Polarization of γ Rays in Ta^{181}†

P. H. STELSON AND F. K. MCGOWAN
Oak Ridge National Laboratory, Oak Ridge, Tennessee
(Received November 13, 1956)

Directional correlation measurements of the γ rays following the decay of Hf^{181} to excited states in Ta^{181} have indicated two possible spin assignments for the 482- and 615-kev states. Calculation of the expected polarization of the 482-kev γ rays in the 133–482 kev decay showed that this property was quite different for the two spin assignments. For the spin assignment 482, $9/2^+$, and 615, $5/2^+$ the calculated value of p is 1.10 where $p\equiv W(90°, 90°)/W(90°, 0°)$ in the notation of Biedenharn and Rose. For the spin assignment 482, $5/2^+$, and 615, $1/2^+$ the value of p is 0.67. The experimental value for liquid HfF_4 source is (0.71 ± 0.05). To check on possible systematic errors, measurements were also made with a dry HfF_4 source for which one expects an attenuated value for p of 0.91. The experimental value is (0.88 ± 0.05). We conclude that the correct spin assignments are 482, $5/2^+$, and 615, $1/2^+$. Reduced γ-ray transition probabilities are obtained. The collective model of the nucleus is used to interpret the low-lying states of Ta^{181}.

INTRODUCTION

EXCITED states at 136.25, 482, 615, and 618.9 kev in Ta^{181} are observed following the β^- decay of Hf^{181}.[1] (See Fig. 1.) Directional angular correlation measurements of the 133–482 kev γ-ray cascade have indicated two possible spin assignments.[2–4] The spin assignment $5/2(E2)9/2(E2+M1)7/2$ for the 133 to 482-kev cascade[2] hinged on the assumption that the angular distribution is strongly perturbed by a time-dependent interaction which could be present for a source in the liquid state. This assignment was also compatible with the measured K-shell internal conversion coefficient[5] of the 482-kev transition. The main objection to this assignment was that one would expect the 615-kev transition to be $M1$ or $E2$ radiation.

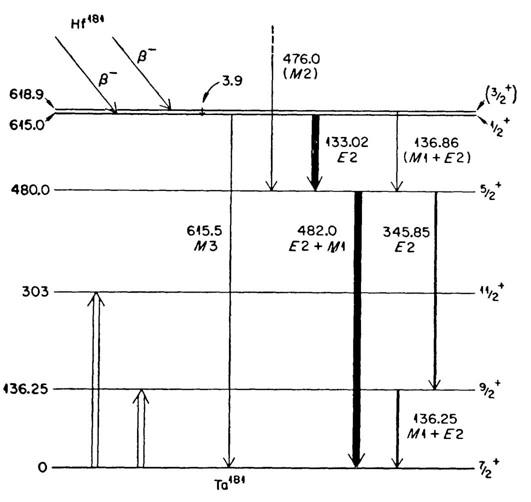

FIG. 1. Energy level diagram of low-lying levels in Ta^{181}.

† Work performed under the auspices of the U. S. Atomic Energy Commission.
[1] F. Boehm and P. Marmier, Phys. Rev. **103**, 342 (1956).
[2] F. K. McGowan, Phys. Rev. **93**, 471 (1954).
[3] H. Paul and R. M. Steffen, Phys. Rev. **98**, 231 (1955).
[4] Heer, Ruetschi, Gimmi, and Kundig, Helv. Phys. Acta **28**, 336A (1955).
[5] F. K. McGowan, Phys. Rev. **93**, 163 (1954).

For an $E2$ transition between states of the independent-particle model, one calculates that the 615-kev transition is 2000 times more probable than the 133-kev transition, but instead it is observed to be about 65 times less probable. The other assignment[3,4] $1/2(E2)5/2(E2+M1)7/2$ for the 133–482-kev cas ade removed the objection concerning the 615-kev transition since the transition would be $M3$ and would not compete favorably with the 133-kev transition. This assignment required only a weak perturbation, if any at all, from the time-dependent interaction for the 482-kev state. However, the ratio $E2/M1$ deduced from this assignment was not in agreement with the measured K shell internal conversion coefficient for the 482-kev transition.

