rather large experimental uncertainties we found no departures from the predictions of this model.

Further evidence was obtained for the drastic change in the type of nuclear excitation spectra as we move from neutron number 88 to 90 in the odd europium isotopes. These last two neutrons are presumably just sufficient to induce a nonspherical equilibrium deformation in Eu^{153}.

V. ACKNOWLEDGMENTS

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Contribution of Coulomb Excitation to Inelastic Scattering between Nuclear Resonances*

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Using the comparison method of Bjerregaard and Huus, we have determined the relative gamma-ray yields for thin targets (~10 kev) of F^{19} (198-kev state) and Na^{23} (446-kev state), with protons and alpha particles over energy intervals corresponding to the same values of ξ, the adiabatic parameter in Coulomb excitation. Between the proton resonances we found that the yield ratios agreed closely with those predicted for $E2$ Coulomb excitation, and differed by about 50% from $E1$ (or $E3$) excitation. Thus the nonresonant inelastic scattering yield between proton resonances in these light nuclei can be entirely accounted for by Coulomb excitation of the $E2$ type. We determined a slight, negative anisotropy in the angular distribution of the 446-kev gamma ray following Coulomb excitation of Na^{23}, fixing the spin of that state at $5/2^+$.

I. INTRODUCTION

BJERREGAARD and Huus,[1] following a suggestion of Bohr and Mottelson, have shown that it is possible to determine the (electric) multipolarity of a given Coulomb-excited transition by a simple comparison of yields obtained with particles of different charge-to-mass ratios. By choosing the bombarding energies judiciously, it is possible to eliminate the usually complicated functions $f_{E\lambda}(\eta,\xi)$,[2,3] λ being the order of the multipolarity, $\eta=Z_1Z_2e^2/\hbar v$, and ξ the well-known adiabatic parameter of Coulomb excitation. The ratio will then only contain known quantities, and will be "quantized" in terms of the parameter λ. Bjerregaard and Huus confirmed the usefulness of this method for the known 0^+–2^+ pure electric quadrupole transitions in the even-even isotopes of wolfram, using protons, deuterons and alpha particles.[1] We have previously reported the use of this method in establishing the $E2$ nature of the excitation of the 128-kev excited state in Mn^{55}.[4] The spin of this state has recently been determined as $7/2^+$ by internal conversion and angular distribution measurements under Coulomb excitation.[5]

It has been shown that for a value of η as low as 2 the semiclassical expression ($\eta\rightarrow\infty$) for the total Coulomb excitation cross section falls only about 10% below the exact quantum-mechanical value,[2,3] as long as the semiclassical expression is slightly modified in accordance with the WKB approximation.[6] We shall therefore assume that the functions $f_{E\lambda}(\infty,\xi)^2$ are adequate for our purposes, since the smallest value of η encountered in any case was 1.63 (0.76-Mev protons of F^{19}). η was always greater than 4 for alpha particles, and the deviations from the semiclassical expression amounted to less than 2%.

For the sake of illustration, let us assume the case of negligible energy transfer, i.e., $\Delta E\ll E$, where E is the bombarding energy. Let us also neglect the center-of-mass motion. If we now choose bombarding energies for protons and alpha particles such that the parameter ξ has the same value ξ^* in both cases, we obtain the following simple expression for the ratio of Coulomb excitation cross sections:

$$\sigma_\alpha(\xi^*)/\sigma_p(\xi^*)=2^{(2\lambda/3)+2}, \tag{1}$$

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[*] A preliminary report of the results in this note can be found in Phys. Rev. **98**, 1198 (1955).

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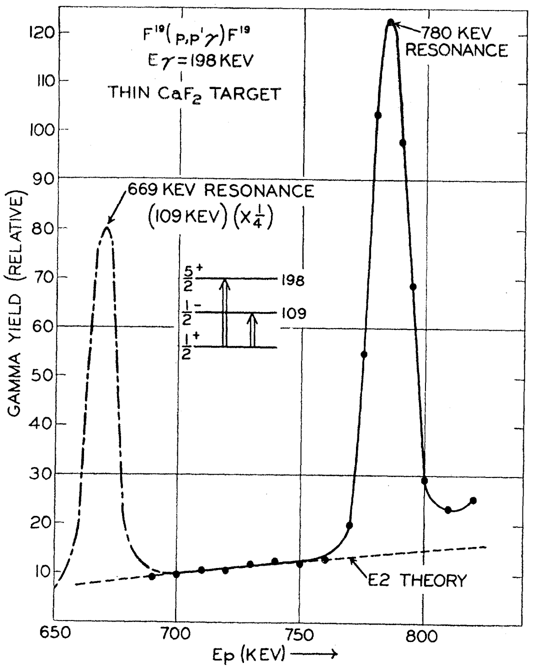

