Coulomb Excitation and Cascade Decay of Rotational States in Odd-Mass Nuclei*

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(Received August 3, 1956)

We have observed the gamma radiation from the first two excited states, as well as the cascade transition between them, by bombarding the following odd-mass rare-earth nuclei with 6-Mev alpha particles: Eu^{151}, Eu^{153}, Gd^{155}, Gd^{157}, Tb^{159}, Ho^{165}, Lu^{175}, Hf^{177}, Hf^{179}, and Ta^{181}. All of these nuclei were available as enriched targets. We also detected x-x, x-γ, and γ-γ coincidences from these nuclei, establishing their previously suggested level schemes. All of them show rotational spectra except Eu^{151} ($N=88$), which was found to have an excitation spectrum very different from that of Eu^{153} ($N=90$) in both the intensity and position of the excited states. From the measured branching ratios of the second rotational states we determined the $M1-E2$ mixture ratios for the cascade radiation, using the theoretical value for the $E2$ component. This is reasonable because within the experimental uncertainties, mainly caused by our lack of information on total internal conversion coefficients, the ratios of $E2$ transition probabilities from ground state to the two rotational states are in agreement with the theory. Combining the mixture ratio with the values of the ground state magnetic moments, we computed the so-called intrinsic and collective g factors from the strong-coupling approximation of the Bohr-Mottelson theory. Two pairs of values are obtained because of the ambiguity in the phase of the mixture ratio. Appreciable deviations from the irrotational value Z/A are observed for the collective g factors. The mixtures we determine also agree with those known from radioactive decay for first-excited state transitions as predicted by the unified model.

I. INTRODUCTION

THE existence of rotational energy levels in nuclei with large static quadrupole moments is by now well established.[1–4] We have previously reported a detailed study of such nuclei in the rare-earth region by Coulomb excitation.[4] As is to be expected from the predominant electric quadrupole nature of the excitation process on the one hand, and the characteristic spin sequence of successive states in rotational bands on the other hand, one can excite but the first excited state in even-even nuclei having character 2^+, while it is possible to populate the first two excited states in nuclei of odd mass, having spins I_0+1 and I_0+2, respectively, and the same parity as the ground state I_0. We should then observe three transitions in general, two of which originate from the second excited state, and one from the first. The cascade transition from the second to the first excited state has been observed only in a few instances so far, mainly because its energy lies rather close to that of the first-excited state transition [differing from it by $100/(I_0+1)\%$], and is always considerably weaker than the latter. Experiments using internal conversion electron detection were found to be particularly well suited for the direct detection of the cascade transitions because of their intrinsically higher energy resolution. However, coincidences between the two cascade events must be detected using scintillation counters, and have previously been reported for Ta^{181} [4,5] and Eu^{153}.[4] In addition to establishing the correctness of decay schemes inferred from the single gamma-ray spectra, such experiments permit a rather detailed study of the consequences of the so-called "strong coupling" approximation of the Bohr-Mottelson model of the nucleus.[1] When combined with a knowledge of the ground-state magnetic moments, the transition probabilities derived from Coulomb excitation and branching ratios from second excited states can be used to evaluate the relative contributions of the odd-particle motion (g_Ω) and the collective motion of the entire nucleus (g_R) to the nuclear gyromagnetic ratio.

We have studied all odd-A nuclei between europium and tantalum which are available as enriched targets or occur naturally as single isotopes. This criterion then excludes the five odd-neutron nuclei of dysprosium, erbium, and ytterbium. We have previously given some tentative conclusions obtained from the spectra of these five nuclei with natural targets.[4]

II. EXPERIMENTAL DETAILS

As in previously published work, we used the He^{++} beam at 6.0 or 6.5 Mev from our electrostatic generator. The detection arrangement was identical to the one previously used[6]; for the coincidence experiments we added another single-channel scintillation detector with a 1 in.×1 in. NaI crystal, whose output was put in slow coincidence ($\tau\sim0.5$ microsecond) with the output from our usual 2 in.×2 in. NaI crystal. The target was inclined at 45° to the beam axis, and the

* Preliminary accounts of this work were presented at the 1956 New York and Washington meetings of the American Physical Society [N. P. Heydenburg and G. M. Temmer, Bull. Am. Phys. Soc. Ser. II, 1, 43 (1956); Ser. II, 1, 164 (1956)].

[1] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 16 (1953).

[2] A. Bohr, doctoral dissertation, Copenhagen, 1954 (unpublished); also A. Bohr and B. R. Mottelson in *Beta- and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (North-Holland Publishing Company, Amsterdam, 1955), Chap. XVII.

[3] Huus, Bjerregaard, and Elbek, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **30**, No. 16 (1956).

[4] N. P. Heydenburg and G. M. Temmer, Phys. Rev. **100**, 150 (1955).

[5] Class, Cook, and Eisinger, Phys. Rev. **94**, 747 (1954).

[6] G. M. Temmer and N. P. Heydenburg, Phys. Rev. **96**, 426 (1954).

two crystals viewed the target from a distance of about one inch, through the 10-mil aluminum backing, one at 0° and one at right angles to the beam; they were shielded from each other by a one-eighth inch thick lead plate to prevent spurious Compton-recoil coincidences. Each crystal was shielded by 10 mils of copper when examining x-x or x-γ coincidences, and by 25 mils of copper when observing γ-γ coincidences. Because of the high internal conversion of these low-energy transitions, it is convenient to observe the K x-rays following internal conversion of one of the cascade transitions in coincidence either with another K x-ray following the conversion of the daughter cascade (x-x), or with the unconverted gamma radiation of the latter (x-γ). The former is particularly helpful if the energies of either or both of the successive gamma rays are not well known, and we are only interested in establishing their cascade nature. Some of the figures below will illustrate these various types of coincidences.

In order to find the relative intensities of cascade and crossover gamma rays from the second rotational states, we found it more reliable to analyze carefully the single-counter spectra. The coincidences served only to establish the correctness of the predicted decay schemes.[4] We kept the beam intensity low enough to limit the accidental coincidence rate to less than 20% of the true rate at the photoelectric peaks. The accidental rate was checked in the usual manner by inserting delay cable. Our coincidence detection efficiency was of the order of one count per several hundred singles counts in the channel with the lower counting rate. As we said above we did not need to know the absolute coincidence efficiency. The availability of enriched targets was essential; we were, for instance, unable to detect coincidences with a natural target of dysprosium because of the high singles rate produced in one channel by the composite peak due to the 2^+-0^+ transitions from all even-even nuclei in the target. The same is of course true of the K x-ray peak.

