Table II. Estimated sources of error in absolute cross section.

Effect	Size of correction %	Error %
Glycerol-tristearate weight and fitting of observed experimental recoil-proton curves	...	1.5
Glycerol-tristearate thickness correction	5	0.5
Weight of U^{235}	...	1.0
Uranium foil thickness correction	1.8	1.1
Room background of neutrons	1.3	0.2
Counter scattering	1.3	0.4
Foil scattering	0.5	0.3
Center-of-mass motion of fission fragments	0.6	0.1
Target scattering	0.2	0.1
Hydrogen contamination of counter	4.0	0.5
Hydrogen content of glycerol-tristearate	...	0.5
Extrapolation of recoil-proton pulse-height distribution	...	2.6
Root-mean-square value	...	3.5

extrapolation of the proton pulse heights to zero pulse height. The counter was tested under various conditions of gas pressure, foil thickness, and neutron energy, and excellent agreement was obtained with theoretical pulse-height distributions. Although there was no indication that there was any malfunction of the equipment, it was thought that in the assignment of errors, a generous allowance should be made for uncertainty in this extrapolation. The results are plotted in Fig. 3. These measurements are in agreement with recent measurements made at the Atomic Energy Research Establishment at Harwell, England.[7]

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Electron Scattering from Neighboring Nuclei*

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A new method of measuring small variations of the charge distributions of neighboring nuclei, such as isotopes and isotones, has been developed. The method is based on a determination of the ratio of electron scattering cross sections near the diffraction dips. Experimental results are given for the combinations Ni^{58}, Ni^{60} and Fe^{56}, Ni^{58}. Sample theoretical interpretations are presented.

I. INTRODUCTION

ELASTIC scattering of electrons from atomic nuclei, in the energy region between 100 and 200 Mev, has proved to be a sensitive method of exploring nuclear charge distributions.[1] For medium and heavy nuclei, two shape parameters can be determined accurately. These parameters characterize the radial extension and surface thickness of the charge distribution.[2] For a charge distribution which is uniform in a central region, and which drops off to zero smoothly at the edge, the following two parameters have been chosen: c, the distance from the center of the nucleus to the point at which the charge density has dropped to one half of its central value, and t, the distance in which the charge density at the edge of the nucleus drops from 90 to 10% of the central value. These parameters are only slightly dependent on the particular analytical form of the two-parameter charge distribution.[2]

An attempt has now been made to detect possible small differences in the charge distributions of neighboring nuclei, i.e., to determine small variations in the parameters c and t as the numbers of protons or neutrons in neighboring nuclei change by small amounts. This has been done by measuring ratios of cross sections. In any experiment of this kind, ratios can always be measured more accurately than individual cross sections. It is the purpose of this paper to describe the central idea of this method, some relevant calculations, and experiments showing that the method is feasible. In addition, certain conclusions may be drawn about nickel and its neighbors.

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[1] R. Hofstadter, Revs. Modern Phys. **28**, 214 (1956). This article is a summary where references to the literature will be found.

[2] See especially Hahn, Ravenhall, and Hofstadter, Phys. Rev. **101**, 1131 (1956).

II. METHOD AND CALCULATIONS

In Fig. 1 are shown three theoretical angular distributions for the nickel nucleus at an energy of 183 Mev. These curves were calculated by an exact partial wave analysis of the scattering.[3] Each corresponds to a different choice of the pair of parameters c and t. The corresponding charge distributions are fairly close to the actual shape of the charge distribution of nickel. It may be noticed that all the curves tend to unite at small angles. This is to be expected since the forward scattering tends to depend only on the total charge (Z) and not on the shape of the nucleus. This corresponds to the fact that at very small angles the scattering is associated with impact parameters so large that the size of the nucleus is unimportant. However, in the neighborhood of the first diffraction dip (60° to 90° in Fig. 1), pronounced differences occur which depend strongly on the assumed shape of the nucleus. Such changes include a shift of the dip to smaller angles when the nuclear dimensions are increased (c increases) and a vertical shift of the level of the valley within the dip. In the case of an increase in c, the shift is upwards for the larger value of c. In the case of t, the shift is downward for the larger values of t. Thus, there are characteristic types of changes in the cross sections for corresponding changes of the two parameters. If the *ratio* R of cross sections is measured in the neighborhood of the dip, it may be expected that small variations of the parameters will result in large changes in the ratios of cross sections. In other words, the ratio is a sensitive measure of small variations of the charge distribution parameters. In the region of Ni, a 1% difference in the radial parameter of two isotopes, and no difference in the surface thickness parameter, produces approximately a 20% variation in the cross-section ratio in the angular region of the dip.

