Two-Nucleon Interaction at High Energies*† and the Lévy Potential

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The predictions of the Lévy potential for nucleon-nucleon scattering and polarization at high energies have been explored. Most of the computations were made at 162 Mev, although some have also been done at 105 and 348 Mev. Various values of the Blatt-Kalos parameters were tried as well as certain modifications in the coupling constants and the range which surrender approximate agreement with the low-energy data. No satisfactory agreement with the unpolarized and polarized cross sections could be found. The need of a spin-orbit term in the two-nucleon interaction is discussed.

1. INTRODUCTION

If one should demand a meson theory of nuclear forces that is completely renormalized in the sense of quantum electrodynamics, then the required interaction Hamiltonian is unambiguously defined to be of the form[1]:

$$H' = iG\psi^* \boldsymbol{\tau}\beta\gamma_5\psi\boldsymbol{\Phi} + \lambda\phi^4.$$

The first term above describes a pseudoscalar meson field with pseudoscalar coupling to the nucleon field. The second term must be added to remove the divergence arising from the scattering of mesons by mesons.[2] Making use of the nonadiabatic Tamm-Dancoff approximation, Lévy attempted to arrive at a solution to the field equations with pseudoscalar-pseudoscalar interaction consistent in that all second and fourth order contributions of major importance were considered.[3] The resulting two-nucleon interaction, which Lévy deduced in the form of a potential with a repulsive core in all states, seemed most promising at first.

Soon after Lévy published his result, Klein clarified and corrected several points in the theory.[4] In particular, he showed that there was no reason to suppose that the second- and fourth-order coupling constants in Lévy's expression for the nuclear potential were the same, and he established that for sufficiently large distances (from μr somewhat less than unity) the approximation series does indeed converge. At the same time, considerable doubt was cast on the assumption that the two-pair term was the dominant part of the fourth-order interaction.

Considering the many qualifications that have since been attached to Lévy's theory by Klein and others,[5] one might question the merit of any attempt to compare predictions based upon it with experiment. Even if the Lévy potential did explain the low-energy data in its essential features, it might not be expected to account for the high-energy scattering data because of the possible importance of velocity-dependent forces at the higher energies. However, the point of view may be taken that the Lévy potential (which is displayed below in the form we have adopted) is a purely phenomenological one suggested by the *ps-ps* meson theory in the Tamm-Dancoff approximation. From this point of view, the potential possesses several promising properties[5] both at low and at high energies and could provide a more suggestive starting point for the correct theory than a straightforward phase-shift analysis of all the experimental data. Considering the low-energy data, the long-range tensor component of the Lévy potential can supply the deuteron quadrupole moment Q, while the short-range Wigner force (the fourth order term) keeps the D-wave admixture small, at the same time allowing Q to remain large. This is possible if the effective range of the central force is less than that of the tensor force. The repulsive core then helps to adjust the singlet and triplet effective ranges. At high energies, the tensor force and repulsive core of the Lévy potential are both useful toward achieving p-p isotropy.[6] In addition, the singularity of the tensor force should lead to the strong p-p polarization found experimentally.[7]

In order to test the validity of the Lévy potential at low energies, Blatt and Kalos investigated the scattering and bound-state properties of the potential with the aid of the ILLIAC electronic computer.[8] Allowing for three arbitrary parameters, the second- and fourth-order coupling constants, and the radius of the repulsive core, they obtained fair agreement with the five experimental numbers: deuteron binding energy, triplet and singlet effective ranges, singlet scattering length, and deuteron quadrupole moment. If one allows the calculated quadrupole moment to differ from the experimental value by as much as 20%, then good agreement might be ascribed to one or two sets of parameters.

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† Based on a Ph.D. thesis submitted to the University of Rochester.
‡ Now with the International Business Machines Research Laboratory, Poughkeepsie, New York.
[1] Schweber, Bethe, and de Hoffmann, *Mesons and Fields* (Row, Peterson, and Company, Evanston, 1955), Vol. 1, pp. 108–117.
[2] G. Bonnevoy, Compt. rend. **238**, 164 (1954).
[3] M. M. Lévy, Phys. Rev. **88**, 72, 725 (1952).
[4] A. Klein, Phys. Rev. **90**, 1101 (1953); **92**, 1017 (1953).
[5] See Chapter by R. E. Marshak and M. M. Lévy in the *Proceedings of the 1954 Glasgow Conference* (Pergamon Press, London, 1955), pp. 10–23.
[6] R. S. Christian and H. P. Noyes, Phys. Rev. **79**, 85 (1951); R. Jastrow, Phys. Rev. **81**, 165 (1951).
[7] L. J. B. Goldfarb and D. Feldman, Phys. Rev. **88**, 1099 (1952).
[8] J. M. Blatt and M. H. Kalos, Phys. Rev. **92**, 1563 (1953).

