only the lowest states n are excited. Let us expand u_{nn} in powers of x, and here (6) gives the result

$$au_{nn}=A_nx+B_nx^2+O(x^3),$$
$$A_n=3(n+\tfrac{1}{2}),\quad B_n=(7/2)[n^2+n+(13/42)]. \quad (8)$$

For the low states $nx\ll 1$, we have

$$E_n\cong\hbar\omega(n+\tfrac{1}{2}),$$
$$u_{nn}=(3x/a)(E_n/\hbar\omega)+O(x^2), \quad (9)$$

and thus at low temperatures ($T<$Debye temperature), Eqs. (7) and (9) give the result

$$\bar{u}(T)=\frac{3x}{a\hbar\omega Z}\sum_n E_n\exp(-E_n/kT)+O(x^2)$$
$$=\frac{3x}{a\hbar\omega}\bar{E}(T)+O(x^2). \quad (10)$$

It follows from this that at low temperatures the coefficient of thermal expansion is given by the expression

$$\alpha=\frac{1}{l_0}\frac{d}{dT}\bar{u}(T)=\frac{3x}{al_0\hbar\omega}\bar{C}(T)+O(x^2), \quad (11)$$

where $\bar{C}(T)$ is the specific heat of an oscillator at temperature T. It is clear that the expansion in powers of x and the approximation (9) is not necessary in the present method, except as a demonstration that the inference (3) is correct for the present model to terms of order x, and also to show how correction terms of order x^2 can be calculated. At high temperatures, where states of large quantum number n are excited so that (9) is no longer a good approximation, a quantum-mechanical calculation is probably not necessary; but it may actually be easier to carry out the present quantum-mechanical calculation than the corresponding classical treatment.[2]

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Resonance and Reversal Phenomena in Ferromagnetic Films*

R. L. Conger and F. C. Essig
United States Naval Ordnance Laboratory, Corona, California
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For fields several times the coercive force, magnetization reversal in thin films takes place by domain rotation. A phenomenological theory based on a modification of the Landau-Lifshitz equation of motion shows that the reversal time for this type of magnetization reversal will be proportional to the damping constant of the modified equation of motion. This damping constant is in turn proportional to the ferromagnetic resonance absorption line width. Experimental measurements on evaporated films of three different alloy compositions demonstrate this predicted proportionality of the reversal time and the resonance line width.

The shift in the value of the resonant field strength at constant frequency as a function of film thickness was also investigated. This shift can be attributed to a decrease in the saturation magnetization, $4\pi M_0$. The results indicate that the saturation magnetization remains reasonably constant down to a film thickness of about 10^{-5} cm, and then decreases rapidly. The resonance line widths of the thickest and the thinnest films tested of a given composition were about the same.

I. INTRODUCTION

In recent years there has been considerable interest in the magnetic properties of thin films and ribbons of the ferromagnetic elements and their alloys. The reason for this current interest is twofold: (1) the physics of domain structure and magnetization reversal mechanisms in thin films is not well understood and existing theories do not explain all observed phenomena; (2) thin films of magnetic material show promise as core materials for gating devices and two-dimensional magnetic memory matrices.

One key to better understanding of these processes is a meaningful correlation between directly observed magnetization reversal times in thin films and reversal times deduced from a phenomenological equation of motion of the magnetization vector. The present paper presents an approach to this problem.

The time required for the reversal of the direction of magnetization in thick sheets of conducting ferromagnetic material is inversely proportional to the square of the thickness because of eddy current effects, but it has been found that the reversal times of films less than about 3×10^{-4} cm thick are independent of film thickness. A ferromagnetic damping effect rather than eddy-current effects appears to control the reversal process of such thin films. It is hypothesized that the absorption of energy in the ferromagnetic resonance experiment is brought about by the same damping mechanism and that, as a result, the reversal time of thin films is related to ferromagnetic resonance line

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width, which is a measure of the resonance energy absorption. Moreover, the reversal time will also depend on the path of the reversal process, that is, the positions and motions of domain walls and the amount of reversal by domain rotation. The path of the reversal process is in general quite complicated, but there are a few cases in which the path is relatively simple. It was demonstrated in a previous experiment[1] that magnetization reversal in very thin films can take place by a simple domain rotation. Later experiments show that this is the mechanism only when the reversing field is greater than a certain critical value.