FIG. 1. Excitation function of 198-kev gamma radiation by protons on F^{19}. Thin target of CaF_2 on nickel. Resonances are identical with those observed by Barnes (reference 8). Theoretical $E2$ curve fits the data between resonances (as would $E1$), but is meaningful only in the light of results shown in Fig. 2.

where

$$E_\alpha(\xi^*) = 2.52 E_p(\xi^*). \qquad (2)$$

This ratio is 6.35 for $\lambda = 1(E1)$, 10.1 for $\lambda = 2(E2)$, and 16.0 for $\lambda = 3(E3)$. That is to say, the ratios for successive electric multipoles differ by about 60%.

If we give up the assumption of negligible energy loss ΔE and also take center-of-mass effects into account, we obtain a slightly more complicated relation for the ratio (1), the numerical factor in (2) is changed and also becomes slightly energy dependent. However, the main property of strong multipole discrimination is not changed, as we shall see below. In Table I we give a summary of the ratios of alpha to proton yields expected for the 198-kev transition in F^{19} and the 446-kev transition in Na^{23} over the energy ranges actually used in our experiments, and for the cases of $E1$ and $E2$ excitation. We also list the proton and alpha energies corresponding to the same value of ξ^* over these intervals.

We have investigated these two transitions in light elements with a twofold purpose in mind: (a) to determine the multipolarity of the excitation, (b) to see whether the gamma-ray yield between the known strong proton resonances could be accounted for by Coulomb excitation. We shall limit ourselves to the cases of $E1$ and $E2$ Coulomb excitation, since the previously determined shape of the excitation curves for these transitions[4,7] already rule out higher multipolarities. Finally, we shall discuss the evidence for the spin and parity of the excited state of Na^{23} at 446 kev.

II. EXPERIMENTAL RESULTS

Our experimental method has been previously discussed.[7] We used thin targets of CaF_2 and NaCl, evaporated onto nickel or niobium backing foils. (The presence of Ca and Cl did not cause any appreciable effect.) The only requirements for these experiments are that the targets be sufficiently thin (as can be ascertained from the width of the proton resonances) and that their relative gamma yields under proton and alpha bombardment can be accurately measured.

An angular distribution experiment was performed for Na^{23}, consisting of the relative measurements of gamma intensity at 0° and 90° to the beam direction. Spurious anisotropies were eliminated by placing a source of Na^{22} at the target position, and normalizing the results accordingly. We shall now discuss the individual transitions.

A. 198-kev Transition in F^{19}

The 198-kev second excited state gamma ray of F^{19} has been previously excited by protons[8] and alpha particles.[7,9,10] The (pure) $E2$ nature of this transition was inferred from its measured lifetime coupled with the $E2$ Coulomb excitation cross section, and the $5/2^+$ character of this state seems quite certain.[9] We have independently established the $E2$ nature of the excitation process, but could not distinguish between assign-

TABLE I. Table of corresponding alpha and proton energies and predicted yield ratios for $E1$ and $E2$ excitation in F^{19} and Na^{23}. All energies are given in the lab system. $\rho(E1) = \sigma_\alpha(E1)/\sigma_p(E1)$ for energies corresponding to equal values of the parameter ξ^*; $\rho(E2) = \sigma_\alpha(E2)/\sigma_p(E2)$ for equal ξ^*. Note slight departures from the values predicted by Eqs. (1) and (2) which neglect energy transfer and center-of-mass motion.