Biedenharn and Rose[6] have expressed in a convenient form the polarization-direction correlation with polarization of the mixed radiation being measured. The correlation function has for a γ-γ cascade the form

$$W(\theta,\phi)=W_{\text{I}}+\delta^2W_{\text{II}}+2\delta W_{\text{III}},$$

where δ is $(E2/M1)^{\frac{1}{2}}$. W_{I}, W_{II}, and W_{III} are the polarization-direction correlation functions for pure 2^{L_1} pole-pure 2^{L_2} pole, pure 2^{L_1} pole-pure 2^{L_2+1} pole, and the interference term, respectively. A calculation of the expected polarization-direction correlation with polarization measurement of the mixed radiation in the 482-kev transition showed that this correlation was quite different for the two spin assignments. These

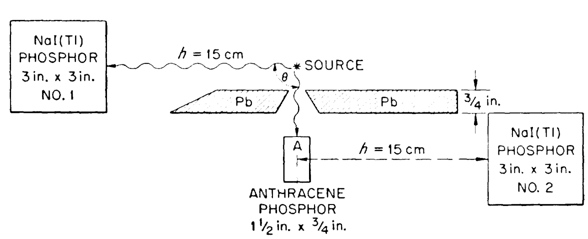

FIG. 2. Diagram of γ-ray polarimeter.

[6] L. C. Biedenharn and M. E. Rose, Revs. Modern Phys. **25**, 729 (1953).

results are tabulated in Table I. The ratio p of the polarization intensities is $W(90°, \phi=90°)/W(90°, \phi=0)$ in the notation of Biedenharn and Rose. The G_2 and G_4 are the attenuation coefficients which represent the effect of the perturbing interaction in the intermediate state of the nucleus. Here ϕ is the angle between the direction of polarization and the normal to the plane defined by the directions of propagation of the two γ rays in cascade.

APPARATUS

A polarimeter based on the Compton scattering mechanism has been constructed and its effectiveness has been determined by measurement of the known polarization of γ rays following Coulomb excitation of 2+ levels in even-even nuclei.[7] A cross section through the polarimeter in the plane defined by the two γ rays is shown in Fig. 2. The anthracene scatterer and the 3 in. ×3 in. NaI crystal which are connected to photomultiplier tubes constitute the polarization sensitive device. The NaI scintillation spectrometer detects the radiation scattered through a mean angle of 90° and

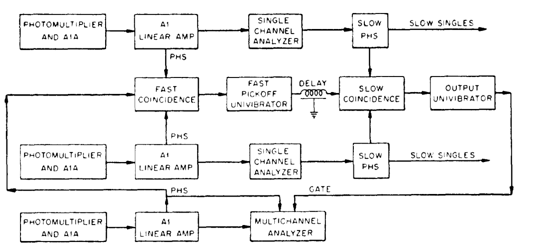

FIG. 3. Block diagram of the fast-slow coincidence system.

the anthracene scintillation spectrometer detects the Compton recoil electron. The detector of the scattered radiation rotates about an axis passing through the scatterer and the source. One measures $N_{\parallel}/N_{\perp}$, the ratio of the triple coincidence rate for the position when the detector of the Compton scattered photons is in the plane of the two γ rays to the triple coincidence rate for the perpendicular position. This ratio $N_{\parallel}/N_{\perp}$ and $p(\theta)$ are connected through the relation[8]

$$N_{\parallel}/N_{\perp}=(p+R)/(pR+1),$$

where R is the sensitivity of the polarimeter. For ideal geometry, R is simply the ratio of differential Compton cross-section average over the polarizations of the scattered photon, i.e., $R=(d\sigma/d\Omega)_{\beta=\pi/2}/(d\sigma/d\Omega)_{\beta=0}$, and β is the angle between direction of polarization of the incident photon and the plane of scattering. The finite extent of the detectors will reduce the value of the asymmetry ratio R and at 482 kev the value of R for our polarimeter is 4.0 ± 0.5.

[7] P. H. Stelson and F. K. McGowan, Bull. Am. Phys. Soc. Ser. II, **1**, 164 (1956).
[8] F. Metzger and M. Deutsch, Phys. Rev. **78**, 551 (1950).

TABLE I. Ratio of polarization intensities.

Spin sequence	$(E2/M1)^{\frac{1}{2}}$	G_2	G_4	p^a
$5/2(E2)9/2(E2+M1)7/2$	−1.7	0.735	0.497	1.10
$1/2(E2)5/2(E2+M1)7/2$	6.3	1.0	1.0	0.67
$1/2(E2)5/2(E2+M1)7/2$	6.3	0.20	0.11	0.91

[a] The calculations include also the effect of the finite extent of detector No. 1.