III. RESULTS

Our results consist of a series of carefully determined pulse-height spectra, all of which reveal the presence of cascade radiation in addition to the previously observed direct ground-state transitions from the first two rotational states.[4] We have also determined x-x, x-γ, and γ-γ coincidence spectra by maintaining one channel on the photopeak of one transition (or K x-ray) and moving the other channel over the spectrum. We used a window width of 2 volts for these measurements.

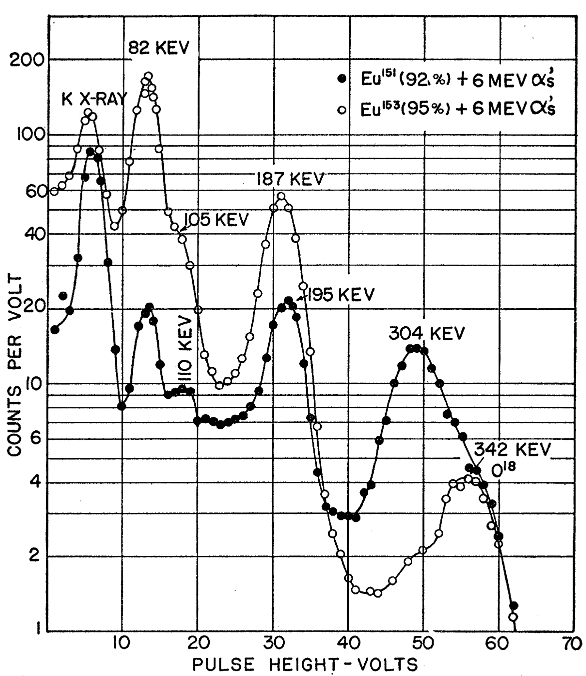

FIG. 1. Pulse-height distributions for Eu^{151} and Eu^{153} Coulomb-excited by 6-Mev alpha particles. Full circles refer to Eu^{151} ($N=88$), having lines at 110, 195, and 304 kev. The first two of these are in coincidence. Open circles refer to Eu^{153} ($N=90$), having lines at 82, 105, and 187 kev, the first two of which are in coincidence. Only Eu^{153} has a rotational spectrum (see level schemes in Fig. 2). The usual 342-kev line due to O^{18} may be seen.

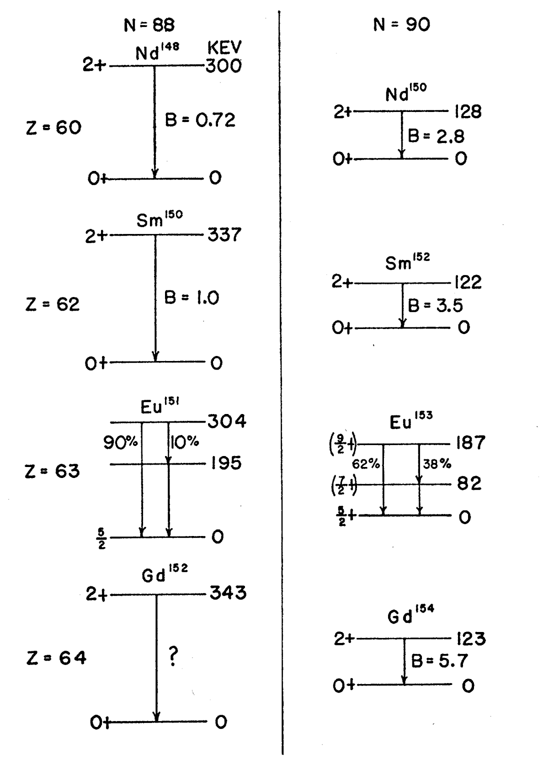

FIG. 2. Level schemes of Coulomb-excited transitions in nuclei straddling the gap between neutron number 88 and 90. In the even-even nuclei we can see a strong *decrease* in the energy of the first-excited states, and a corresponding *increase* in the reduced (upward) transition probabilities $B(E2)$, given in the diagram as B in units of 10^{-48} cm^4. (Gd^{152} has not been excited.) Note increase of B with increasing atomic number Z. Branching percentages given for Eu nuclei refer to gamma transitions only; for further data on these nuclei, see Table I.

TABLE I. Transition from nonrotational to rotational behavior in europium between $N=88$ and $N=90$. $\epsilon B(E2)$ = reduced gamma transition probability as defined in reference 15; $B(E2)$ = reduced transition probability, corrected for branching and internal conversion. $\tau_\gamma(E2)$ = mean life for partial $E2$ decay.

Nucleus	E_γ (kev)	$I_0 - I^*$	$\epsilon B(E2)$ (10^{-48} cm^4)	$\sim B(E2)$ (10^{-48} cm^4)	$\tau_\gamma(E2)$ (sec)
$_{63}Eu_{88}^{151}$	195	$5/2^- - ?^-$	0.067	0.051	2.8×10^{-8}
	304	$5/2^- - ?^-$	0.22	0.28	6.8×10^{-10}
	110[a]		0.024		
$_{63}Eu_{90}^{153}$	82	$5/2^+ - (7/2)^+$	0.60	2.8	5.2×10^{-8}
	187	$5/2^+ - (9/2)^+$	0.29	0.83	8.5×10^{-9}
	105[a]		0.18		...

[a] Transition from second excited state to first excited state.

We shall now discuss the various nuclei individually.

