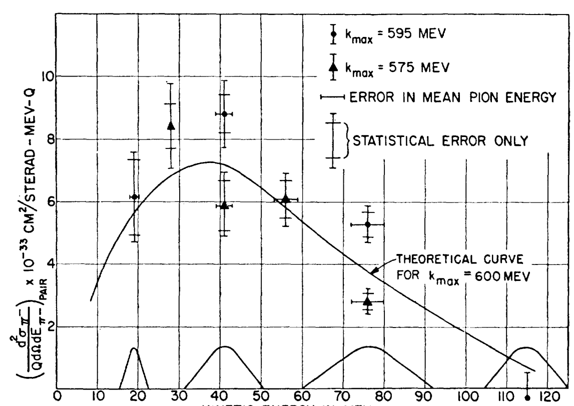

FIG. 9. The spectrum of negative pions from hydrogen in Setup II (60° lab angle). The results of Runs 10 and 11(b) from Table II have been plotted. In the former case, the neutral hydrogen background has been subtracted out using the value indicated in Fig. 8. The curves at the bottom of the graph indicate the width of the energy acceptance spectrum for the spectrometer setting.

[35] G. F. Chew and F. Low, Phys. Rev. **101**, 1570, 1579 (1956).

[36] A. M. Bincer, Massachusetts Institute of Technology doctoral dissertation (unpublished); see also A. Petermann, Phys. Rev. **103**, 1053 (1956).

[37] See Fig. 2, reference 30.

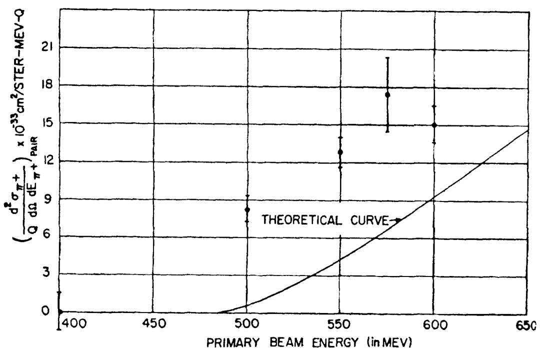

FIG. 10. Pion-pair excitation function based on the yield of 76±4 Mev positive pions in Setup II (60° lab angle). The average values from Table III are plotted. The standard deviations shown are calculated by external consistency. The theoretical curve includes the contribution from $(\pi^+ + \pi^0)$ pairs.

threshold used in their calculations higher than that obtained from a proper kinematical treatment including the proton recoil.

Figure 10 shows the excitation function for 76-Mev positive pions from pairs at 60° in the lab from the data of Table III. The cross section for 76-Mev positive pions from pair production is approximately three times as large as that for negative pions of the same energy. The pair π^+ energy spectrum has not been measured since for low energies the pair cross section decreases relative to the single-production cross section, making detection by the subtraction technique much more difficult.

A shift of the theoretical excitation curves of the order of 50 Mev toward lower bremsstrahlung energies would bring the theory into agreement with the data. This is an indication of the possible effect on the theory of neglecting recoil and is not surprising. The theory also ignores any pion-pion interaction which might also affect the absolute scale for the curves. Unfortunately, the approximate nature of the theory combined with our limited accuracy precludes any experimental conclusions concerning the importance of such an interaction.

In conclusion, the results agree with the theory of Cutkosky and Zachariasen in regard to (1) the slow initial rise in the excitation function, indicating that at least one pion is emitted in a P state; (2) the general shape of the π^- energy spectrum; and (3) the favored emission of positive pions at high energies. This suggests that there is a strong resonance between the π^+ in a P state and the nucleon.

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