A block diagram of the fast-slow coincidence system is shown in Fig 3. The resolving time 2τ of the fast coincidence stage is 0.278 μsec. The window of one single-channel analyzer included the full-energy peak of the 133-kev γ ray and the other single channel analyzer with a window width of 40 kev selected the Compton recoil electrons. The triple coincidence rate was displayed as a pulse-height spectrum of the Compton scattered γ rays in 20 channels of a 20-by-120-channel analyzer[9] which was gated by the output of the fast-slow coincidence system. A record of the slow-singles was taken with a printing timer and a traffic counter. The energy resolution at 661-kev was 8.1% for detector No. 1 and 7.6% for detector No. 2. The anthracene detector had an energy resolution of 10% at 976 kev.

RESULTS AND DISCUSSION

Sources of Hf^{181} were prepared from a sample of Hf metal irradiated with pile neutrons. The polarization-direction correlation was measured with a source of HfF_4 in $27N$ HF and with a source of polycrystalline HfF_4. In Table II the experimental results are summarized for two runs for each type of source. The triple coincidence rate was of the order of 1.2 counts per minute and the random rate was 12% of this rate. About 99% of the random rate came from the term $2\tau N_1C_{A2}$ in the expression for the random rate

$$C_R=2\tau(N_1C_{A2}+N_2C_{A1}+N_AC_{12})+3\tau^2N_1N_2N_A,$$

where C_{A2}, C_{A1}, and C_{12} are the true double coincidence rates. The measurements with a dry polycrystalline source of HfF_4 were made to check on possible systematic errors. With this source the directional angular correlation of the 133- to 482-kev cascade is known to be attenuated[2] to the "hard core" value for static interactions in the intermediate state. As a result one

TABLE II. Summary of data for polarization direction correlation of the 133–482 kev γ-ray cascade.

Form of source	$N_{\parallel}/N_{\perp}$	P_{exp}
HfF_4 in $27N$ HF	1.24 ± 0.03	0.71 ± 0.05
HfF_4 in $27N$ HF	1.19 ± 0.05	
Polycrystalline HfF_4	1.09 ± 0.03	0.88 ± 0.05
Polycrystalline HfF_4	1.07 ± 0.03	

[9] Kelley, Bell, and Goss, Oak Ridge National Laboratory Physics Division Quarterly Progress Report ORNL-1278, 1951 (unpublished).

TABLE III. Summary of reduced transition probabilities for gamma-ray transitions in Ta^{181}.

$E\gamma$ (kev)	$(E2/M1)^{\frac{1}{2}}$	α_T	$B(E2)_d\times10^{48}$ (cm^4)	$\left(\frac{B(E2)_d}{B(E2)_{SP}}\right)$	$B(M1)_d$
136.25	0.5	1.53	1.63	270	8.6×10^{-2}
166	0.5	0.79	2.50	412	1.9×10^{-1}
303	∞	0.079	0.363	59	
345.85	∞	0.057	1.58×10^{-4}	2.6×10^{-2}	
482	6.3	0.024	1.82×10^{-4}	3.0×10^{-2}	7.4×10^{-7}
133	∞	1.34	3.0×10^{-5}	5.0×10^{-3}	

expects an attenuated value for p which is the last entry in Table I.

The agreement between the experimental and the expected values of p is quite good for the two types of sources and we conclude that the results are consistent with a spin sequence $1/2(E2)5/2(E2+M1)7/2$ for the 133–482 kev cascade.

After these measurements were completed we received a preprint of the paper by Boehm and Marmier.[1] Their high-resolution work showed that two close-lying states at 615.0 and 618.9 kev are excited in the decay of Hf^{181} whereas it had previously been supposed that there was only one state at 615 kev. The presence of the weakly excited state at 618.9 kev was not taken into account in the analysis of the above results nor in the previous interpretations of the directional angular correlation results. The experiments have actually measured a composite correlation and polarization of 96% 133-482 cascade and 4% 136.82-482 cascade. We believe that the 4% contribution of the 136.82-482 cascade cannot alter the conclusions deduced from the polarization-direction measurements. If one takes rather extreme values (2 and 0.5) for the polarization of the unknown 4% contribution one finds that it alters $N_{\parallel}/N_{\perp}$ by about the amount of the statistical uncertainty of the measurements.