There is some theoretical justification for this, since the quadrupole moment calculation did not take into consideration contributions to the charge distribution by the charged pion fields. Estimates of the magnitude of this effect range from a few percent to about twenty percent.[9]

It was felt, then, that an accurate evaluation of the predictions of the Lévy potential at high energies was worth while in order to distinguish between those errors which arise from the insufficiency of the physical assumptions underlying the postulated potential and those due to the approximations made in solving the scattering equation. Since there is ample evidence that the Born approximation, and even variational calculations using Born approximation trial functions are unreliable for the calculation of scattering by singular potentials, the "exact" numerical solution of the nonrelativistic Schrödinger equation for nucleon-nucleon scattering by an arbitrary potential was coded for the IBM 650 Magnetic Drum Electronic Computer.[10] The program treats exactly the coupling between states of the same parity introduced by the tensor force. In the present work, all coupled phase shifts to $J=4$ and uncoupled phase shift to $L=3$ are included. The author feels reasonably certain that all phase shifts presented in the following section are within 1% of their exact values; those calculated for states with $L<2$ are probably within 0.1% of the exact value.[11] Our program was to examine Blatt and Kalos' favorable parameters at high energies (chiefly around 150 Mev) and, lacking agreement with experiment, to attempt to find some set of parameters to give agreement at the specified energy.

2. "PHENOMENOLOGICAL" LÉVY POTENTIAL AT LOW ENERGIES

The form of the Lévy potential adopted for comparison with experiment by Blatt and Kalos and the author was dictated by the considerations outline below and may be written:

$$V(x)=-(\mu c^2)\left\{\left(\frac{G^2}{4\pi}\right)^2\left(\frac{3}{2\pi}\right)\left(\frac{\mu^2}{M^2}\right)\frac{K_1(2x)}{x^2}-\tfrac{1}{3}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g^2}{4\pi}\right)\left(\frac{\mu}{2M}\right)^2\frac{e^{-x}}{x}\times\left[\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+S_{12}\left(1+\frac{3}{x}+\frac{3}{x^2}\right)\right]\right\}$$

for $x\equiv\mu r\geqslant\mu r_{\text{core}}$, and

$$V(x)=+\infty \quad \text{for} \quad x<\mu r_{\text{core}}.$$

Greene and Feldman, in a calculation of higher-order contributions to the nuclear force in pseudoscalar theory,[12] have shown that the behavior of the complete potential cannot be estimated at the present time for distances less than about $1/2\mu$. We have included, therefore, only those contributions which are expected to be important at greater distances than that given. We have then the second-order term (with coupling constant $g^2/4\pi$) in static approximation, and the fourth-order two-pair term (with coupling constant $G^2/4\pi$) given in an approximation somewhat more restrictive than the static. The second- and fourth-order coupling constants are allowed to differ from one another in order to simulate the effects of radiative corrections, and the second-order repulsive core, which Lévy finds only in S states, was extended to all states in order to permit the existence of solutions to the Schrödinger equation for these states. The effect on the final results of introducing other methods for removing the singularity at the origin for states with $L\geqslant 1$ will be discussed below.

One would expect our adaptation to be a reasonable approximation to the true potential at moderate energies and at distances $\gtrsim 1/2\mu$. As Greene and Feldman point out, the behavior of the potential for smaller distances is unknown; our assumption was to continue the same analytic form until the core, which was made repulsive and infinite for all states, was reached.

By making use of a prepared program for the ILLIAC, Blatt and Kalos examined the Lévy potential in the form we have described. Their program solves the coupled differential equations for the ${}^3S_1-{}^3D_1$ ground state of the deuteron and also solves the ${}^1S$ scattering equation. In every case, they demand that the deuteron binding energy be given exactly. With this constraint, they then are able to achieve the agreement shown in Table I with four additional parameters: Q, the deuteron quadrupole moment; ρ_t, the triplet effective range; ρ_s, the singlet effective range; and a_s, the triplet scattering length. Three parameters were allowed to vary, the second- and fourth-order coupling constants and the core radius. In addition, all cases quoted above give a D-state probability within the range indicated by qualitative considerations concerning the deuteron magnetic moment. In Table I,

TABLE I. Results of Blatt and Kalos.

r_{core}	$G^2/4\pi$	$g^2/4\pi$	Q	ρ_t	ρ_s	a_s
0.38	11.452	5.853	0.008519	0.3877	0.5836	−5.132
0.43	13.234	7.102	0.009998	0.4150	0.6362	−5.530
0.58	19.142	11.601	0.01470	0.4839	0.7924	−5.493
Experimental value			0.0147	0.395	0.63	−5.488

[9] F. Villars, Phys. Rev. **86**, 476 (1952); S. Deser, Phys. Rev. **92**, 1542 (1953); A. M. Sessler, Phys. Rev. **96**, 793 (1954).