The ferromagnetic resonance experiment is very useful in the study of ferromagnetism because the large magnetic field used in the experiment causes the specimen to be magnetized as a single domain, and thereby simplifies the interpretation of the experimental results. The experiment is particularly well suited to the study of thin films of conducting ferromagnetic materials because of the small skin depth of the microwave fields used in the experiment. Since the microwaves penetrate only a short distance, the thinness of the film does not cause any difficulty.

Although a considerable amount of comprehensive work[2,3] has been done on magnetization reversal in thin rolled ribbon, very little is applicable to this work on evaporated films, since the thinnest rolled ribbon is an order of magnitude greater in thickness than the films employed in these measurements. There exist few published references to the magnetics of evaporated films, but the authors acknowledge the definitive work on the theory of domain structure in thin films by Kittel,[4] some interesting measurements published by Griffiths on the ferromagnetic resonance of thin films of nickel evaporated onto mica substrata,[5] the theory of Klein and Smith on the magnetization of thin films,[6] and the work of Fowler and Fryer on the domain structure of thin films.[7]

In this paper, a theory is presented which correlates the ferromagnetic resonance line width with the reversal time of thin films for reversing fields greater than the above critical value. Experimental results of measurements of reversal times and resonance line widths are presented for a series of alloys of iron, cobalt, and nickel. These experiments give support to the theory.

In addition to the relationship between the reversal time and the resonance line width, the effect of film thickness on the resonance absorption curve is presented and the effect of magnetic anisotropy on the reversal times and hysteresis loops of the film is considered.

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II. PREPARATION OF FILMS

The thin films of magnetic material were prepared by vacuum evaporation from premelted slugs of the alloy under investigation. Reagent grade metals (iron, nickel, and cobalt) were combined in the pre-melt in the proper ratio to yield the desired percentages in the film. The percentages in the slug must be different from those desired in the film since the constituents of the alloy have different distillation rates. The evaporation technique has been described in detail by Blois[8]; therefore, the authors will present only an outline of the process in this paper.

An alumina crucible is charged with the pre-melt slug and placed within the coil of a 4-kw radio-frequency induction heater. A soft glass substrate is fixed in position 10 inches above the crucible, flush against a substrate heater. The apparatus is enclosed in a bell jar which is subsequently pumped down to a pressure of from 10^{-5} to 10^{-4} mm Hg, this pressure being adequate to assure that the mean free path of the evaporated metal atoms is considerably longer than the distance from crucible to substrate. Once the system has reached the given pressure, the substrate is heated to a uniform temperature of from 300°C to 400°C. It is found that this increases the adherence of metal to glass and tends to anneal the film. The thickness obtained during the course of the evaporation is monitored by observing the change in resistivity of the slide as the film builds up. An evaporating time the order of a minute is required to build a film 5000 angstroms in thickness. The films thus obtained and employed in the following experiments have a mirror-like surface, and the electrical conductivity measurements mentioned above seem to indicate that the films, at least at the thicknesses employed here, are continuous layers. Thickness and chemical composition of the films are determined by colorimetric chemical analysis. A few thickness measurements were checked by optical interference methods. These optical measurements were within a few percent of those made by chemical analysis. Hysteresis loops of the film are determined by the use of an apparatus similar to that used by Crittenden.[9] The films have coercive forces of a few oersteds.