It is desirable to determine to what extent the weak 136.82–482 contribution to the angular correlation measurement might alter the mixture of $E2$ and $M1$ for the 482-kev transition. Now, it is quite likely that the 618.9-kev state is $3/2^+$. It is observed to decay by $M1+E2$ (from internal conversion measurement) to the $5/2^+$ state at 482-kev. The $M1$ character of the radiation eliminates spin 1/2 while spins of 5/2 and 7/2 are unlikely because the state would be able to decay by low multipole order γ rays to states below the 482-kev state. Therefore we have taken the 136.82-482 cascade to be of the type $3/2(E2+M1)5/2(E2+M1)7/2$ and have investigated the possible error in the value of δ_2 for the main cascade which is of the type $1/2(E2)5/2\times(E2+M1)7/2$. The value of δ_2 is $+6.3$ when the 136.82–482 is neglected. By taking the largest positive and negative values of A_2 (coefficient of P_2 in angular correlation) for a cascade of the type $3/2(M1+E2)5/2\times(M1+E2)7/2$, where we allow δ_1 to vary but keep δ_2 near $+6$, we find that the δ_2 for the main cascade may be as low as $+4.7$ or as high as $+6.6$. The value of $+4.7$ for δ_2 leads to an α_K (482-kev) of 0.019 which is still considerably smaller than the previously measured values. However, a recent measurement of Sunyar[10] gives $\alpha_K(482)=(2.1\pm0.2)\times10^{-2}$ which is in agreement with the predicted value.

REDUCED TRANSITION PROBABILITIES

The transition probabilities are known either from direct lifetime measurements or from Coulomb excitation for many of the transitions between the low-lying states in Ta^{181}. We list in Table III the reduced transition probabilities for decay and compare these to those expected for transitions between states of the independent particle model. The values for $B(E2)$ and $B(M1)$ are actually those for the quantities $B(E2)/e^2$ and $B(M1)/(eh/2Mc)^2$. $B(M1)_{SP}$ is approximately unity and $B(E2)_{SP}$ is given approximately by $(1/4\pi)|\frac{3}{5}R_0^2|^2$ where we have taken R_0 equal to $1.2\times10^{-13}\,A^{\frac{1}{3}}$ cm. The total internal conversion coefficients α_T are taken from Rose *et al.*[11] However, a 30% reduction has been applied to the β_1 total values to take into account the finite nuclear size.[12]

We have assigned the 20-μsec half-life to the 615-kev state rather than to the recently identified 618.9-kev state. Boehm and Marmier found that most of the β decay directly excites the 615-kev state. The delayed-coincidence measurements show that most of the 480-kev γ rays exhibit the 20-μsec decay. This could not be the case if the weakly excited 618.9-kev state were the 20-μsec state. The 615-kev cross-over γ ray is an $M3$ transition and the value deduced for $B(M3)_d$ from the lifetime and branching ratios is 1.2×10^{-49} cm^4. The $B(M3)_d$ for a transition between independent-particle states is approximately 1.4×10^{-48} cm^4 so that the observed rate is about a factor of 10 slower than that for the independent-particle transition.

INTERPRETATION BY COLLECTIVE MODEL

The Ta^{181} nucleus is a well-known case of the permanently deformed nucleus which exhibits rotational excited states. The ratio of excitation energies, the very fast $E2$ transition rates, and the proper ratio of the $E2$ transition rates identify the 136.25 and 303-kev states as rotational excited states of the ground-state configuration. It is therefore of interest to see how well the collective model can interpret what is known about the low-lying states of Ta^{181}.

The $E2$ transition rates determined from Coulomb excitation give $Q_0=7.5\times10^{-24}$ cm^2 or $\beta=0.30$ (alternatively $\eta=+5$). The classification of nucleonic states in

[10] A. W. Sunyar (private communication).

[11] Rose, Goertzel, Spinrad, Harr, and Strong, Phys. Rev. **83**, 79 (1951) and M. E. Rose (privately circulated tables).

[12] L. A. Sliv, Zhur. Eksptl. i Teort. Fiz. **21**, 77 (1951); L. A. Sliv and M. A. Listengarten, Zhur. Eksptl. i Teort. Fiz. **22**, 29 (1952); A. H. Wapstra and G. J. Nijgh, Nuclear Phys. **1**, 245 (1956); Nordling, Siegbahn, Sokolowski, and Wapstra, Nuclear Phys. **1**, 326 (1956); F. K. McGowan and P. H. Stelson, Phys. Rev. **103**, 1133 (1956).

deformed nuclei[13] suggests a spin of 7/2 for this deformation. The measured spin is 7/2. From the measured $B(M1)_d$ for the 136.25- or the 166-kev transitions and the ground state magnetic moment (+2.1 nm) one can calculate g_Ω, the gyromagnetic ratio of the particle configuration and g_R, the gyromagnetic ratio of the collective motion. The $B(M1)_d$ for the 136.25-kev transition gives +0.69 and +0.29 for g_Ω and g_R, respectively. Similarly, the $B(M1)_d$ for the 166-kev transition gives +0.71 and +0.22 for g_Ω and g_R, respectively.[14] From the model one calculates (curve 25, $\eta=+5$) a g_Ω of +0.41. The g_Ω for this state is insensitive to the exact value of the deformation ($g_\Omega=0.42$ for $\eta=+4$ and $g_\Omega=0.40$ for $\eta=+6$). It is expected that g_R is approximately Z/A or +0.4. Now, the $B(M1)_d$ depends on the quantity $|g_\Omega-g_R|$ and the measured values of $B(M1)_d$ give $|g_\Omega-g_R|\simeq 0.45$. On the other hand, the model gives $|g_\Omega-g_R|\approx 0$.