[10] Details of the coding will be found in the author's doctoral thesis.

[11] All results and conclusions given in this paper supersede those presented at the 1956 New York meeting of the American Physical Society [H. Gelernter and R. E. Marshak, Bull. Am. Phys. Soc. Ser. II, **1**, 37 (1956)], which, through an error in calculation, were incorrect in part.

[12] J. M. Greene and D. Feldman (to be published).

TABLE II. Phase shifts for Blatt-Kalos parameters. Phase shifts are in degrees; the coefficients of admixture are pure numbers; the σ_{n-p} are in millibarns; and r_{core} is in pion Compton wavelengths.

Energy (Mev) = / r_{core} = / Phase shift	105 / 0.43	105 / 0.58	162 / 0.38	162 / 0.43	162 / 0.58	348 / 0.43	348 / 0.58
δ_0	17.8	2.1	8.9	3.0	−16.9	−27.6	−55.6
δ_1	18.1	17.9	21.8	22.2	16.0	14.8	−4.0
δ_2	5.5	10.9	8.3	10.9	19.3	22.6	25.7
δ_3	−0.60	−0.71	−0.18	−0.06	0.64	4.0	8.5
δ_1^0	58.3	58.9	55.7	56.2	48.0	38.3	17.7
$\delta_1^\alpha\equiv\delta_0^1$	37.6	24.7	26.8	22.2	9.4	−23.7	−49.3
$\delta_1^\beta\equiv\delta_2^1$	5.4	9.9	−6.3	−8.1	−17.3	4.0	−3.4
$\eta_1^\alpha\equiv\eta_0^1$	0.148	0.283	0.228	0.300	0.657	−0.799	−0.537
$\delta_1^\gamma\equiv\delta_1^1$	13.6	14.2	16.1	16.7	11.9	9.7	−7.4
$\delta_2^\alpha\equiv\delta_1^2$	30.3	33.4	32.0	33.3	29.0	60.0	18.8
$\delta_2^\beta\equiv\delta_3^2$	0.96	1.8	1.8	2.3	4.5	8.5	2.0
$\eta_2^\alpha\equiv\eta_1^2$	−0.0616	−0.0905	−0.0647	−0.0751	−0.156	−0.0454	−2.31
$\delta_2^\gamma\equiv\delta_2^2$	16.5	34.2	22.5	29.9	51.2	49.9	53.4
$\delta_3^\alpha\equiv\delta_2^3$	4.8	10.2	7.4	10.0	19.2	22.3	28.3
$\delta_3^\beta\equiv\delta_4^3$	−1.1	−1.6	−1.3	−1.5	−2.2	−1.8	−2.8
$\eta_3^\alpha\equiv\eta_2^3$	0.377	0.300	0.316	0.286	0.238	0.205	0.232
$\delta_3^\gamma\equiv\delta_3^3$	−0.28	−0.18	0.04	0.20	1.0	3.5	7.7
$\delta_4^\alpha\equiv\delta_3^4$	0.86	1.7	1.6	2.1	4.4	7.7	15.1
$\delta_4^\beta\equiv\delta_5^4$	−0.03	0.01	0.05	0.08	0.22	0.83	1.7
$\eta_4^\alpha\equiv\eta_3^4$	−0.358	−0.291	−0.282	−0.226	−0.203	−0.146	−0.123
$\sigma_{n-p}^{\text{total}}$	109	138	70.0	80.5	111	75.6	71.6

r_{core} is given in units of the pion Compton wavelength ($1/\mu=1.40\times10^{-13}$ cm). The other lengths characteristic of the deuteron, ρ_t, ρ_s, a_s, and Q, are given in units of the deuteron radius ($r_d=4.3\times10^{-13}$ cm). Note, however, that the ILLIAC program calculates the quadrupole moment from the ground state wave functions alone, neglecting those contributions due to the charged pion cloud in the nuclear force field.

3. SCATTERING AND POLARIZATION AT HIGH ENERGIES WITH BLATT-KALOS PARAMETERS

The three sets of parameters given in Table I were inserted into the Lévy potential, and all uncoupled

