TABLE I. Comparison of the parameters for the eight available atomic systems. μ is the mass ratio, h the trapping parameter, η the macroscopic cross section or mean-free-path ratio, and E_T the sputtering threshold as predicted by Wehner. Note the internal consistency of h, except for systems of large and small mass ratio. In the new calculation η seems to increase with the mass ratio, rather than remain constant as previously reported.

System	μ	h	η	E_T (ev)
He−Pb	0.0193	0.05	0.04	336
He−Ag	0.0371	0.30	0.05	275
Ne−Ag	0.1871	0.30	0.16	72
A−Pb	0.1926	0.30	0.16	46
A−Ag	0.3700	0.30	0.26	48
A−Cu	0.6264	0.30	0.23	57
Kr−Ag	0.7766	0.30	0.31	39
Kr−Cu	1.3155	0.65	0.15	56

for the remaining sets of data in this series. For convenience the new parameters are tabulated in Table I. We note that a single value of h applies over a considerable range in the mass ratio. This effect is somewhat more pronounced here than it was in the earlier calculations. The value of h is larger, but 0.30 seems a more realistic value for $\sigma_s/(\sigma_s+\sigma_t)$ than the value 0.10 previously reported. As we would expect, the cross section for trapping is considerably reduced when the mass ratio exceeds unity, and the value of h increases.

Except for the system A−Cu, the mean free path ratio shows a steady increase with the mass ratio for values of μ less than unity.

There is no apparent correlation of either h or η with the velocity of sound or elastic constants of the metals.

As before, the system He−Pb requires special treatment. The fitting is not as satisfactory because the "constant h" approximation fails in the manner described in I. The degree of failure is about the same in each calculation.

In conclusion, it appears that there is no significant change in the theoretical interpretation, but the new parameters are slightly more realistic.

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Hall and Drift Mobilities; Their Ratio and Temperature Dependence in Semiconductors

F. J. BLATT
Department of Physics and Astronomy, Michigan State University, East Lansing, Michigan
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It is shown that the Hall and drift mobilities in the temperature range in which scattering by ionized impurities dominates should exhibit a temperature dependence less rapid than $T^{\frac{3}{2}}$. This conclusion is in agreement with experimental observation. Furthermore, the mobility ratio, μ_H/μ_d, is not equal to $315\pi/512$ nor is it independent of temperature. Instead, μ_H/μ_d only approaches $315\pi/512$ at very high temperatures and decreases monotonically to $3\pi/8$ as the temperature is lowered.

INTRODUCTION

IN a recent paper[1] it was shown that at *low temperatures* the usual expression for the mobility in the range where ionized impurity scattering predominates is not valid because it is based on the Born approximation. This approximation was shown to fail at low temperatures. Moreover, it was found in I that even at *higher temperatures* where the Born approximation *is* valid the Hall and drift mobilities, which were calculated by numerical integration of the relaxation times over the distribution function, did not obey the generally accepted $T^{\frac{3}{2}}$ dependence. Instead, the calculated mobilities followed no simple power law behavior, although at high temperatures a $T^{\frac{3}{2}}$ law appeared to be approached. Over the entire range of temperature investigated the mobilities increased with temperature less rapidly than $T^{\frac{3}{2}}$, $d(\log\mu)/d(\log T)$ increasing monotonically with temperature and approaching the value $\frac{3}{2}$ asymptotically.

It was suggested in I that the $T^{\frac{3}{2}}$ dependence is generally obtained theoretically because certain factors which depend on temperature and carrier energy have not been considered in sufficient detail in the past. In particular, it was thought that the difference between the $T^{\frac{3}{2}}$ and the true temperature dependence of the mobility may be due to a certain approximation which is commonly employed when performing the necessary integrations over the distribution function.

In order to clarify this point we have performed detailed calculations of the mobilities in the impurity scattering range using the Born approximation. We have carried out the necessary integrations exactly for a particular case and have compared the results with those obtained by a slightly modified form of the usually accepted expression for the mobility.