(1) *Europium* ($Z=63$).—Separated isotopes of the two stable species of europium, Eu^{151} and Eu^{153}, have recently become available and have yielded rather striking results.[7] We had previously studied the singles spectrum of a natural europium (oxide) target and established the existence of coincidences between the 105-kev and 82-kev radiations.[4] Figure 1 shows the pulse-height distributions obtained from the two highly enriched isotopic samples.[8] We see that transitions at 195 kev, 110 kev, and 304 kev occur in Eu^{151}; we also detected coincidences between the first two gamma-ray lines whose energies add to 304 kev. A transition of ~300 kev had been tentatively assigned to Eu^{151} previously.[4] From a study of the yield as a function of bombarding energy we could establish that the 195-kev transition leads to the ground state. Note that the energies of the two cascade gamma rays nearly coincide with the energies of the cascade and cross over transitions in Eu^{153}; hence all previous measurements based on natural targets gave wrong relative as well as absolute intensities.[3,4] Figure 2 shows the decay schemes and branching ratios for the two Eu isotopes, along with the 0^+–2^+ transitions in even-even nuclei (all Coulomb excited except Gd^{152}), all of which straddle the gap between neutron number 88 and 90. One can see the abrupt change from rotational to nonrotational spectra, as well as a sudden drop in excitation energies, as we cross the gap. Not only do the positions of the levels show this pronounced break, but their transition probabilities as well, as can be seen in Fig. 2 for the even nuclei and in Table I for Eu.[9] The complete disappearance of rotational characteristics in Eu^{151}, in spite of a 5/2 ground state spin, identical to Eu^{153} (which has a well-developed rotational band) represents convincing evidence for the existence of the 88–90 break from odd-A nuclear spectra. The great difference in spectroscopic quadrupole moments between the two europium isotopes has long been known, the ratio of the quadrupole moments being 2.1.[10] Recent evidence from the decay of Eu^{152} seems to indicate[11] that the break really occurs between neutron numbers 89 and 90.

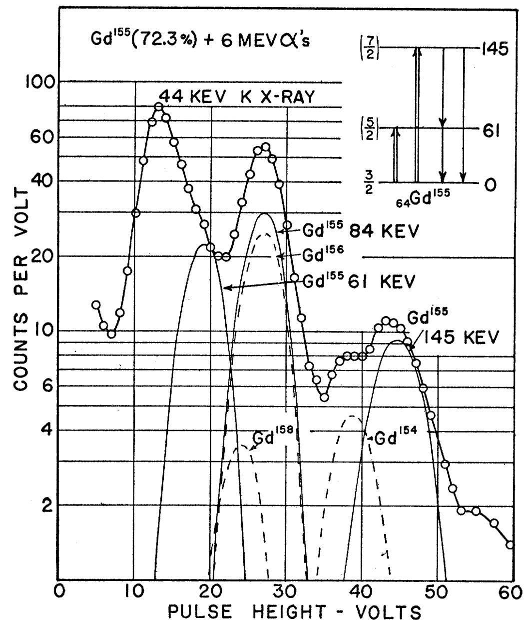

FIG. 3. Pulse-height spectrum from enriched Gd^{155} under 6-Mev alpha bombardment. Contributions from even-even isotopes Gd^{154}, Gd^{156}, and Gd^{158} are shown as dashed parabolas, and were evaluated in turn from targets enriched in these nuclei. Parabolas in solid line at 61, 84, and 145 kev belong to Gd^{155}; their positions in the rotational level scheme are shown in the insert. Coincidences between x-rays and gamma rays were also observed (not shown). Analogous results were obtained for enriched Gd^{157}. For a summary of these results, see Table II.

(2) *Gadolinium* ($Z=64$).—We had previously found excited states in Gd^{155} and Gd^{157} at 145 kev and 131 kev, respectively, and had predicted, from their positions and transition probabilities, that they represented the *second* rotational states.[4] In the meantime two groups, following up this prediction, independently[12,13] found the first excited state as well as cascade transitions by internal conversion electron detection. Figure 3 shows our gamma-ray spectrum for one of these nuclei (Gd^{155}), showing some indication of this highly-converted transition. A similar spectrum was obtained for Gd^{157}. We have also found x-x and x-γ coincidences for both odd Gd nuclei, confirming the level scheme shown in the insert of Fig. 3. We also have found evidence for the cascade transitions in the Gd nuclei, using Gd^{155} as an example, the large peak at 88 kev in Fig. 3 cannot entirely be accounted for by the presence of the even-

[7] Similar results on Eu^{151} have been obtained independently by C. M. Class and U. Meyer-Berkhout using internal conversion electron detection (private communication).

[8] Kindly lent to us by L. Grodzins, Brookhaven National Laboratory.

[9] For a plot illustrating this break in transition probabilities for even-even nuclei, see Fig. 9 of reference 14.

[10] H. Kopfermann, *Kernmomente* (Akademische Verlagshandlung, Frankfurt am Main, 1955), second edition, p. 441.

[11] L. Grodzins, Bull. Am. Phys. Soc. Ser. II, 1, 163 (1956), and private communication.

[12] J. H. Bjerregaard and U. Meyer-Berkhout, Z. Naturforsch. **11a**, 273 (1956).

[13] E. M. Bernstein and H. W. Lewis, Bull. Am. Phys. Soc. Ser. II, 1, 41 (1956), and private communication.

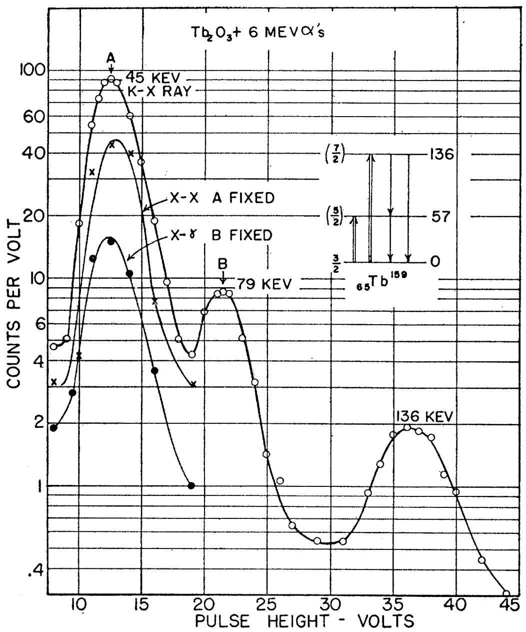

FIG. 4. Gamma-ray obtained from Tb^{159} with 6-Mev alpha particles. Open circles denote singles spectrum, showing K x-ray, 79-kev cascade and 136-kev second-excited state radiation. Crosses denote coincidence spectrum obtained with one channel fixed at A (x-x coincidences); full circles denote coincidence spectrum with one channel fixed at B (x-γ coincidences). First-excited state radiation at 57 kev is not resolved. Insert shows the rotational level scheme of Tb^{159}.

even nuclei Gd^{156}, Gd^{158}, and Gd^{160}. We recently obtained enriched targets of the latter three isotopes and found that we could account for only about 47% of the peak height by the even-even contributions, as may be seen from the analysis in Fig. 3; the rest must come from Gd^{155}. Analogous results were found in Gd^{157}. Incidentally, this explains the obviously discordant points in Fig. 1 of a previous publication[14] for even-even nuclei. Since we used the odd-A enriched targets (which are also somewhat enriched in adjacent even-even isotopes) to determine transition probabilities, we erroneously ascribed the entire photopeaks at around 88 kev and 78 kev to the even-even nuclei. (See also Fig. 5 of reference 4.)