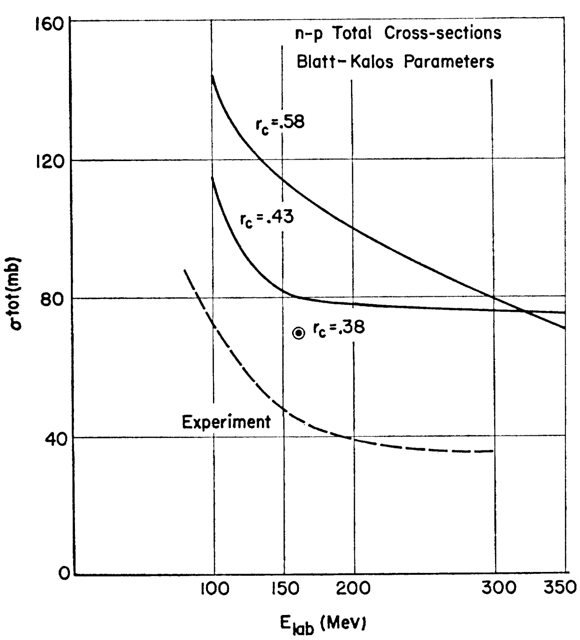

FIG. 1. *n-p* total cross sections for Lévy potential scattering with Blatt-Kalos parameters. r_c is the core radius in pion wavelengths for the given set of parameters.

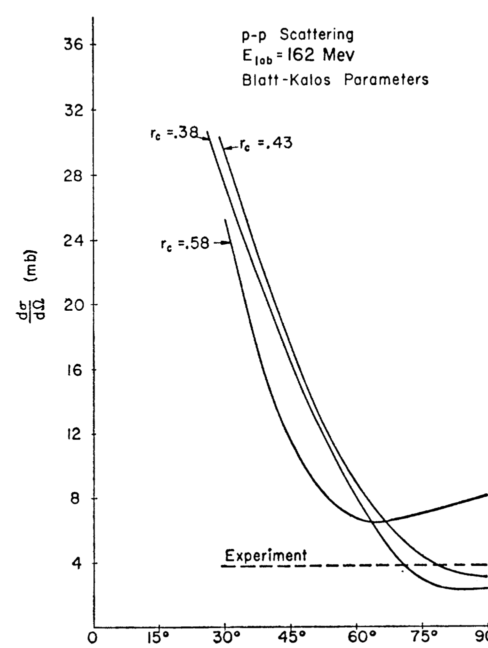

FIG. 2. *p-p* differential cross sections for Lévy potential scattering with Blatt-Kalos parameters at a lab energy of 162 Mev.

phase shifts to $L=3$ and coupled phase shifts to $J=4$ were calculated at laboratory energies of 105, 162, and 348 Mev. These phase shifts are given in Table II. The singlet phase shifts are denoted by δ_L. For the coupled triplet phase shifts, we shall adopt a nomenclature which has become standard in the literature, namely that of identifying the larger phase shift in the set $(\delta_J{}^\alpha,\delta_J{}^\beta)$ with the $L=J-1$ state, the other with the $L=J+1$ state. Thus, if $|\delta_J{}^\alpha|>|\delta_J{}^\beta|$, we shall write $\delta_J{}^\alpha\equiv\delta_{J-1}{}^J$, $\delta_J{}^\beta\equiv\delta_{J+1}{}^J$, and $\eta_J{}^\alpha\equiv\eta_{J-1}{}^J$, where η is the coefficient of admixture.

In each case, the total *n-p* cross section, the *p-p* differential cross section, and the *p-p* polarization was computed. In addition, the *n-p* differential cross section and *n-p* polarization were computed for one set at 162 Mev.

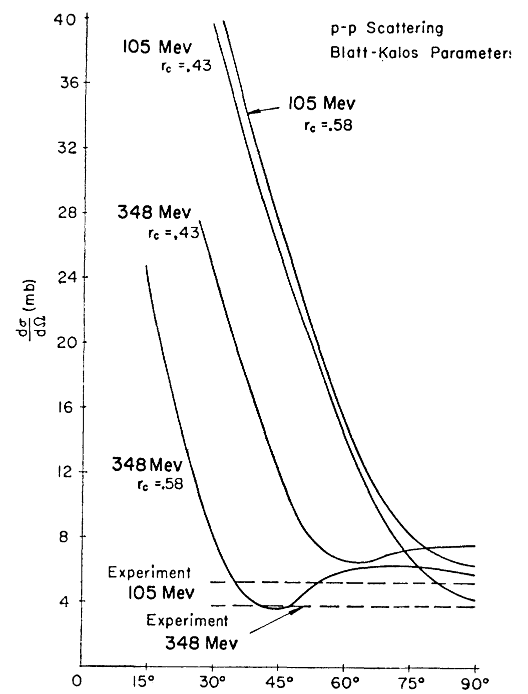

FIG. 3. *p-p* differential cross sections with Blatt-Kalos parameters at lab energies of 105 and 348 Mev.

In every case, over-all agreement with experiment was shown to be extremely poor. Total cross sections for *n-p* scattering are too large and exhibit an incorrect energy dependence (Fig. 1). Differential cross sections for *p-p* scattering exhibit none of the isotropy characteristic of experiment at these energies (Figs. 2–3); *p-p* polarizations at 105 and 162 Mev are peaked in the wrong region, and even go negative at 348 Mev (Figs. 4–5). Finally, the *n-p* differential cross section considered is strongly peaked in the forward direction, indicating that the amount of exchange force in the interaction is far too small to give the required approximate symmetry (Fig. 6). The only result which is not directly contradicted by experiment is the *n-p* polarization (Fig. 7). However, the magnitude of the errors involved in the experimental determination make any agreement with *n-p* polarization data rather inconclusive.