In I, curves of the mobility ratio as a function of

[1] F. J. Blatt, Intern. J. Phys. Chem. Solids (to be published); hereafter referred to as I.

temperature were presented. A significant feature of these curves was that even at rather high temperatures they fell significantly below the generally quoted result $\mu_H/\mu_d=315\pi/512$. This difference between the true value of the mobility ratio and $315\pi/512$ appears again, and, as we shall show, is to be expected.

As in I, we shall proceed under the assumption that the surfaces of constant energy in $\mathbf{k}$ space are spheres, although this is generally a rather poor assumption. However, the points which will be raised in this paper do not depend on the exact form of the energy bands.

RESULTS AND DISCUSSION

If the Born approximation is used to calculate the differential cross section for scattering of a charge carrier by an ionized impurity in a semiconductor, the relaxation time is given by the expression[2]

$$\frac{1}{\tau}=\frac{\pi e^4 N_I}{(m^*)^{\frac{1}{2}}(2\epsilon)^{\frac{3}{2}}\kappa^2}\left[\ln(1+b)-\frac{b}{1+b}\right]=\frac{\pi e^4 N_I[g(b)]}{(m^*)^{\frac{1}{2}}(2\epsilon)^{\frac{3}{2}}\kappa^2}, \quad (1)$$

in which

$$b=8\pi m^*\kappa kT\epsilon/n'e^2h^2. \quad (2)$$

The symbols have the following meaning: N_I is the number of ionized impurity centers; κ is the dielectric constant; ϵ is the kinetic energy of the carrier; n' is given by

$$n'=n+(n+N_A)[1-(n+N_A)/N_D]$$

if the sample is n type; for p-type materials a similar expression is valid with N_A and N_D interchanged and n replaced by p.

The drift and Hall mobilities are given by[2]

$$\mu_d=(e/m^*)\langle\tau\rangle, \quad (3)$$

$$\mu_H=(e/m^*)\langle\tau^2\rangle/\langle\tau\rangle, \quad (4)$$

where

$$\langle\tau^n\rangle=(4/3\sqrt{\pi})\int_0^\infty \tau^n(x)x^{\frac{3}{2}}e^{-x}dx, \quad x=\epsilon/kT. \quad (5)$$

The integrations indicated in Eq. (5) are complicated by the factor $g(b)$ of Eq. (1). In fact, it is not possible to carry out these integrations analytically if that factor is retained in the integrand. An accepted approximate procedure is the following.[2] Since over the range of interest, and also in the range where the Born approximation is valid,[1] b is much larger than unity, it is reasonable to replace $g(b)$ by $\ln(b)-1$. It has been assumed further that $\ln(b)$ is large compared to unity, although this is not always justified. We shall, therefore, not make this additional approximation. In any case, however, for large b $g(b)$ is a slowly varying function of energy, and a reasonable approximation is to remove this factor from the integrands which appear in Eqs. (3) and (4); in the argument of $g(b)$, ϵ is then set equal to that energy at which the remaining integrand attains its maximum value. In the case of $\langle\tau\rangle$ the simplified integrand attains a maximum when $\epsilon=3kT$. The drift mobility is, in this approximation, given by the following expression

$$\mu_d=2^{7/2}\pi^{-\frac{3}{2}}(kT)^{\frac{3}{2}}(\kappa)^2e^{-3}(m^*)^{-\frac{1}{2}}N_I^{-1}\times\left\{\ln\left[\frac{24\pi m^*\kappa(kT)^2}{n'e^2h^2}\right]-1\right\}^{-1}. \quad (6)$$

The simplified integrand for $\langle\tau^2\rangle$, however, reaches a maximum when $\epsilon=(9/2)kT$, a point which appears to have been overlooked in the past. The Hall mobility, therefore, is not simply the drift mobility multiplied by a temperature independent constant, namely $315\pi/512$, but is given by

$$\mu_H=\frac{315(kT)^{\frac{3}{2}}\kappa^2}{2^{11/2}\pi^{\frac{1}{2}}\epsilon^3m^{*\frac{1}{2}}N_I}\cdot\frac{\{\ln[24\pi m^*\kappa(kT)^2/n'e^2h^2]-1\}}{\{\ln[36\pi m^*\kappa(kT)^2/n'e^2h^2]-1\}^2}. \quad (7)$$