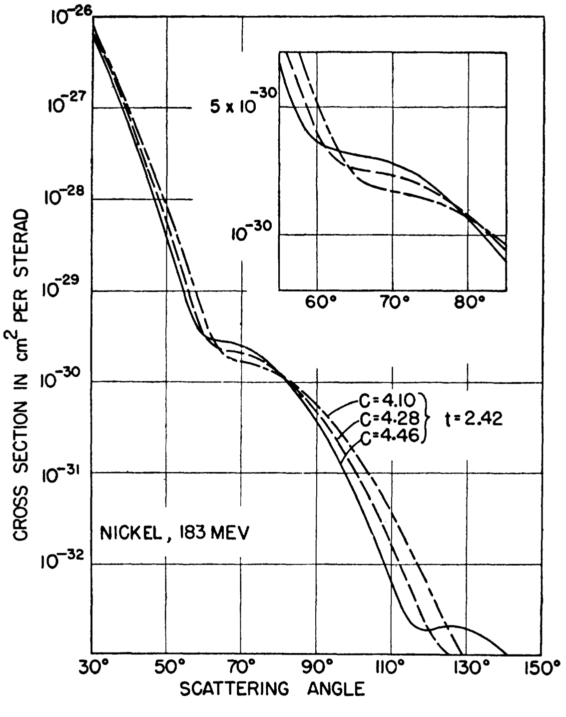

FIG. 1. Theoretical cross sections for the scattering of 183-Mev electrons by nickel ($Z=28$), obtained using the Fermi shape for the nuclear charge distribution. The three curves correspond to shapes with the same surface thickness, but varying radii; the numerical values of the parameters are given in units of 10^{-13} cm. The curves are intended to illustrate the fact that, because of the diffraction structure, small changes in the charge distribution can lead to pronounced variations in the cross-section ratio.

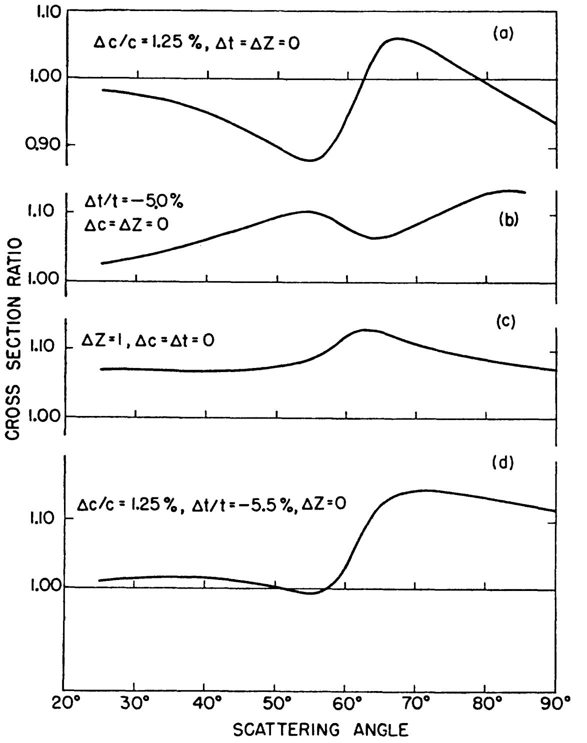

FIG. 2. Theoretical cross-section ratios based on the Fermi charge distribution whose dimensions are given in the text. The curves (a), (b), and (c) show the effect of changing a single parameter, and correspond to changes in c, t, and Z, respectively. The curve (d) illustrates the effect of a combined change in c and t.

Further examples of this sensitivity of the ratio to small variations in the parameters are shown in Fig. 2. The basic charge distribution was selected to be

$$\rho(r)=\rho_0/\{\exp[(r-c)/z]+1\}, \qquad (1)$$

the Fermi distribution; by fitting to the experimental results for Ni^{58}, the parameters were determined to be $c=4.28\times10^{-13}$ cm, $z=0.56\times10^{-13}$ cm$[t=4.40z=2.49\times10^{-13}$ cm$]$. The cross section in this case is close to the middle curve of Fig. 1. The ratio of cross sections calculated from charge distributions with radii differing by 1.25%, but with equal surface thicknesses, is shown in Fig. 2(a). Figure 2(b) illustrates the effect of a change in surface thickness of 5%, and no change in radius. The ratio obtained for nuclei with identical radii and surface thicknesses, but with Z differing by unity, is shown in Fig. 2(c). For rough estimates, the effect of a

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[3] Yennie, Ravenhall, and Wilson, Phys. Rev. **95**, 500 (1954).

combined change in all three of the parameters c, t, and Z, if small, can be approximated by compounding these three curves in the appropriate proportions. Figure 2(d) illustrates the effect of a combined change in c and t. The necessity of using the exact partial wave analysis in calculating cross sections is clear from these curves, since the first Born approximation would yield a constant ratio in the case of Fig. 2(c). This approximation would also give ratios going to zero and infinity at the angles where the ratios of Figs. 2(a) and 2(b) have minima and maxima.

From these examples it may be seen that certain characteristic types of behavior of the ratio R can lead to a determination of the type of change occurring in the parameters c, t as one passes from isotope to isotope. Moreover, calculations with charge distributions of trapezoidal shape[2] show that the characteristic behavior of R does not depend vitally on the form of the charge distribution so long as the latter has the general features of the model described by Eq. (1) or of equivalent models given in reference 2.