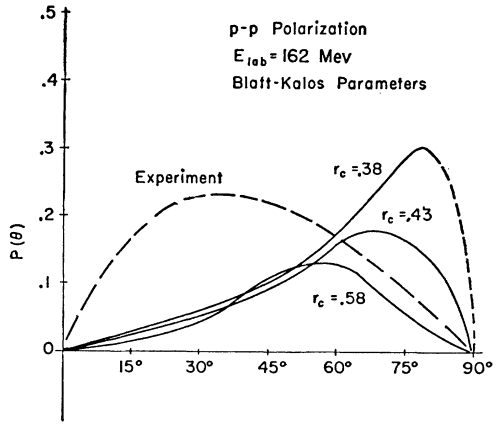

FIG. 4. *p-p* polarization at 162 Mev for the Lévy potential with Blatt-Kalos parameters.

Unfortunately, the extreme complexity of the relationship between the large number of phase shifts considered and the cross sections and polarizations which one must compare with experiment makes it extremely difficult to establish any connection between the poor agreement and any particular phase shift or group of phase shifts. However, it is likely that the incorrect location of the *p-p* polarization maxima at angles greater than $\sim 45°$ is related to the excessive values of the *p-p* differential cross sections for angles less than $\sim 45°$, since the differential cross section appears in the denominator of the expression for the polarization.

Only the kinematic relativistic correction was considered in calculating the wave number k in the calculations described. One still does not know how to make

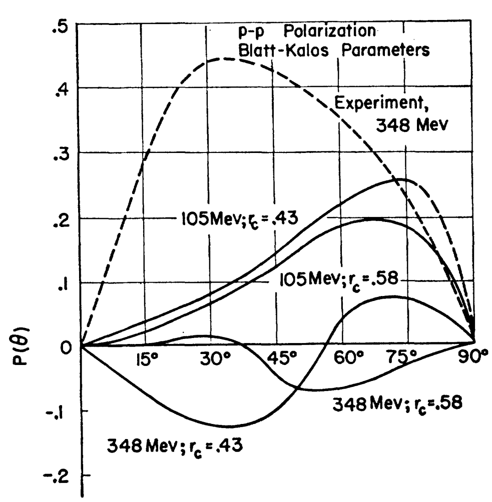

FIG. 5. *p-p* polarization at 105 and 348 Mev with Blatt-Kalos parameters.

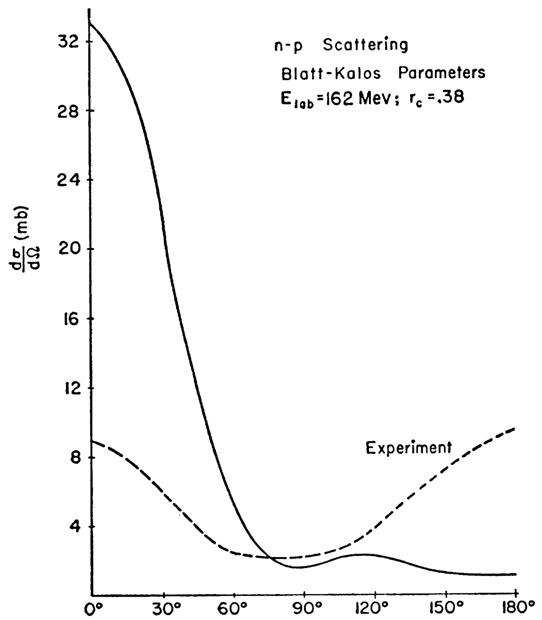

FIG. 6. *n-p* differential cross section at 162 Mev for the set of Blatt-Kalos parameters $G^2/4\pi=11.45$, $g^2/4\pi=5.853$, and $r_c=0.38$.

the dynamic correction for the relativistic reduced mass, and so the effect has been ignored, even though it has been estimated to be of the order 10 to 20% for the energies considered.[13] Noyes and Camnitz point out however, that the S phase shifts are quite sensitive to the dynamical correction, although they may be compensated by modifying the phenomenological core. In any event, such corrections are clearly not called for in evaluating our results, considering the gross departure from experiment that they exhibit.

The last remark applies as well to Coulomb effects in the case of *p-p* scattering. Ohnuma and Feldman[14] have shown these to be of the order of a few percent at angles greater than about 30°.