TABLE II. Summary of results on even and odd gadolinium nuclei. I_0 and I^* are spins of ground state and excited states, respectively; E_γ = transition energy in kev; $\epsilon B(E2)$ = reduced gamma transition probability, as defined in reference 15 (not corrected for branching), in 10^{-48} cm^4. Abundances are given for enriched isotopes (except Gd^{154}). $\beta_B{}^2$ = square of the nuclear deformation parameter (for definition, see reference 14; note that points in Fig. 1 for Gd isotopes now follow the general trend).

Nucleus	Abundance (percent)	I_0	I^*	E_γ (kev)	$\epsilon B(E2)$ (10^{-48} cm^4)	$\beta_B{}^2$
Gd^{154}	2.2[a]	0^+	0^+	123	1.93	0.12
Gd^{155}	72.3	$3/2^-$	$(5/2)^-$	61[b]	...	...
				84[c]	0.32	...
			$(7/2)^-$	145	0.085	...
Gd^{156}	80.2	0^+	2^+	88	0.88	0.12
Gd^{157}	69.7	$3/2^-$	$(5/2)^-$	55[b]	...	...
				76[c]	0.22	...
			$7/2^-$	131	0.61	...
Gd^{158}	92.9	0^+	2^+	78	0.63	0.12
Gd^{160}	95.4	0^+	2^+	73	0.50	0.11

[a] Natural target.
[b] Transition not resolved; see references 12 and 13.
[c] Cascade transition.

In Table II we summarize the information obtained with both odd and even enriched isotopes of Gd. The values given there supersede those in Table II of reference 4.

(3) *Terbium* ($Z=65$).—Our previous results on Tb^{159} showing the cascade and crossover transitions from the second rotational state at 136 kev[4] were extended to include both x-x and x-γ coincidences, as shown in Fig. 4. The first-excited state has been observed both by internal conversion detection[3] and by inelastic proton detection.[15,16] The cascade radiation is the most clearly resolved such transition we have observed. The level scheme shown in the insert of Fig. 4 is firmly established[4] (excepting spins of excited states).

(4) *Holmium* ($Z=67$).—Figure 5 shows the spectrum obtained from Ho^{165},[4] showing the three transitions we expect. x-x, x-γ, and γ-γ coincidence spectra were also

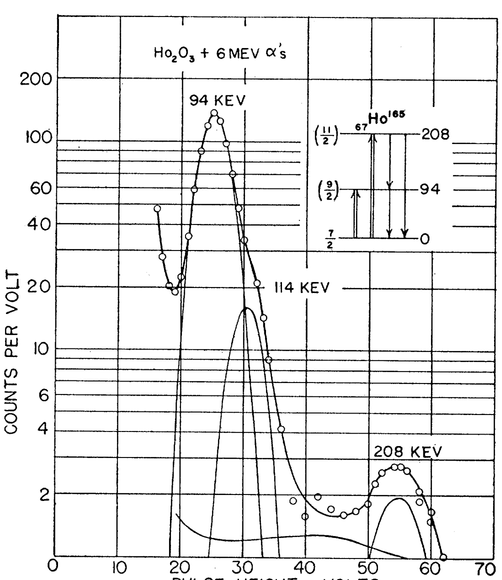

FIG. 5. Gamma-ray spectrum obtained from Ho^{165} under 6-Mev alpha bombardment. Cascade radiation at 114 kev can be seen, in addition to previously observed direct ground-state transitions at 94 and 208 kev. x-x, x-γ, and γ-γ coincidences were observed but are not illustrated. Insert shows rotational level scheme established for Ho^{165}.

[14] G. M. Temmer and N. P. Heydenburg, Phys. Rev. **98**, 1609 (1955). Most of the data displayed in Figs. 1 and 2 of this reference have been revised. For revised plots, see N. P. Heydenburg and G. M. Temmer, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1956), Vol. 5, Figs. 9 and 10.
[15] Alder, Bohr, Huus, Mottelson, and Winther, Revs. Modern Phys. (to be published).
[16] R. D. Sharp (private communication).

observed (not shown). The ratio of x-γ to γ-γ coincidences was about 8, reflecting the K-conversion coefficient of the 94-kev transition (fluorescence yield $\sim$1). The cascade transitions have also been observed by means of their conversion electrons.[3] The level scheme shown in the insert is now established (excepting spins of excited states).

(5) *Thulium* ($Z=69$).—Because of the ground state spin $I_0=\frac{1}{2}$ of Tm^{169}, an anomalous rotational spectrum obtains.[3,4] This spectrum has been observed in the decay of Yb^{169}, showing a first-excited state at about 10 kev.[17] As previously reported,[4] we see only a 110-kev (cascade) transition from the second rotational state. A detailed theoretical discussion for this nucleus has been given by Mottelson and Nilsson.[18]

(6) *Lutetium* ($Z=71$).—Figure 6 shows the spectrum obtained from Lu^{175}, showing evidence for a cascade at 136 kev in addition to previously observed transitions.[4] We also show the x-γ and γ-γ coincidence spectra in Fig. 7. This establishes the level scheme in the insert (except for spins of excited states) as observed in Coulomb excitation.[4,19] Internal conversion electrons associated with some of these transitions have also been seen.[3]

(7) *Hafnium* ($Z=72$).—Figure 8 illustrates the singles spectrum obtained from an enriched target of Hf^{177},[4] revealing the cascade radiation at 138 kev. An analogous spectrum was found for Hf^{179}. We again observed x-γ and γ-γ coincidences in both nuclei. Note