It is to be noticed that the shape of the ratio curve is important in determining the actual variation of parameters c and t, but no less important is the absolute value of the ratio. It is possible to have a variety of values of Δt for which the ratios have roughly the same character over the angular range 40° to 80°. However, the behavior of R for small scattering angles will eliminate this uncertainty. Ratios for larger angles will show up any inadequacies in this simple picture.

III. EXPERIMENTAL RESULTS AND CONCLUSIONS

We have studied experimentally the isotopes $_{28}\mathrm{Ni}_{30}{}^{58}$ and $_{28}\mathrm{Ni}_{32}{}^{60}$ and the isotones $_{26}\mathrm{Fe}_{30}{}^{56}$ and $_{28}\mathrm{Ni}_{30}{}^{58}$, differing by two neutrons and two protons, respectively. Nickel has the magic proton number 28. The targets, which have been supplied by Oak Ridge, have an isotopic purity higher than 99%. Differential cross-section ratios $\sigma(\mathrm{Ni}^{60})/\sigma(\mathrm{Ni}^{58})$ and $\sigma(\mathrm{Ni}^{58})/\sigma(\mathrm{Fe}^{56})$ *versus* scattering angle have been measured at 183 Mev. As a measure for the cross-section ratio, we have taken the relative peak heights and also the areas of the elastic peaks. In measuring the elastic peaks, the targets have been switched alternatively at each magnet current setting of the analyzing magnet. Special care has been taken to separate elastic scattering from inelastic scattering (excitation of nuclear levels), since for the determination of charge distributions we were interested only in elastic scattering.

The experimental results obtained at 183 Mev for Ni^{58}-Ni^{60}, and Fe^{56}-Ni^{58} are shown in Fig. 3. The cross-section ratios are plotted in arbitrary units *versus* scattering angle. The errors attached to the experimental points are due to counting statistics only.

No accurate absolute cross-section ratios have been measured up to now. It is therefore not possible at this stage to distinguish between the possibilities illustrated in Figs. 2(a) and 2(d), for example, so that a complete interpretation of the experiment is not possible at present. The following preliminary statements can be made, however:

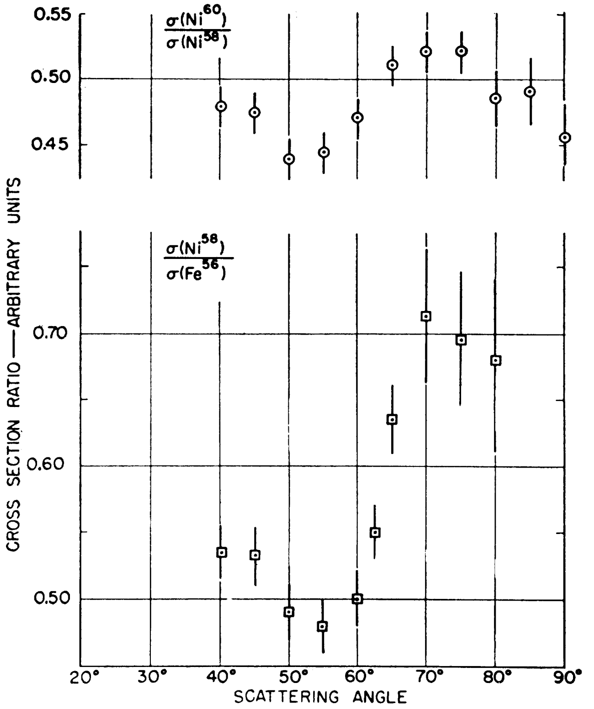

FIG. 3. Experimental cross-section ratios for the pairs of nuclei Ni^{58}, Ni^{60} and Fe^{56}, Ni^{58}, in arbitrary units.

(1) Since the measured cross-section ratios Ni-Ni and Fe-Ni are not constant with scattering angle, the charge distributions of the two members of the pairs of compared nuclei must be different. This means in the Ni-Ni case that the two extra neutrons in Ni^{60} affect the closed-shell proton structure of Ni.

(2) The observed variation of the cross-section ratio for Ni^{60}-Ni^{58} and Ni^{58}-Fe^{56} with scattering angle can be satisfactorily interpreted by a change of the radial charge distribution parameter by approximately a factor of $(A+2)/A^{\frac{1}{3}} \cong 1.01$ (A = mass number of the lighter member of a pair), assuming a constant surface thickness t. A change of 5% or 10% in t, in addition to the change in c, is, however, not excluded. A change in t alone, with constant c, yields no good fit to the experiment. Knowledge of the absolute value of the ratio in the forward direction will resolve this uncertainty.

(3) The experiments presently described represent a new and sensitive method for studying small differences in the proton distributions of neighboring atomic nuclei. It might be of special interest to study systematically a larger number of neighboring nuclei near the magic numbers.

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