III. RESONANCE MEASUREMENTS

Ferromagnetic resonance measurements were performed in order to determine the value of the phenomenological damping constant, α, in the Landau-Lifshitz equation[10]

$$d\mathbf{M}/dt=\gamma(\mathbf{M}\times\mathbf{H})-(\alpha\gamma/M)[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})], \quad (1)$$

[8] M. S. Blois, J. Appl. Phys. **26**, 975 (1955).
[9] Crittenden, Hudimac, and Strough, Rev. Sci. Instr. **22**, 872 (1951).
[10] L. Landau, and E. Lifshitz, Physik. Z. Sowjetunion **8**, 153 (1955).

where γ is the gyromagnetic ratio, **M** is the saturation magnetization, and **H** is the effective magnetic field. Equation (1) is assumed to be the basic equation of motion. A calculation similar to that of Yager *et al.*[11] shows that the damping constant, α, is related to the half-width of the μ_2 *vs* H curve at half its maximum amplitude, ΔH, by

$$\alpha \approx \gamma \Delta H/\omega. \tag{2}$$

μ_2 is the imaginary part of the isotropic permeability $\mu=\mu_1-j\mu_2$ and ω is the resonant frequency. However, due to the skin effect, the microwave field penetrates only 10^{-5} to 10^{-4} cm into the film, and as a result the resonance measurements yield the dissipative part, μ_r, of the effective permeability rather than the imaginary part, μ_2, of the isotropic permeability on which Eq. (2) is based. Kittel[12] shows that the relationship between μ_r and the two terms μ_1 and μ_2 is given by

$$\mu_r=[\mu_1^2+\mu_2^2]^{\frac{1}{2}}+\mu_2. \tag{3}$$

The relationship between μ_1 and μ_2 near resonance can be obtained from the equation of motion (1). From this result and (3) it can be shown that when the μ_2 curve drops to half its maximum amplitude near resonance, the μ_r curve drops to approximately 0.60 of its maximum amplitude if it is assumed that the resonance value of μ_2 and also the values of μ_1 and μ_2 at half-resonance are much greater than 1.0. Experimental evidence[13] indicates that this is the case.

The determination of the experimental (μ_r) resonance curve will now be described. The ferromagnetic resonance apparatus is similar to that used by Bloembergen[13] in his work with supermalloy and nickel. The measurements to be described have been made at X band (9300 mc/sec). The use of two high-grade directional couplers and a ratio meter have allowed direct and continuous measurement of the fraction of the energy reflected from the cavity. The reflection coefficient is defined as

$$\rho_i=(P_r/P_i)^{\frac{1}{2}}, \tag{4}$$

where P_r and P_i are the reflected and incident power respectively. The ratio meter measures the quantity $|\rho_i|$. The voltage standing wave ratio is defined as

$$\beta=(1+|\rho_i|)/(1-|\rho_i|). \tag{5}$$

It can be shown by following in detail the text of Slater[14] that the voltage standing-wave ratio (VSWR) is related to the external Q and the unloaded Q of the cavity into which the microwave system is feeding power. Specifically, at microwave resonance,

$$\beta=Q_x/Q_a, \tag{6}$$

where Q_x is the external Q of the cavity and Q_a is the unloaded Q of the cavity. Q_a takes into account losses in the sample as well as in the other components of the cavity. The VSWR, β, is determined from the experimentally determined values of ρ_i, with the aid of Eq. (5).

To establish a base line for the resonance curve, a value β' is taken at a high field, H_z, where no ferromagnetic resonance is occurring. Again referring to the work of Yager,[11] the value $1/Q_a-1/Q_a'$ represents the energy dissipated in the ferromagnetic medium, where Q_a' is measured at the high magnetic field. This quantity is proportional to μ_r, the imaginary part of the effective permeability.

$$1/Q_a-1/Q_a'=K\mu_r=(\beta-\beta')/Q_x, \tag{7}$$

where K is a constant. The increase in field strength needed to decrease μ_r from $(\mu_r)_{\max}$ to 0.6 $(\mu_r)_{\max}$ is needed in order to calculate α. Equation (7) shows that μ_r will be a maximum for β maximum, and will decrease proportionally with β. Experimental values of α were determined from measurements of the half-width of the β curve at 0.60 of maximum resonance amplitude, ΔH. Table I shows the values of α calculated for three alloy films, each of which was between 10^{-4} and 2×10^{-4} cm in thickness. The microwave frequency was 9300 mc/sec. At this frequency the microwave skin depth is less than 10^{-4} cm. The values of g for these alloys, needed to determine the gyromagnetic ratio, γ, are not known precisely. For the (80% nickel, 20% iron) film, $g=2.16$, as obtained by Bloembergen[13] for mopermalloy, was used. For the (85% cobalt, 15% iron), g for pure cobalt,[15] 2.22, was assumed. No data on g factors for the perminvar system (FeCoNi) were found in the literature, but it is reasonable to assume that it will be close to that for permalloy (2.16), and the value is used here for the (48% nickel, 22% cobalt, 30% iron) film.