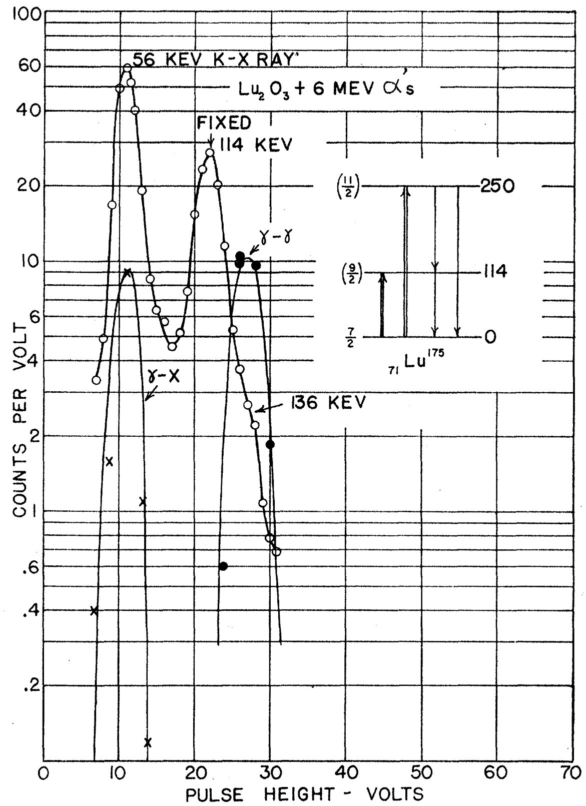

FIG. 7. Coincidence spectra from Lu^{175}. Open circles denote portion of the singles spectrum containing the K x-ray and two cascade lines. Coincidences obtained with one channel fixed as indicated; crosses denote x-γ coincidences; full circles denote γ-γ coincidences. Rotational level scheme established by these measurements is shown in the insert.

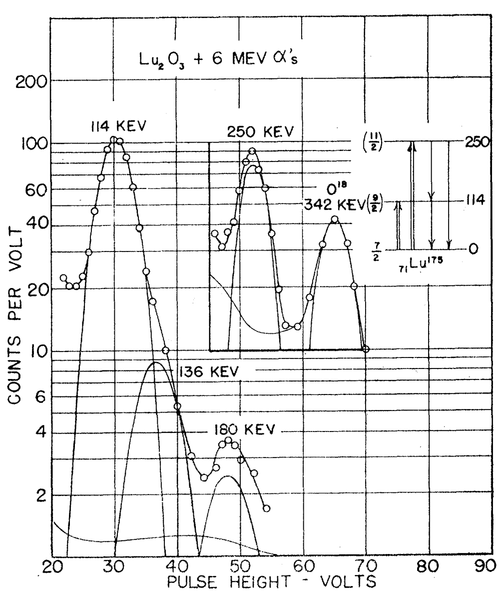

FIG. 6. Gamma-ray spectrum obtained from Lu^{175} under 6-Mev alpha bombardment. Cascade radiation at 136 kev can be seen, in addition to previously observed direct ground-state transitions at 114 and 250 kev. Line at 180 kev is assigned to odd-odd Lu^{176} until further notice. 342-kev line due to O^{18} is again present for this oxide target. Rotational level scheme shown in the insert.

the considerable difference in the branching ratios of these two nuclei (Table III). Internal conversion electron detection of some of these transitions has also been successful.[3]

(8) *Tantalum* ($Z=73$).—Much work has been reported on the cascade decay of Ta^{181},[3–5,20,21] and the decay scheme of rotational states is well known. We have determined the branching ratio for the 303-kev state mainly for the sake of comparison with other workers.[22]

IV. DISCUSSION

In Table III we summarize our results for the cascade decay of strongly deformed odd-A nuclei. We shall presently describe how we obtained the quantities in Table III.

It is relatively straightforward to determine the experimental reduced transition probabilities $\epsilon B(E2)$ [15]

[17] See, e.g., S. A. E. Johansson, Phys. Rev. **100**, 835 (1955).

[18] B. R. Mottelson and S. G. Nilsson, Z. Physik **141**, 217 (1955).

[19] McClelland, Mark, and Goodman, Phys. Rev. **97**, 1191 (1955).

[20] W. I. Goldburg and R. M. Williamson, Phys. Rev. **95**, 767 (1954).

[21] P. H. Stelson and F. K. McGowan, Phys. Rev. **99**, 112 (1955).

[22] γ-γ coincidences in some of the rare-earth nuclei discussed above have recently also been observed under proton bombardment by G. Goldring and G. T. Paulissen (private communication).

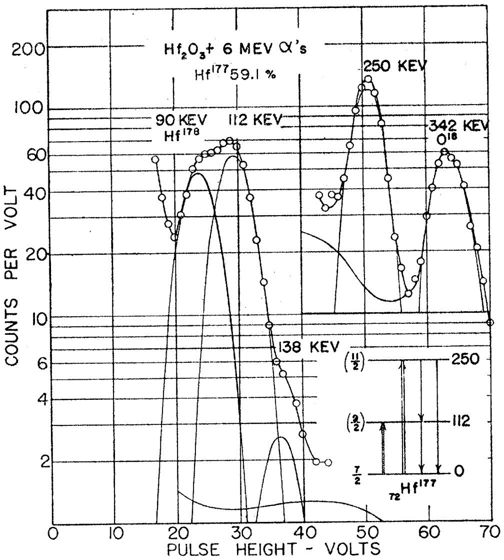

FIG. 8. Gamma-ray spectrum obtained from enriched Hf[177] under 6-Mev alpha bombardment. Cascade radiation at 138 kev can be seen, in addition to previously observed direct ground-state transitions at 112 and 250 kev. 90-kev transition is due to even-even contaminant (Hf[178]). 342-kev line is the usual O[18] line due to oxide target. x-x, x-γ and γ-γ coincidences were also observed (not shown). Analogous results were obtained with enriched Hf[179]. Rotational level scheme established by these measurements are shown in the insert.

for the two rotational states excited in most cases, in the manner described previously.[4] Since all gamma rays lie below 260 kev, the photopeak efficiency of our crystal is high and can be determined reliably from semiempirical procedures. However, to remove the factor ϵ in front of the transition probabilities we must know the total population of each state. This involves a knowledge of the total internal conversion coefficients for all transitions. In the absence of experimental results on these coefficients, we are forced to use theoretical values.[23] Now the transitions between the rotational states differing by one unit of angular momentum take place by magnetic dipole-electric quadrupole mixtures. These mixtures must be known in order to be able to interpolate properly between the total internal conversion coefficients for pure $M1$ and pure $E2$ radiation. Fortunately, it turns out that most transitions are predominantly of the $M1$ type, the exact amount of $E2$ admixture then being of little consequence in the determination of the total internal conversion coefficient. However, there is recent experimental[24] and theoretical[25,26] evidence to the effect that the available theoretical values for $M1$ internal conversion coefficients, computed for a point nuclear charge,[23] may be inadequate in the heavier elements, the discrepancy being possibly as high as 40% for high Z. Because of lack of better information, we have used Rose's values in the rare-earth region, making an arbitrary (downward) correction varying from 15 to 25%. Since only some K and L coefficients are available, rather uncertain interpolation and extrapolation procedures must be followed to obtain the coefficients; furthermore, the conversion in the M shell may become quite important for the very low-energy transitions, and only calculations for unscreened nuclei are available for that shell. Following Sunyar,[27] we have added about 30% of the L-shell coefficient to allow for M conversion. Admittedly, the final values for $B(E2)$ contain considerable uncertainties for these various reasons.