For the ratio of Hall-to-drift mobilities, one finds

$$\frac{\mu_H}{\mu_d}=\frac{315\pi}{512}\left\{\frac{\ln[24\pi m^*\kappa(kT)^2/n'e^2h^2]-1}{\ln[36\pi m^*\kappa(kT)^2/n'e^2h^2]-1}\right\}^2. \quad (8)$$

For reasonable temperatures and carrier concentrations the correction term in the curly brackets is not negligible. For example, if we take $\kappa=16$, $m^*=0.2m_e$, $n'=10^{16}$, and $T=70°\text{K}$, the correction factor reduces

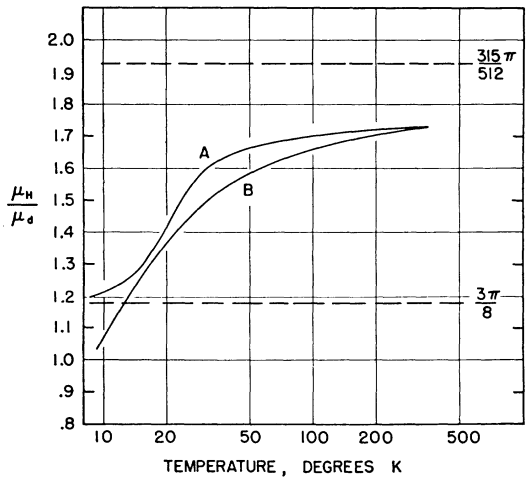

FIG. 1. The ratio of Hall-to-drift mobilities as a function of temperature. $m^*=0.2m_e$, $n'=10^{16}$, $\kappa=16$. Curve A was obtained by performing the averages over the carrier distribution exactly by numerical methods. Curve B was obtained by removing the slowly varying function of energy, $g(b)$ of Eq. (1), from the integrands and then setting ϵ in $g(b)$ equal to that value of the energy at which the remaining portion of the integrand attains its maximum; that is, curve B was obtained from Eq. (8). Also shown are the values $315\pi/512$ and $3\pi/8$, corresponding to the assumption that the relaxation time depends on energy as $\epsilon^{\frac{1}{2}}$ and as $\epsilon^{-\frac{1}{2}}$, respectively.

[2] H. Brooks, in *Advances in Electronics and Electron Physics* (Academic Press, Inc., New York, 1955), Vol. 7, p. 87. See also, for example, R. Mansfield, Proc. Phys. Soc. (London) **B69**, 76 (1956), as well as references cited in I.

the mobility ratio from 1.93 to 1.63. For these choices of κ, m^*, and n' we have plotted the mobility ratio as a function of temperature, curve B of Fig. 1. That curve is simply a graphical presentation of Eq. (8).

Curve A of Fig. 1 was obtained without use of the approximations leading to Eqs. (6) and (7). Instead, the integrals $\langle\tau\rangle$ and $\langle\tau^2\rangle$ were evaluated numerically. Curves A and B show the same general behavior. At low temperatures, however, curve A approaches the value $3\pi/8$ whereas curve B continues to decrease monotonically. In this temperature range b is no longer large compared to unity, and the approximation leading to Eqs. (6) and (7) fails. If $g(b)$ is expanded for small values of b, one finds that the energy dependence of τ is $\epsilon^{-\frac{1}{2}}$, that is, the same as for lattice scattering. The mobility ratio then must approach the value corresponding to an energy-independent mean free path, namely $3\pi/8$.