Figure 1 shows in detail the resonance curve for (80% Ni, 20% Fe).

TABLE I. Phenomenological damping constants as calculated from ferromagnetic resonance data for three evaporated film alloys.

Alloy	ΔH (gauss)	α
80% Ni, 20% Fe	60	0.0195
48% Ni, 30% Fe, 22% Co	90	0.0292
85% Co, 15% Fe	330	0.1102

IV. SATURATION MAGNETIZATION OF EVAPORATED FILMS OF MAGNETIC ALLOYS

The saturation magnetization of minute samples of magnetic materials may be obtained directly from ferromagnetic resonance data. A principal result of the ferromagnetic resonance theory is the expression relating the static magnetic field, the magnetization, and

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[13] N. Bloembergen, Phys. Rev. **78**, 572 (1950).
[14] J. C. Slater, *Microwave Electronics* (D. Van Nostrand and Company, New York, 1950).
[15] R. M. Bozorth, *Ferromagnetism* (D. Van Nostrand and Company, Inc., New York, 1951).

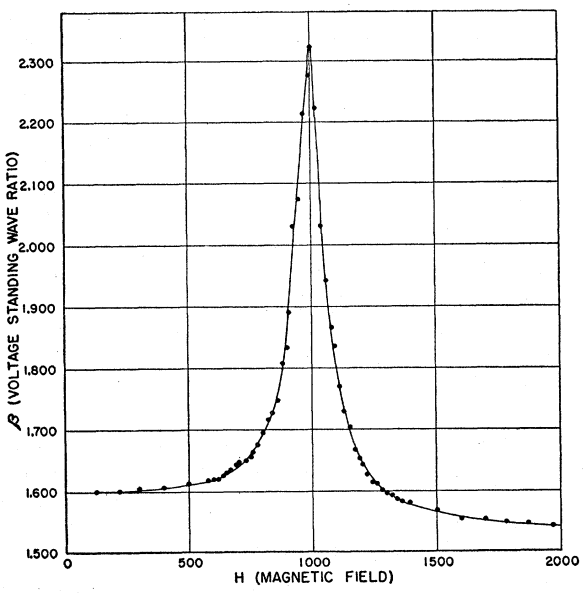

FIG. 1. Ferromagnetic resonance line for evaporated film of (80% nickel, 20% iron) 10 000 angstroms thick. (The magnetic field was taken to 7000 gauss, with β remaining the same as that shown for $H=2000$ gauss.)

the Larmor (applied rf) frequency at the point of resonance:

$$\omega^2=\gamma^2H_0(H_0+4\pi M_0). \tag{8}$$

This expression is admittedly an approximation, since it does not take into account the effects of crystalline anisotropy. It is felt that the anisotropy is small for the alloys considered.

Ferromagnetic resonance curves were obtained for five different thicknesses of the (80% nickel, 20% iron) alloy. The measurements were made in the X-band region ($f=9305\pm2$ mc/sec). The purpose of this set of measurements was to determine the way in which saturation magnetization and ferromagnetic resonance line width vary as a function of film thickness. Figure 2 shows $4\pi M_0$ as a function of film thickness. It will be noticed that the magnetization is fairly constant down to thicknesses the order of 1000 A; below this value $4\pi M_0$ decreases very rapidly.