There are several independent means for the determination of the mixture ratios in the cascade transitions, such as angular distribution measurements[5,20,28] and the determination of empirical K to L to M conversion ratios.[3,29] No results are presently available from the former method for the rare earths. Experimenters at Copenhagen[3] and at Duke University[13] have deduced some $M1-E2$ mixtures from K/L conversion ratios, obtained in Coulomb excitation. For some of the first-excited transitions of interest, information is available from measurements with radioactive sources, (Lu[175],[30,31] Hf[177],[32] Ta[181]).[33]

We have derived values of the $E2/M1$ mixture ratios for the cascade transitions from the observed branching ratio λ, coupled with the theoretically predicted amount of $E2$ radiation in the cascade transition. A check on how justifiable this assumption might be is obtained by comparing the quadrupole moments Q_0 derived from the first and second rotational states. If they are in reasonable agreement, it is not unlikely that the same theoretical expressions[1,2] used in this case should yield the correct $E2$ transition probabilities for the cascades. Any excess of cascade radiation over the predicted $E2$ amount is then ascribed to $M1$ radiation, and a ratio can readily be obtained from the expression

$$\delta'^2 = \lambda/(\lambda^* - \lambda), \tag{1}$$

where λ^* is the theoretically predicted branching ratio which would obtain if no magnetic dipole cascade radiation occurred. λ^* is given by

$$\lambda^* = \left(\frac{2I_0+3}{I_0+2}\right)^5 \left(\frac{(2I_0+1)(I_0+3)}{2I_0^2(2I_0+3)}\right), \tag{2}$$

where we have used the theoretical (and empirically

[23] Rose, Goertzel, Spinrad, Harr, and Strong, Phys. Rev. **93**, 79 (1951); Rose, Goertzel, and Swift (privately circulated tables).
[24] A. H. Wapstra and G. J. Nijgh, Nuclear Phys. **1**, 245 (1956).
[25] L. A. Sliv, J. Exptl. Theoret. Phys. (U.S.S.R.) **21**, 770 (1951).
[26] L. A. Sliv and M. A. Listengarten, J. Explt. Theoret. Phys. (U.S.S.R.) **22**, 29 (1952).
[27] A. W. Sunyar, Phys. Rev. **98**, 653 (1955).
[28] F. K. McGowan and P. H. Stelson, Phys. Rev. **99**, 127 (1955).
[29] E. M. Bernstein and H. W. Lewis, Phys. Rev. **100**, 1345 (1955).
[30] Mize, Bunker, and Starner, Phys. Rev. **100**, 1390 (1955).
[31] Boehm, Hatch, Marmier, and Dumond, Bull. Am. Phys. Soc. Ser. II, **1**, 170 (1956).
[32] P. Marmier and F. Boehm, Phys. Rev. **97**, 103 (1955).
[33] See, e.g., F. Boehm and P. Marmier, Phys. Rev. **103**, 342 (1956).

TABLE III. Summary of results on cascade decay of rotational states populated by Coulomb excitation. μ_0=static magnetic moment of the ground state in nuclear magnetons; $E_1/E_{2-1}/E_2$=first rotational state/cascade transition/second rotational state, all in kev; λ=branching ratio (crossover cascade); δ'^2=intensity ratio $E2/M1$; α_t=assumed total internal conversion coefficient; $B(E2)$=reduced upward $E2$ transition probability in barns^2 (corrected for cascade decays); Q_0 intrinsic quadrupole moment (in barns) deduced from $B(E2)$; g_Ω=intrinsic g factor; g_R=collective g factor. Because of the uncertainty in the sign of δ', two pairs of values are given for the last two quantities. For relations used to obtain these values, see text.

Nucleus	I_0	μ_0	$E_1/E_{2-1}/E_2$	λ	δ'^2	$(1+\alpha_t)$	$B(E2)$	Q_0	$\delta'>0$ g_Ω	$\delta'>0$ g_R	$\delta'<0$ g_Ω	$\delta'<0$ g_R
$_{63}Eu^{153}$	5/2+	+1.5[a]	82 105 187	1.62	0.38	5.1 2.7 1.2	2.8 0.83	7.6 7.1	0.68	0.41	0.52	0.79
Gd^{155}	3/2−	−0.30[b]	(61) 84 145	0.26	0.027	... 3.5 1.9	... 1.3	... 6.8	0.19	−0.78	−0.59	0.38
Gd^{157}	3/2−	−0.37[b]	(55) 76 131	0.27	0.028	... 4.3 2.2	... 1.1	6.2	0.066	−0.72	−0.56	0.22
$_{65}Tb^{159}$	3/2+	+1.5[c]	(57) 79 136	0.13	0.013	4.3 2.0	... 2.2	... 8.7	1.66	0.01	0.34	1.99
$_{67}Ho^{165}$	7/2−	+3.3[d]	94 112 206	0.16	0.039	3.4 2.3 1.2	2.5 0.52	7.6 6.9	1.10	0.39	0.78	1.49
$_{71}Lu^{175}$	7/2+	+2.9[a]	114 136 250	0.50	0.135	3.1 3.0 1.1	2.5 0.78	7.6 8.5	0.92	0.50	0.73	1.16
$_{72}Hf^{177}$	7/2−	+0.61[e]	112 138 250	2.62	1.64	3.7 2.1 1.1	3.4 0.56	8.9 7.2	0.21	0.06	0.14	0.29
$_{72}Hf^{179}$	9/2+	0.47[e]	119 141 260	0.44	0.153	3.5 3.0 1.1	2.6 0.23	8.3 5.5	−0.04	−0.40	−0.17	0.20
$_{73}Ta^{181}$	7/2+	+2.1[a]	136 167 303	0.36	0.13	3.1 2.1 1.0	2.10 0.37	7.1 5.9	0.73	0.15	0.47	1.05