Turning now our attention to the temperature dependence of the mobilities themselves, we see from Fig. 2 that the Hall and drift mobilities increase with temperature less rapidly than $T^{\frac{3}{2}}$. In that range of temperature in which the mobilities can be approximated roughly by a power law, it appears that the exponent of T is intermediate between 1.5 and 1.0. This result is in good agreement with the findings of Debye and Conwell.[3] In fact, if tangents are drawn to the mobility curves at 100°K, their slopes are about 1.1, showing that indeed "the exponent of T should be closer to 1.0 than 1.5"[3] in this range of temperature and carrier concentration. The dashed curves, labeled I and II in Fig. 2, are graphical presentations of Eqs. (6) and (7), respectively. The solid curves were obtained by numerical integration of the integrals $\langle\tau\rangle$ and $\langle\tau^2\rangle$. Except at very low temperatures the solid and dashed curves are in very good agreement, indicating that the approximation in which $g(b)$ is removed from under the integral sign is reliable when $b>1$. This conclusion contradicts an earlier surmise[1] that the difference between the "predicted" $T^{\frac{3}{2}}$ dependence and the actual temperature dependence found in I appears to be due to having removed $g(b)$ from the integrand.

The curves of Fig. 2 do show that the temperature variations of $g(b_1)=g(b)_{\epsilon=3kT}$ and of $g(b_2)=g(b)_{\epsilon=9kT/2}$ are sufficiently rapid that they cannot be ignored. Since these functions increase monotonically with temperature they reduce the temperature dependence of the mobilities.

Of course, the results presented here should not be used in the region of low temperatures where the mobility curves show significant curvature. First, we took no account of the change of n' with temperature.

[3] P. P. Debye and E. M. Conwell, Phys. Rev. **93**, 693 (1954).

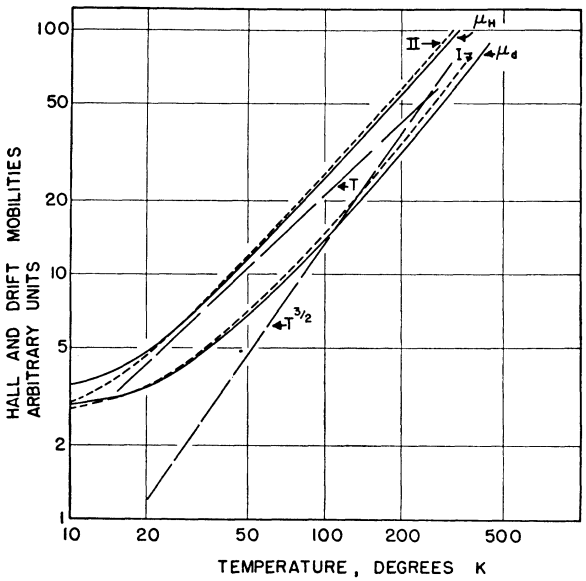

FIG. 2. The Hall and drift mobilities as functions of temperature. $m^*=0.2m_e$, $n'=10^{16}$, $\kappa=16$. The solid curves were obtained by performing the averages over the distribution function exactly by numerical methods. The dashed curves I and II show the temperature dependence of the drift and Hall mobilities, respectively, using the approximate expressions Eqs. (6) and (7). Also shown are the straight lines corresponding to a $T^{\frac{3}{2}}$ and a linear temperature dependence.

In practice n' decreases with decreasing temperature as carriers "freeze out" on impurity centers. Secondly, at low temperatures the relaxation times predicted by the Born approximation are not reliable.[1]

CONCLUSIONS

The results presented here show that (1) the approximate procedure for evaluating the integrals $\langle\tau\rangle$ and $\langle\tau^2\rangle$ is justified except at very low temperatures where b is of the order or less than unity; (2) the functions $g(b_1)$ and $g(b_2)$, defined above, vary with temperature sufficiently rapidly that their influence on the temperature dependence of the mobility must not be neglected; (3) when the mobilities are plotted as functions of temperature using the approximate theoretical expressions Eqs. (6) and (7), the temperature dependences of the Hall and drift mobilities are less rapid than $T^{\frac{3}{2}}$; (4) the usual assumptions concerning scattering of carriers by ionized impurities are capable of explaining the results obtained by Debye and Conwell[3] without involing a temperature-dependent effective mass or some other scattering mechanism; (5) the ratio of Hall-to-drift mobilities decreases monotonically from a value somewhat less than $315\pi/512$ to $3\pi/8$ as the temperature is decreased.