4. RESULTS OF SCATTERING AT HIGH ENERGIES WITH ARBITRARY PARAMETERS

In an attempt to determine if one could fit the experimental data with the analytical form of the Lévy potential regardless of parameters, a number of cases were considered at an energy of 162 Mev in which the following changes were introduced:

(1) The second and fourth order coupling constants were decreased by varying amounts.

(2) The range of the potential was decreased. This was accomplished by consistently multiplying the meson mass μ by a constant $\lambda>1$ wherever μ appears in the expression for the potential.

(3) A run was made with the singular fourth order central potential removed.

In all of the above runs, the core radius was kept constant at $0.38/\mu$. The *p-p* cross sections are given in

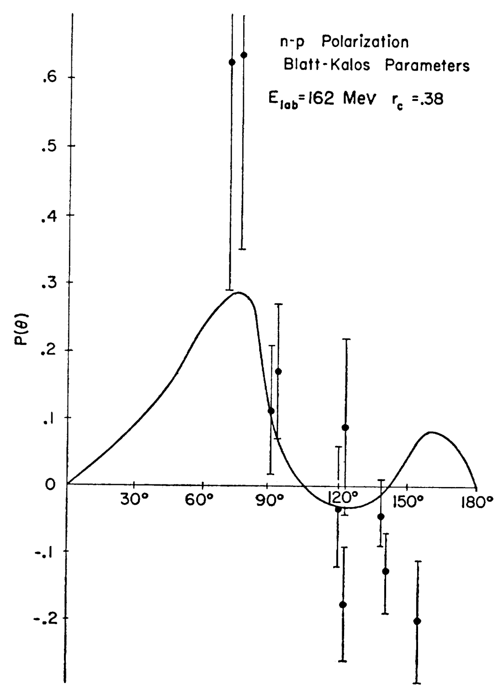

FIG. 7. *n-p* polarization corresponding to the differential cross section given in Fig. 6.

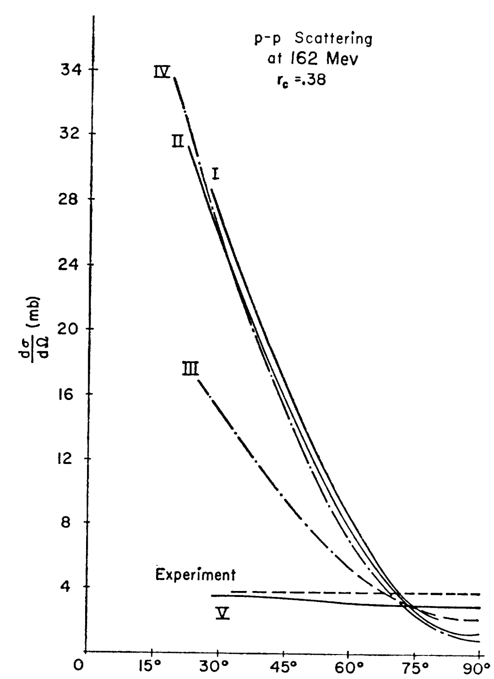

FIG. 8. *p-p* differential cross sections at 162 Mev for arbitrarily chosen parameters which surrender approximate low-energy agreement with experiment. The individual cases are described in the text.

[13] H. P. Noyes and H. Camnitz, Phys. Rev. 88, 1206 (1952).

[14] S. Ohnuma and D. Feldman, Phys. Rev. 102, 1641 (1956).

Fig. 8 for the most interesting of those runs for which the total *n-p* cross section lies within 30% of the experimental value. The corresponding *p-p* polarizations appear in Fig. 9. In each graph, curve I is the Blatt-Kalos curve discussed in the preceding section, which has been included for purposes of comparison. For curve II, the second-order Blatt-Kalos coupling constant was decreased to 3/4 its value for curve I. For curve III, parameters were the same as those of curve I, but the range was decreased by a factor of 1.13. Curve IV is the result of removing the singularity from the tensor force by setting the tensor potential equal to a constant, i.e., its value at $x=1$, for $r_c \leqslant x \leqslant 1$, again retaining the original Blatt-Kalos parameters. Finally, the singular fourth-order central potential was entirely removed to produce curve V. Here, the entire second-order potential with the Blatt-Kalos parameter was retained. Phase shifts for these five cases are given in Table III.

Although case V seems to be quite promising because of the isotropy of the *p-p* differential cross section, one finds that the *p-p* polarization is not only much too small, but of the wrong sign, as well. The *p-p* isotropy arises here from the fact that most of the scattering is in the ^1S state. Clearly such scattering cannot give the magnitude of polarization required.

It is evident from the examination of cases II and IV that the strong forward peak in the *p-p* cross section is a consequence of the extremely singular fourth-order central potential, since for those cases where only the second-order potential was altered, the *p-p* curves are hardly affected at all. On the other hand, the magnitude

TABLE III. Phase shifts at 162 Mev with $r_{\text{core}}=0.38/\mu$ for variations on the Blatt-Kalos parameters. Case designation is explained in the text. Phase shifts are in degrees.