E_p	ξ^*	E_α	E_α/E_p	$\rho(E1)$	$\rho(E2)$
(a) 198-kev transition in F^{19}					
0.690	0.336	1.72	2.495	6.41	9.31
0.700	0.328	1.75	2.500	6.40	9.30
0.710	0.319	1.78	2.503	6.39	9.29
0.720	0.311	1.81	2.509	6.38	9.28
0.730	0.303	1.84	2.515	6.37	9.26
0.740	0.296	1.86	2.518	6.36	9.24
0.750	0.289	1.89	2.520	6.35	9.22
0.760	0.282	1.92	2.522	6.34	9.21
(b) 466-kev transition in Na^{23}					
0.900	0.801	2.00	2.223	7.19	10.75
0.910	0.782	2.03	2.231	7.18	10.71
0.920	0.763	2.06	2.236	7.16	10.68
0.930	0.745	2.09	2.245	7.14	10.65
0.940	0.728	2.12	2.250	7.12	10.62
0.950	0.711	2.14	2.258	7.10	10.59
0.960	0.695	2.17	2.264	7.08	10.56
0.970	0.679	2.20	2.270	7.06	10.53
0.980	0.664	2.23	2.273	7.04	10.50

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[7] N. P. Heydenburg and G. M. Temmer, Phys. Rev. **94**, 1252 (1954).

[8] C. A. Barnes, Phys. Rev. **97**, 1226 (1955).

[9] Sherr, Li, and Christy, Phys. Rev. **96**, 1258 (1954).

[10] G. A. Jones and D. H. Wilkinson, Phil. Mag. **45**, 230 (1954).

ments of 3/2+ and 5/2+ for the state without angular distribution measurements.

We have previously reported an alpha-particle excitation curve[4] for the energy region of interest, namely, 1.72 Mev to 1.92 Mev. In that interval no resonances are apparent (see Fig. 4 of reference 7), and the excitation is undoubtedly purely electric. The conjugate energy interval for protons, i.e., the region of equal ξ values, ranges between 0.69 and 0.76 Mev. Figure 1 shows the thin-target excitation curve for F^{19} under proton bombardment; the strong resonances to either side of the interval of interest show the target thickness ($\sim$10 kev). These resonances have been previously observed.[8]

In Fig. 2 we have displayed the relative yields for 198-kev radiation under alpha-particle and proton bombardment. All curves are plotted on a common abscissa of ξ; the gamma yields are meaningful on a relative scale. Note that bombarding energies increase toward the left. Theoretical $E1$ and $E2$ excitation curves are plotted, *all four* curves being normalized as shown. We cannot attach any significance to the small difference in shape in the $E1$ and $E2$ theoretical curves shown in the upper half of the diagram (alpha yield); what is significant, however, is that the experimental points for the *proton* yield between resonances fall very close to the $E2$ curve. Any proton yield over and above Coulomb excitation due to possible compound-nucleus formation would tend to move the experimental points up toward the $E1$ curve in the lower half of Fig. 2. This of course happens on the strong resonances (points not shown). There are six independent pairs of points plotted, each of which confirms the $E2$ nature of the excitation of the 198-kev state in F^{19}. Furthermore, we have shown that the nonresonant yield produced by protons between resonances is just the amount expected for proton Coulomb excitation.

We were unfortunately unable to find an energy region suitably free of resonances over which to test the $E1$ nature of the 109-kev transition to the first excited state of F^{19}.

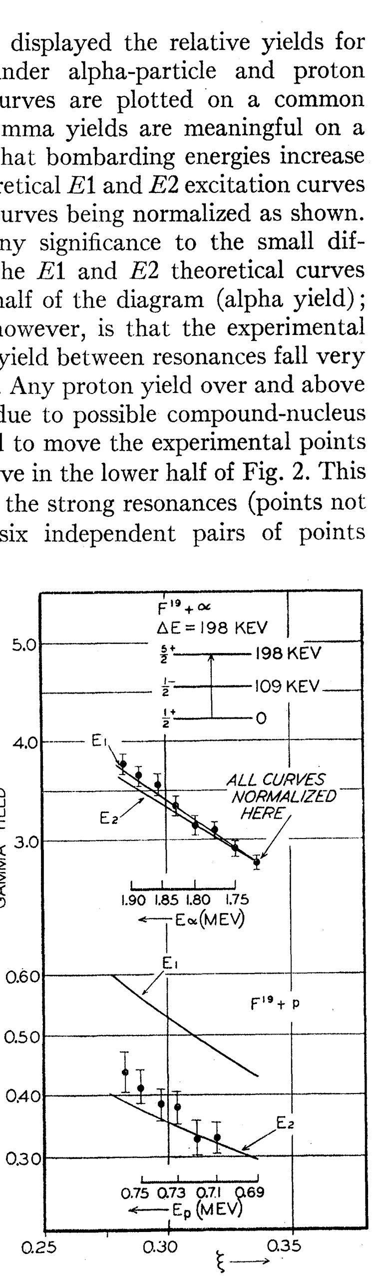

FIG. 2. Comparison of thin-target yields of 198-kev radiation from F^{19} under alpha-particle and proton bombardment. All curves plotted on a common scale of ξ; note that energies E_α and E_p increase from right to left. All four theoretical curves are normalized as shown. All yields are plotted on the same relative scale. Note gap in ordinate values. Small differences between $E1$ and $E2$ shapes for one type of projectile are not experimentally distinguishable; however, the six experimental points in the lower part of the figure prove the $E2$ nature of the excitation.