[a] H. Hopfermann, reference 10, p. 438.
[b] D. R. Speck, Phys. Rev. **101**, 1725 (1956).
[c] J. M. Baker and B. Bleaney, Proc. Phys. Soc. (London) **A68**, 257 (1955).
[d] J. M. Baker and B. Bleaney, Proc. Phys. Soc. (London) **A68**, 1090 (1955).
[e] D. R. Speck and F. A. Jenkins, Phys. Rev. **101**, 1831 (1956), and private communication.

confirmed) energy ratios of rotational states. The theoretically predicted mixture ratios for the first excited state (δ) and second excited state cascades (δ') are very nearly the same. In Table IV we give the quantities λ^* as well as $\rho=\delta^2/\delta'^2$ for all ground state spins $I_0>\frac{1}{2}$ encountered:

$$\rho=\left(\frac{I_0+1}{I_0+2}\right)^3\left(\frac{I_0+3}{I_0}\right). \tag{3}$$

We also list the factors, K_{I_0+1} and K_{I_0+2}, by which the quantities $[B(E2)]^{\frac{1}{2}}$ of the first rotational state $(B_{I_0+1})^{\frac{1}{2}}$, and second rotational state $(B_{I_0+2})^{\frac{1}{2}}$, must be multiplied to obtain the intrinsic quadrupole moments Q_0:

$$Q_0=\left[\frac{16\pi(I_0+1)(I_0+2)}{15I_0}\right]^{\frac{1}{2}}(B_{I_0+1})^{\frac{1}{2}};$$
$$Q_0=\left(\frac{8\pi(2I_0+3)(I_0+2)}{15}\right)^{\frac{1}{2}}(B_{I_0+2})^{\frac{1}{2}}. \tag{4}$$

For the cascade from the second rotational state, the theoretical expression for the downward reduced magnetic dipole transition probability, $B(M1)$, for a nucleus of ground state spin $I_0>\frac{1}{2}$ is given by[1,2]

$$B(M1)=\frac{3}{4\pi}\left(\frac{e\hbar}{2Mc}\right)^2(g_\Omega-g_R)^2\left(\frac{4I_0^2(I_0+1)}{(I_0+2)(2I_0+5)}\right), \tag{5}$$

where $e\hbar/2Mc$ is the nuclear Bohr magneton, and g_Ω and g_R are the intrinsic and collective g factors, respectively. Combining this expression with that for $B(E2)$, we obtain a convenient expression of the amplitude mixture ratio δ' of the cascade:

$$\delta'=\pm\frac{0.933EQ_0}{(g_\Omega-g_R)[(I_0+1)(I_0+3)]^{\frac{1}{2}}}, \tag{6}$$

where E is the energy of the cascade transition in Mev, and Q_0 is expressed in barns. Since we have no information on the *sign* of δ' (only angular correlation experi-

TABLE IV. Summary of numerical factors derived from the unified model for strongly deformed nuclei with $I_0>1/2$. λ^* = theoretical branching ratio (crossover/cascade) if cascade were pure $E2$ [see Eq. (2)]; ρ = ratio of mixture ratios for upper and lower cascade transitions, δ^2/δ'^2, where $\delta^2=\text{Int.}(E2)/\text{Int.}(M1)$ [see Eq. (3)]; for definitions of K_{I_0+1} and K_{I_0+2}, see Eqs. (4) and text.

Ground state spin	λ^*	ρ	K_{I_0+1}	K_{I_0+2}
3/2	9.88	1.093	4.42	5.94
5/2	5.88	1.035	4.59	7.76
7/2	4.22	1.017	4.87	9.60
9/2	3.31	1.010	5.16	11.42

ments can yield the sign), we can determine only the absolute value of $|g_\Omega-g_R|$. According to the strong-coupling approximation of the unified model.[1,2] on which all the above relations are based, the ground-state static magnetic moment is given by

$$\mu_0=\frac{I_0}{I_0+1}(g_\Omega I_0+g_R). \tag{7}$$

Upon solving Eqs. (6) and (7) explicitly for g_Ω and g_R, we obtain

$$\begin{aligned} g_\Omega&=\pm\frac{0.933EQ_0}{[(I_0+1)(I_0+3)]^{\frac{1}{2}}(I_0+1)\delta'}+\frac{\mu_0}{I_0},\\ g_R&=\frac{\mu_0}{I_0}\mp\frac{0.933EQ_0I_0}{[(I_0+1)(I_0+3)]^{\frac{1}{2}}(I_0+1)\delta'},\end{aligned} \tag{8}$$

where the upper sign applies for $\delta'>0$, and the lower sign for $\delta'<0$. Since we do not know the sign of δ', two pairs of solutions will be obtained and are given in Table III. The values for the g factors are in general not very sensitive to the values of Q_0 and δ', since the quantity μ_0/I_0 is usually considerably larger than the other terms in Eqs. (8).

From Table III we can see that the values Q_0 derived for the two rotational states are in fair agreement, considering the uncertainties involved in the internal conversion corrections discussed earlier. We have used the average of the two values for Q_0 in Eqs. (8). Our values for δ'^2 agree rather well with this quantity for the first rotational transition as obtained from radioactivity and Coulomb excitation. Table V compares these values; some experimentally somewhat uncertain values are also available[13] for the upper cascade from K/L ratios in Coulomb excitation; they seem to be somewhat smaller than ours. On the whole the agreement with the theoretically predicted values is quite satisfactory. This indicates (a) that the amount of $E2$ radiation in the upper cascade is correctly given in terms of the same intrinsic quadrupole moment Q_0 as the other two transitions and (b) that the amounts of $M1$ radiation in both cascades are quite comparable, as seen from Eq. (3) and Table IV.

It is in principle possible to calculate g_Ω theoretically

TABLE V. Comparison of mixture ratios for odd-A cascade transitions where other methods are available. $\delta^2=\text{Int.}(E2)/\text{Int.}(M1)$ for $I_0+1\rightarrow I_0$ transition; δ'^2 = same quantity for $I_0+2\rightarrow I_0+1$ cascade transition (I_0 = ground state spin). I.C. by internal conversion electron ratio measurement (K/L or $L_I/L_{II}/L_{III}$); A.D. by angular distribution measurement; B.R. by gamma-branching ratio for second excited state (see text).