Klein and Smith[6] have calculated by means of the Bloch spin-wave theory the dependence of the spontaneous magnetization of thin films upon the number of atomic layers. Below a critical thickness, which depends on the temperature and film dimensions, the magnetization decreases rapidly. This is verified by the results obtained with ferromagnetic resonance as shown in Fig. 2. Crittenden and Hoffman[16] show a plot of normalized magnetization *vs* thickness (number of atomic layers) for Ni films. From their data the value of $4\pi M_0$ becomes essentially equal to that of bulk material in the neighborhood of 200 atomic layers. The results obtained with ferromagnetic resonance show that for (80% Ni, 20% Fe) films, the knee of the curve would occur around 1000 angstroms for the (80% Ni, 20% Fe) films, corresponding to about 500 atomic layers. The reason that this value of critical thickness differs from that obtained by other techniques for 100% nickel films is not known, except that the film composition may be a factor. It can be seen, however, that there is qualitative agreement in that both show a sharp decrease in saturation induction for very thin films.

[16] E. C. Crittenden and R. W. Hoffman, Revs. Modern Phys. **25**, 310 (1953).

V. DIRECT MEASUREMENT OF MAGNETIZATION REVERSAL TIMES

The magnetization reversal times of thin evaporated films of some alloys of iron, cobalt, and nickel as a function of the applied reversing field have been measured, and some of the results are reported here. In this experiment the films are first magnetized in a certain direction, and then a reversing magnetic field in the form of a rectangular pulse is applied in the opposite direction. The time required for the magnetization of the film to be reversed by the action of this pulse is the reversal time. The reversal and reset pulses had rise times much less than the magnetization reversal time of the magnetic film. In view of this the reversal pulse may be considered a step function.

A pulse generator is connected to a 50-ohm terminating resistor in series with a driving coil. The driving coil consists of a 12-turn solenoid wound around a $\frac{1}{4}$ in. by $\frac{1}{4}$ in. square cut from the evaporated film and its glass substrate. The rise time of the current pulse through the drive coil is increased by the inductance of the coil to several millimicroseconds. A field in the opposite direction to return the core to its initial condition after each pulse is provided by a reset pulse of opposite polarity supplied by the same pulse generator.

When the direction of magnetization in the film reverses, a voltage pulse is induced in a one-turn pickup winding placed around the film. The voltage from this pickup winding is amplified and then observed on a

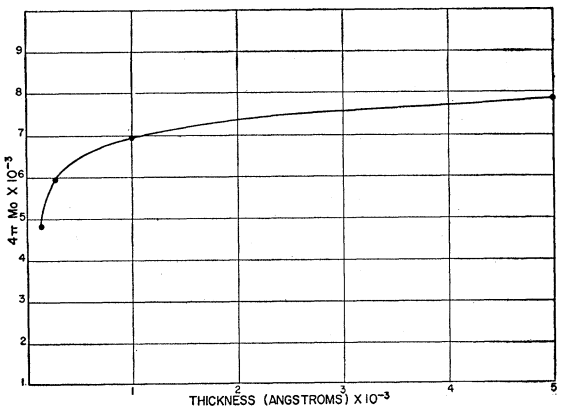

FIG. 2. Saturation magnetization as a function of film thickness for the (80% nickel, 20% iron) composition.

cathode-ray oscilloscope, from which the reversal time measurements are obtained directly.

In using this apparatus, the oscilloscope trace is observed first with the reset pulse omitted. For this case there will be no magnetization reversal. The difference between this result and the trace when the reset pulse is used is the reversal pulse.

Figure 3 shows the type of result obtained with this apparatus for a film of (85% Co, 15% Fe), 2500 A in thickness. Trace (a) shows the reversal pulse; trace (b) the nonreverse condition. Since the calculations indicate that the reversal time is actually infinite [see Eq. (16)], the experimental switching time is defined as the pulse width at half-amplitude.

VI. PHENOMENOLOGICAL THEORY OF MAGNETIZATION REVERSAL

This theory is applicable only to thin sheets of ferromagnetic alloys in which eddy-current effects can be ignored. Eddy-current effects cause the reversal times of thick sheets of conducting ferromagnetic material to be inversely proportional to the square of the thickness of the sheet, but it has been found that the reversal times of films less than about 3×10^{-4} cm thick are independent of film thickness. Ferromagnetic relaxation effects rather than eddy-current effects appear to control the reversal process of such thin films.