Phase shift	Case I	Case II	Case III	Case IV	Case V
δ_0	8.9	8.4	1.1	8.9	−41.3
δ_1	21.8	23.5	13.7	21.8	−10.5
δ_2	8.3	8.0	6.3	8.3	0.63
δ_3	−0.18	0.18	−0.51	−0.18	−1.4
δ_1^0	55.7	48.5	45.2	42.0	7.2
$\delta_1^\alpha \equiv \delta_0^1$	26.8	20.8	19.7	14.0	−34.9
$\delta_1^\beta \equiv \delta_2^1$	−6.3	−3.9	−7.8	−4.1	−4.7
$\eta_1^\alpha \equiv \eta_0^1$	0.228	0.313	0.330	0.285	−0.419
$\delta_1^\gamma \equiv \delta_1^1$	16.1	19.1	9.8	20.7	−12.4
$\delta_2^\alpha \equiv \delta_1^2$	32.0	30.8	23.6	29.8	−5.4
$\delta_2^\beta \equiv \delta_3^2$	1.8	1.7	1.3	1.8	1.2
$\eta_2^\alpha \equiv \eta_1^2$	−0.0647	−0.0499	−0.0866	−0.0529	0.316
$\delta_2^\gamma \equiv \delta_2^2$	22.5	17.9	18.3	19.1	9.6
$\delta_3^\alpha \equiv \delta_2^3$	7.4	7.0	5.6	7.3	−3.2
$\delta_3^\beta \equiv \delta_4^3$	−1.3	−0.80	−1.3	−1.3	1.6
$\eta_3^\alpha \equiv \eta_2^3$	0.316	0.260	0.376	0.311	−1.01
$\delta_3^\gamma \equiv \delta_3^3$	0.04	0.34	−0.22	0.05	−1.2
$\delta_4^\alpha \equiv \delta_3^4$	1.6	1.5	1.2	1.6	0.55
$\delta_4^\beta \equiv \delta_5^4$	0.05	0.08	−0.01	0.05	−0.26
$\eta_4^\alpha \equiv \eta_3^4$	−0.282	−0.224	−0.335	−0.281	−0.881
$\sigma_{n-p}^{\text{total}}$(mb)	70.0	60.4	41.5	55.4	56.8

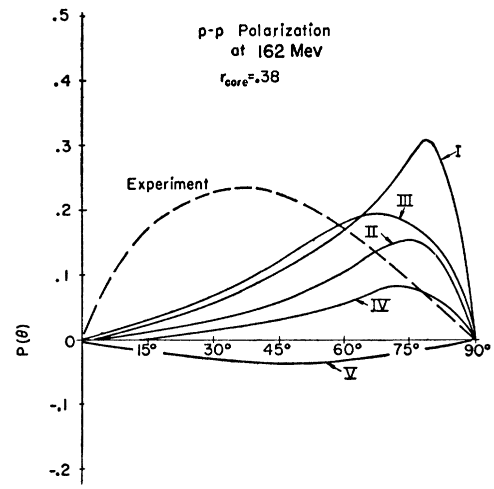

FIG. 9. *p-p* polarization curves corresponding to the differential cross sections given in Fig. 8.

of the *p-p* polarization is seen to be strongly dependent on these changes. In agreement with Goldfarb and Feldman,[7] we find that most of the polarization arises from the singularity of the tensor force, for in case IV, where the singularity in question has been removed while the coupling constant is unchanged, the magnitude of the polarization has been decreased by a factor of about four.

Little needs to be said concerning case III, except that it is clear that varying the range of the potential in the manner indicated will not give the desired results. Decreasing the range beyond the value of case III decreases the *n-p* cross section seriously below the experimental value.

It is illuminating to discuss the effects of modifications introduced into the potential on the predicted cross sections in terms of the behavior of the phase shifts. Thus, one can learn about the effect of the repulsive core by examining the variation of the singlet phase shifts at 162 Mev with respect to the radius of repulsive core (Fig. 10). One must recall here that in order to obtain agreement with the low-energy data, increasing the core radius requires that one increase the strength of both the second- and fourth-order potential. The ^1S phase shift is seen to be so strongly affected by the repulsive core, that despite the corresponding increase in strength of the attractive well, increasing the core radius produces a rapid decrease in δ_0. In effect, the increase in the negative repulsive part of the phase shift is much greater than that of the positive attractive part. The ^1P potential is such that the attractive and repulsive contributions are just about compensated at a core radius of 0.40; the repulsive increase is more important at greater radii, the attractive increase is greater at lesser radii. The ^1D and ^1F phase shifts, on the other hand, are barely affected by the core increase, responding mostly to the increased strength of the potential. The same effects are exhibited by the triplet