	δ^2		δ'^2		
Nucleus	I.C.[a]	I.C.	I.C.[a]	A.D.	B.R.
$_{63}Eu^{153}$	0.48	>1[b]	$\sim$0.08	...	0.38
$_{64}Gd^{155}$	$\sim$0.05	$\sim$0.06[b]	...	...	0.027
$_{64}Gd^{157}$	$\sim$0.04	...	...	...	0.028
$_{67}Ho^{165}$	0.044	...	$\sim$0	...	0.039
$_{71}Lu^{175}$	0.11	0.18[c]	$\sim$0.08	...	0.135
$_{72}Hf^{177}$	...	$\sim\infty$[d]	...	...	1.64
$_{73}Ta^{181}$	0.05	0.11[e]	$\sim$0	0.26[f]	0.13

[a] From K/L ratio measured in Coulomb excitation. Gd nuclei from reference 12, all others from reference 13.
[b] E. L. Church, Bull. Am. Phys. Soc. Ser. II, 1, 180 (1956) (radioactive decay).
[c] References 30 and 31 (radioactive decay).
[d] Reference 32 (radioactive decay).
[e] Reference 33 (radioactive decay).
[f] Reference 28 (Coulomb excitation).

from the ground-state wave functions for spheroidal nuclei[34,35] so that the correct pair of values of g_Ω and g_R could then be selected. One of the main factors limiting the accuracy of these quantities is our knowledge of μ_0 (from optical spectroscopy or paramagnetic resonance).

For irrotational flow of uniformly charged nuclear matter we expect that $g_R\cong Z/A\sim 0.4$; it is well established, however, that the flow is far from irrotational,[36] and it is not surprising to find deviations of g_R from this value. The interpretation of the observed values of g_Ω and g_R is uncertain at present; attempts have been made to correlate these quantities with the effective moments of inertia[37]; the odd particle plays an essential role in these considerations (as do the other particles outside of closed shells) since it seems to contribute considerably more than its proportionate share of moment of inertia to the motion of deformed nuclei. Within the accuracy of available measurements it is not clear whether a correlation exists between the nature of the odd particle and the direction of the deviation of g_R from Z/A.

To sum up, then, we have put the level sequences in seven rare-earth nuclei of odd mass on a firm basis by various coincidence experiments. We have also determined the gamma-ray branching ratios in the decay of second rotational states; using estimates of the total internal conversion coefficients, we determined the total reduced $E2$ transition probabilities to the first two rotational states, as well as the $M1$ contribution to the cascade de-excitation of the second state, using the strong-coupling Bohr-Mottelson model. Within the

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[35] K. Gottfried, Phys. Rev. **103**, 1017 (1956).
[36] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **30**, No. 1 (1955).
[37] S. A. Moszkowski, Phys. Rev. **103**, 1328 (1956), and private communication.

rather large experimental uncertainties we found no departures from the predictions of this model.

Further evidence was obtained for the drastic change in the type of nuclear excitation spectra as we move from neutron number 88 to 90 in the odd europium isotopes. These last two neutrons are presumably just sufficient to induce a nonspherical equilibrium deformation in Eu^{153}.

V. ACKNOWLEDGMENTS

We are indebted to L. Grodzins (Brookhaven) for his generous loan of the europium isotopes and to C. P. Keim (Oak Ridge) for all separated isotopes used in our work. We are grateful to S. A. Moszkowski for informative discussion, and to E. M. Bernstein and G. Goldring for sending us results in advance of publication.

PHYSICAL REVIEW VOLUME 104, NUMBER 4 NOVEMBER 15, 1956

Contribution of Coulomb Excitation to Inelastic Scattering between Nuclear Resonances*

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(Received August 3, 1956)

Using the comparison method of Bjerregaard and Huus, we have determined the relative gamma-ray yields for thin targets (~10 kev) of F^{19} (198-kev state) and Na^{23} (446-kev state), with protons and alpha particles over energy intervals corresponding to the same values of ξ, the adiabatic parameter in Coulomb excitation. Between the proton resonances we found that the yield ratios agreed closely with those predicted for $E2$ Coulomb excitation, and differed by about 50% from $E1$ (or $E3$) excitation. Thus the nonresonant inelastic scattering yield between proton resonances in these light nuclei can be entirely accounted for by Coulomb excitation of the $E2$ type. We determined a slight, negative anisotropy in the angular distribution of the 446-kev gamma ray following Coulomb excitation of Na^{23}, fixing the spin of that state at $5/2^+$.

I. INTRODUCTION

BJERREGAARD and Huus,[1] following a suggestion of Bohr and Mottelson, have shown that it is possible to determine the (electric) multipolarity of a given Coulomb-excited transition by a simple comparison of yields obtained with particles of different charge-to-mass ratios. By choosing the bombarding energies judiciously, it is possible to eliminate the usually complicated functions $f_{E\lambda}(\eta,\xi)$,[2,3] λ being the order of the multipolarity, $\eta=Z_1Z_2e^2/\hbar v$, and ξ the well-known adiabatic parameter of Coulomb excitation. The ratio will then only contain known quantities, and will be "quantized" in terms of the parameter λ. Bjerregaard and Huus confirmed the usefulness of this method for the known 0^+–2^+ pure electric quadrupole transitions in the even-even isotopes of wolfram, using protons, deuterons and alpha particles.[1] We have previously reported the use of this method in establishing the $E2$ nature of the excitation of the 128-kev excited state in Mn^{55}.[4] The spin of this state has recently been determined as $7/2^+$ by internal conversion and angular distribution measurements under Coulomb excitation.[5]

It has been shown that for a value of η as low as 2 the semiclassical expression ($\eta\rightarrow\infty$) for the total Coulomb excitation cross section falls only about 10% below the exact quantum-mechanical value,[2,3] as long as the semiclassical expression is slightly modified in accordance with the WKB approximation.[6] We shall therefore assume that the functions $f_{E\lambda}(\infty,\xi)^2$ are adequate for our purposes, since the smallest value of η encountered in any case was 1.63 (0.76-Mev protons of F^{19}). η was always greater than 4 for alpha particles, and the deviations from the semiclassical expression amounted to less than 2%.

For the sake of illustration, let us assume the case of negligible energy transfer, i.e., $\Delta E\ll E$, where E is the bombarding energy. Let us also neglect the center-of-mass motion. If we now choose bombarding energies for protons and alpha particles such that the parameter ξ has the same value ξ^* in both cases, we obtain the following simple expression for the ratio of Coulomb excitation cross sections:

$$\sigma_\alpha(\xi^*)/\sigma_p(\xi^*)=2^{(2\lambda/3)+2}, \tag{1}$$

* A preliminary report of the results in this note can be found in Phys. Rev. **98**, 1198 (1955).
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