The reversal time will depend on the mechanism of the reversal process. As shown by one of the authors,[1] magnetization reversal in thin films appears to take place by domain rotation for sufficiently large fields. This theory is based on the model of reversal by domain rotation with the reversal time controlled by ferromagnetic relaxation effects.

When a large magnetic field is applied to a ferromagnetic material, the magnetization vector of the material executes a damped precessional motion about an axis parallel to the direction of the applied field. A large damping term in the equation of motion for magnetization will damp out the precession quickly and allow the magnetization vector to approach the direction of the applied field rapidly. But if a small field rather than a large field is applied to the ferromagnetic material, the system may become overdamped and precession will not take place. Under these circumstances a large damping term will decrease the rate at which the magnetization vector approaches the direction of the applied field. Thus the action of the damping term at low fields is opposite to its action at high fields.

The demagnetizing field normal to the surface of a flat plate of ferromagnetic material will tend to restrain the magnetization vector to the plane of the film. As a result, the precessional motion at high field strengths has a superimposed wobble while at low field strengths the motion is almost completely a rotation in this plane. Therefore, the equation developed for the low-field case is an equation for rotation in the plane of the film.

The effect of anisotropy can be ignored for isotropic materials like permalloy but for other materials it cannot be ignored. It is shown that the effect of anisotropy is to increase the experimentally determined reversal time at low fields.

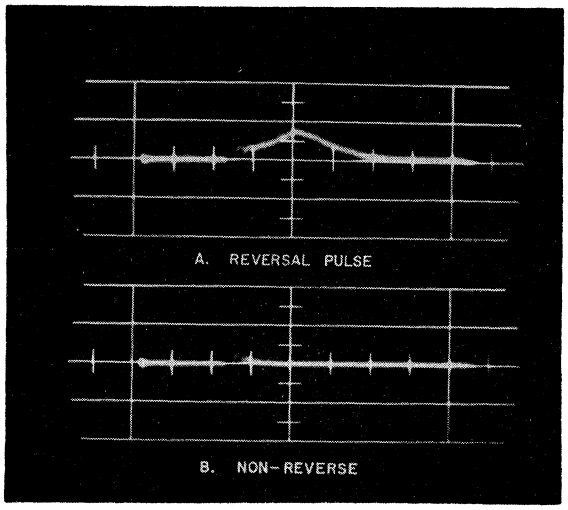

FIG. 3. Reversal signal for 10 000 A (85% cobalt, 15% iron) film. (a) Reversal signal; (b) nonreversal condition.

The mathematical details are now presented.

Let the film be in the x, z plane and the reversing field H_r be in the z direction. There will be a demagnetizing field $-4\pi M_y$ normal to the film surface. The total field H, to be used in the Landau-Lifshitz equation of motion (1), is

$$\mathbf{H}=-4\pi M_y\mathbf{j}+H_r\mathbf{k},$$

where $\mathbf{j}$ and $\mathbf{k}$ are unit vectors in the y and z directions. The magnetization will stay almost completely in the plane of the specimen. In this case,

$$M_x^2+M_z^2\approx M_0^2. \tag{9}$$

The introduction of the angle θ gives the direction of magnetization in the plane of the specimen:

$$M_x=M_0\sin\theta, \tag{10}$$

$$M_z=M_0\cos\theta. \tag{11}$$

Then

$$\dot{M}_x^2+\dot{M}_z^2=M_0^2\dot{\theta}^2. \tag{12}$$

Values of M_x and M_z can be substituted from the equations of motion. Making use of (9) and $H_r\ll4\pi M_0$, one finds

$$\dot{\theta}=\gamma(4\pi M_y-\alpha H_r M_x/M_0).$$

This expression is differentiated, and again use is made of the Landau-Lifshitz equation, where it is assumed that

$$4\pi M_y^3\ll4\pi M_0^2M_y \quad\text{and}\quad M_xH_rM_y\ll4\pi M_0^2M_y.$$

In this way one obtains finally

$$\ddot{\theta}=-4\pi\gamma^2M_0H_r(1+\alpha^2)\sin\theta-\alpha\gamma\dot{\theta}4\pi M_0.$$