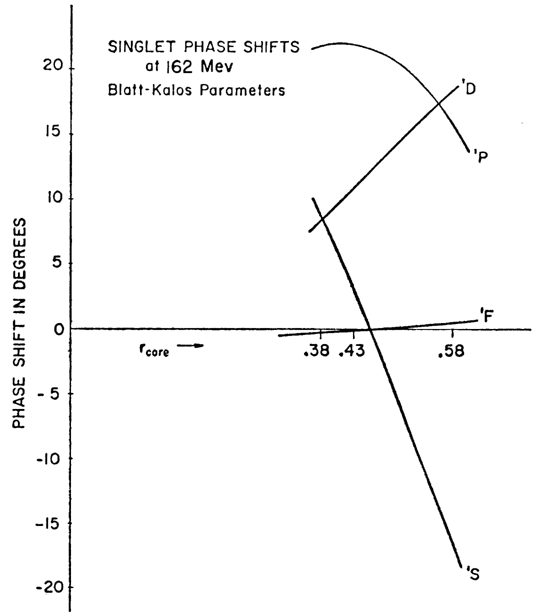

FIG. 10. Typical behavior of singlet phase shifts with variation of the core radius at a given energy.

phase shifts, although they are somewhat masked by the coupling due to the tensor potential. Hence, the S wave is the only one strongly affected by a hard core of any radius, the P wave is somewhat affected when the radius is large, while higher waves barely get to "see" the core at all.

5. CONCLUDING REMARKS

Because of the rather fortuitous fact that at 162 Mev, the coupling between ${}^3P_2-{}^3F_2$ states is quite small and the ${}^3F_4-{}^3H_4$ phase shifts are negligible, it is instructive to compare our results with the phase shifts of Ohnuma and Feldman at approximately the same energy[14] which were deduced from the p-p experimental data, even though they neglected possible coupling between triplet states of the same parity. The fact that their ${}^3P_0$ phase shifts are for the most part negative, or small if positive, is particularly interesting, since our results are all characterized by large positive $\delta_1{}^0$.

The original program for this work called for testing the possibility of improving agreement by removing the respulsive core for some states, other than S, to simulate a velocity-dependence for the potential. This was to be done by replacing the core with either a zero cutoff or square-well cutoff; first in all states with $L\geqslant 1$, and then in all odd states only. However, the considerations described above indicate that changing the core would have negligible effect on states higher than P states, and would have just the opposite effect from that desired on the ${}^3P_0$ state since removing the repulsive core would make the ${}^3P_0$ phase shift even larger than its already excessive value.

If one should attempt to salvage the Lévy potential, the results of this investigation indicate the following possible points of departure. First, one might allow the core radius to differ in the singlet and triplet states, a situation that is not all unlikely from the theoretical standpoint, since those singularities which we approximate with a hard core might very well be spin-dependent. The additional parameter should make it relatively easy to fit the low-energy data, and varying the different core radii will enable one to manipulate the S phase shifts, and to some extent, the P phase shifts. The difficulty with the ${}^3P_0$ phase shift still remains, however, unless one allows the triplet core radius to become large enough to cut it down. A second possible way out is to introduce an $\mathbf{L}\cdot\mathbf{S}$ dependence into the potential. Such a term does indeed appear in the fourth-order ps-ps theory, but is of order $(\mu/2M)^2$ with respect to $V_4{}^{(a)}$. On the other hand, Wolfenstein[15] finds that if he analyzes the scattering data into five independent scattering amplitudes, that amplitude which in Born approximation is due to an $\mathbf{L}\cdot\mathbf{S}$ dependence accounts for from 35% to 70% of p-p scattering at 300 Mev and 90°. The term corresponding to a tensor force accounts for only 2% to 20%.

The eigenvalues of the operator $\mathbf{L}\cdot\mathbf{S}$ are $J(J+1)-L(L+1)-S(S+1)$, which vanish for all singlet states. If one then introduced an $\mathbf{L}\cdot\mathbf{S}$ potential with negative sign, it would clearly be inoperative in S states, and introduce a strong repulsion in the ${}^3P_0$ state, which would certainly be beneficial toward obtaining agreement with the p-p data at 162 Mev. The net effects of such a potential for all states remain to be seen. It is most interesting to note, however, that for almost every set of acceptable phase shifts that Ohnuma and Feldman find to fit the p-p data at both 150 and 300 Mev, the ${}^3P_2$ phase shift is positive and the ${}^3F_2$ phase shift is negative. This is exactly the effect to be expected on these states from the $\mathbf{L}\cdot\mathbf{S}$ potential above.

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[15] L. Wolfenstein, Bull. Am. Phys. Soc. Ser. II, 1, 36 (1956).