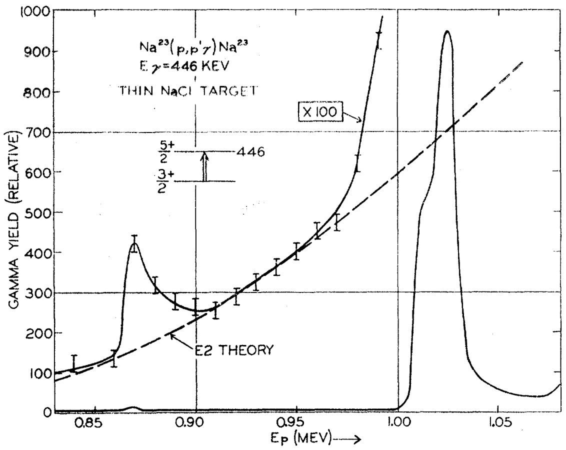

FIG. 3. Excitation function of 446-kev gamma radiation by protons on Na^{23}. Thin target of NaCl on niobium. Resonances are identical with those observed by Burling (867 kev, reference 13) and, e.g., Newton (1.008 and 1.017 Mev, reference 12). Theoretical $E2$ curve fits the data between resonances (as would $E1$), but is meaningful only in the light of results shown in Fig. 4.

B. 446-kev Transition in Na^{23}

Most of what we said above for F^{19} applies to Na^{23} as well. Figure 3 shows the inelastic proton excitation between 0.83 and 1.07 Mev, again showing resonant structure. The unresolved doublet of states at about 1.02 Mev has been previously observed by Stelson and Preston[11] and by Newton,[12] while the weak resonance at 867 kev was discovered by Burling.[13] The useful proton energy range for comparison purposes extended between 0.90 and 0.98 Mev. The corresponding energy interval for alpha particles lies between 2.00 and 2.23 Mev. We have previously studied thin-target excitation functions covering this range (see Figs. 3 and 4 of reference 4) and found the smooth behavior characteristic of Coulomb excitation. Figure 4 summarizes all information obtained with protons and alpha particles on Na^{23} consisting of six independent thin-target ratio determinations. We again claim no discrimination between $E1$ and $E2$ shapes in the upper portion of the figure; the only significant result consists, as in the case of F^{19}, in the location of the experimental points in the lower half of Fig. 4 with respect to the theoretical $E1$ and $E2$ curves, establishing without question the $E2$ nature of the ex-

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[11] P. H. Stelson and W. M. Preston, Phys. Rev. **95**, 974 (1954).

[12] J. O. Newton, Phys. Rev. **96**, 241 (1954), and private communication.

[13] R. L. Burling, Phys. Rev. **60**, 340 (1941).