The spin of 5/2 for the 482-kev state and the inhibited $E2$ transitions from this state to the ground state and 136.25-kev state indicate that this state is a different particle configuration from that of the ground state. The classification of Mottelson and Nilsson predicts that the next particle configuration is a $5/2^+$ state (if one assumes roughly the same deformation for the new configuration). Raboy and Krone[15] have determined the gyromagnetic ratio of this state as $+(1.20\pm0.12)$ from a measurement of the rotation of the angular correlation when the source is placed in a magnetic field. One calculates a gyromagnetic ratio of +1.46 from the model (curve 31, $\eta=+5$).

Nilsson has given formulas for the calculation of reduced electric and magnetic transition probabilities for transitions between states belonging to different rotational bands. We have calculated the $B(E2)_d$ for the transitions from the 482-kev state to the ground state and from the 482-kev state to the 136.25-kev state. Taking $\eta=5$ for both nucleonic states (No. 31 and No. 25), we calculate $B(E2)_d$ to be 1.0×10^{-52} cm^4 and 0.80×10^{-52} cm^4 for the 482-kev and 346-kev transitions, respectively.[16] Comparison of these calculated values with the experimental values listed in Table III shows good agreement. We have also calculated the $B(M1)_d$ for the 482-kev transition to be 2×10^{-3}. The model thus predicts an inhibited $M1$ transition and this is in qualitative agreement with what is observed.

The spin of 1/2 and the small $E2$ transition probability to the 482-kev state suggest that the 615-kev state belongs to a third nucleonic state. The two possible $1/2^+$ orbits available are No. 43 and No. 51. However, it would be surprising to find orbit No. 51 occurring at this low excitation energy. Furthermore this orbit does not exhibit the observed characteristics. For example, the calculated $B(E2)$ for decay from the 615- to 482-kev state is 5×10^{-51} cm^4 whereas the observed value is 3×10^{-53} cm^4. The 619-kev state ($3/2^+$) is probably a rotational state of this third nucleonic state. Orbit No. 51 predicts a large splitting of the 1/2 and 3/2 states instead of the small splitting observed.

Orbit No. 43 ($1/2^+$) has been used to give a rather complete description of the low-lying states of Tm^{169}.[17] A natural explanation of the 615-kev state in Ta^{181} is that this state results from the lifting of one of the protons in the filled orbit No. 43 ($1/2^+$) to fill orbit No. 25 ($7/2^+$), i.e., two protons in $7/2^+$ orbits and one proton in the $1/2^+$ orbit. This orbit does exhibit the observed characteristics of the 615- and 618-kev states. It predicts a small 1/2–3/2 splitting. The calculated $B(E2)$ for the 615- to 482-kev transition is quite small (1×10^{-53} cm^4) and is in approximate agreement with the observed transition rate. The calculated value of $B(M3)$ for the 615-kev transition is approximately 100 times less than the independent-particle estimate. As mentioned above, the observed value for $B(M3)$ is about 10 times smaller than the independent-particle model estimate. On the basis of the independent-particle estimates it is expected that the predominant mode for decay of the 619-kev ($3/2^+$) state is by an $E2$ transition to the $7/2^+$ ground state. However this γ ray has not been observed. An explanation for the absence of this γ ray is that this is a "K-forbidden" transition since $\Delta K=3$ and $L=2$.

---

[13] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **29**, No. 16 (1955); B. R. Mottelson and S. G. Nilsson, Phys. Rev. **99**, 1615 (1955).

[14] There are actually two sets of values for g_Ω and g_R; the other set is probably the spurious solution ($g_\Omega\approx-0.45$, $g_R\approx+0.95$).

[15] S. Raboy and V. E. Krone, Phys. Rev. **95**, 1689 (1954).

[16] We used the eigenfunctions listed in Table Ib of Nilsson (reference 13).

[17] B. R. Mottelson and S. C. Nilsson, Z. Physik **141**, 217 (1955).