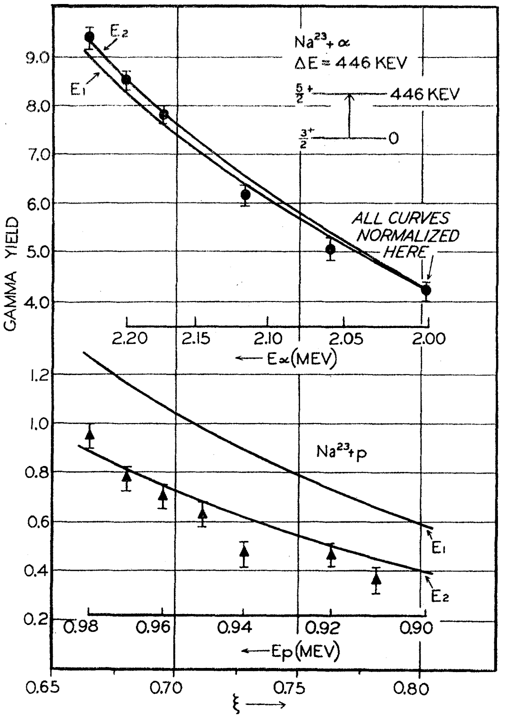

FIG. 4. Comparison of thin-target yields of 446-kev radiation from Na^{23} under alpha-particle and proton bombardment. All curves plotted on a common scale of ξ; note that energies E_α and E_p increase from right to left. All four theoretical curves are normalized as shown. All yields are plotted on the same relative scale. Note gap in the ordinate values. Small differences between $E1$ and $E2$ shapes for one type of projectile are not experimentally distinguishable; however, the seven experimental points in the lower part of the figure prove the $E2$ nature of the excitation.

citation of the 446-kev state. As we shall see below, this state has spin $5/2^+$, so that it decays predominantly by $M1$ radiation. Once again, we have shown that the nonresonant yield between proton resonances is satisfactorily ascribed to $E2$ Coulomb excitation.

C. Spin Parity of the 446-kev State of Na^{23}

The $E2$ nature of the transition connecting the ground state ($I_0 = 3/2^+$) with the 446-kev state still allows spins $1/2^+$, $3/2^+$, $5/2^+$, and $7/2^+$ for this state.

Krone and Read[14] have found an anisotropic distribution for this gamma ray when bombarding Na^{23} with protons at 1.288 Mev, which is a strong resonance corresponding to a 1^- state in Mg^{24}.[15] This immediately rules out spin $1/2^+$ for the 446-kev state. These authors are able to account for their angular distribution by assignments of either $3/2^\pm$ or $5/2^\pm$, but not by $7/2^\pm$. The latter possibility is definitely ruled out by a comparison of the *partial* mean lifetime for $E2$ decay of this state, found from our Coulomb excitation cross section [$\tau_\gamma(E2) = 4.3\times 10^{-10}$ sec[16]] and the upper limit placed on the total lifetime by a recoil experiment under proton bombardment [$\tau_\gamma < 2.5\times 10^{-11}$ sec[17]]. This shows that the decay must take place by at least 95% of $M1$ radiation, and rules out spin $7/2^+$. This now leaves us with only $3/2^+$ and $5/2^+$ as possible choices. In order to see if we could eliminate $3/2^+$, we performed an angular distribution measurement of the 446-kev gamma radiation following Coulomb excitation with 2.5-Mev alpha particles. Now the angular distribution following Coulomb excitation has the character of a gamma-gamma angular correlation which in our case is given by the sequence $3/2^+(E2)I^*(M1+E2)3/2^+$. It fortunately turns out that the anisotropy accidentally vanishes for the case $I^* = 3/2^+$,[18] while it may take on positive or negative values for $I^* = 5/2^+$, depending on the amount of $E2$ admixture in the deexcitation, and the relative phase of $M1$ and $E2$ components.

We obtained the following value for the anisotropy at 2.5 Mev:

$$A \equiv \frac{W(0) - W(\pi/2)}{W(\pi/2)} = -0.078 \pm 0.040.$$

This rules out the possibility of $I^* = 3/2^+$, and establishes the spin of the 446-kev state as $5/2^+$. This is in keeping with the predictions of the shell model and probably represents the spin which the ground state of this nucleus with a $(d_{5/2})^3$ configuration was expected to have. The negative sign of the slight anisotropy once again eliminates the possibility of a pure $E2$ transition and $I^* = 7/2^+$.

III. CONCLUSIONS

The method of Bjerregaard and Huus was found to be very useful to establish the $E2$ excitation for the case of pure $E2$ (F^{19}) as well as mixed $M1$-$E2$ transitions in odd-mass nuclei (Mn^{55}, Na^{23}). A by-product of these measurements was the discovery that the nonresonant yield of inelastic proton scattering could be entirely accounted for by Coulomb excitation. The $E2$ excitation in the case of mixtures containing a preponderant amount of magnetic dipole radiation illustrates the intrinsic smallness of $M1$ Coulomb excitation.

Existing evidence coupled with our angular distribution measurements following Coulomb excitation in Na^{23} uniquely assigns a spin of $5/2^+$ to the 446-kev excited state of that nucleus.

No evidence for any major interference effects between compound-nucleus formation and Coulomb excitation is apparent within our accuracy; this is not unexpected in view of the fact that only one out of the large number of partial waves participating in the Coulomb excitation process can contribute to the formation of a given compound state.

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