Assume, in agreement with experimental observation, that α is so large that the system is overdamped. The term $\ddot{\theta}$ may then be dropped, and

$$\dot{\theta}+[\gamma H_r(\alpha^2+1)\sin\theta]/\alpha=0. \tag{13}$$

This equation describes the magnetization reversal. Since experimentally the quantity $\dot{M}_z$ is measured, Eq. (13) is rewritten as

$$\dot{M}_z/(M_0^2-M_z^2)=\gamma H_r(\alpha^2+1)/M_0\alpha. \tag{14}$$

Integration gives

$$M_z=M_0\tanh\left\{\frac{\gamma H_r}{\alpha}(\alpha^2+1)(t-t_0)\right\}, \tag{15}$$

where t_0 is the time that the z component of magnetization passes through zero. The derivative

$$\dot{M}_z=\frac{\gamma H_r M_0(\alpha^2+1)}{\alpha}\operatorname{sech}^2\left[\frac{\gamma H_r(\alpha^2+1)(t-t_0)}{\alpha}\right] \tag{16}$$

has a maximum at $t=t_0$ and is approximately Gaussian around this point. The tails are exponential. The reversal time T will be defined as the width of the curve (16) where $\dot{M}_z$ has been dropped to half its maximum value,

$$T=1.76\alpha/[\gamma H_r(\alpha^2+1)]. \tag{17}$$

In all the previous calculations the effects of anisotropy have been ignored. This is justified for low-anisotropy alloys like permalloy, but is not justified for some other alloys.

For example, it has been found that films of (85% Co, 15% Fe) and (48% Ni, 30% Fe, 22% Co) evaporated in the presence of a magnetic field have very definite hard and easy directions of magnetization, even though x-ray diffraction patterns show that the films are polycrystalline with small crystals oriented randomly. The direction of easy magnitization is rotated 90° with respect to the hard direction, in the plane of the film.

This uniaxial anisotropy is probably the result of an interaction of strain and magnetostriction. The films are prepared by evaporating magnetic alloys onto soft glass substrates placed in a magnetic field of a few oersteds and heated to between 300°C and 400°C. The magnetic field will cause the films to be magnetized in one direction as they form. As the films cool they will be strained because of the different coefficients of expansion of the soft glass and the film, but this strain will be isotropic in the plane of the film as long as the material is magnetized in the initial direction. If the film is magnetized in any other direction, magnetostriction will cause an additional strain to be introduced, and stress associated with such a strain will tend to prevent the film from being magnetized in any direction other than the initial direction.

Such a uniaxial anisotropy can be expressed by an equation of the form

$$f=K\sin^2\theta,$$

where f is the anisotropy energy, K is the anisotropy constant, and θ is the angle between $\mathbf{M}$ and the axis of easy magnetization. The case in which the film is in the (X,Z) plane and the easy direction of magnetization is in the Z direction will be considered first. The effect of uniaxial anisotropy can be represented by an anisotropy field defined by the equation:

$$\partial\mathbf{f}/\partial\theta=\mathbf{M}\times\mathbf{H}_A, \tag{18}$$

where θ is the angle between $\mathbf{M}$ and the direction of easy magnetization and $\mathbf{H}_A$ is the anisotropy field. Since this equation does not define $\mathbf{H}_A$ completely $\mathbf{H}_A$ will be chosen to be in the Z direction. θ will then be the angle between $\mathbf{M}$ and the Z axis. For this case Eq. (18) can be written in the form:

$$2K\sin\theta\cos\theta=M_0H_A\sin\theta,$$

so that

$$H_A=(2K/M_0)\cos\theta. \tag{19}$$

For the case in which the direction of easy magnetization is in the X direction, H_A will be given by a somewhat different equation:

$$H_A=-(2K/M_0)\sin\theta', \tag{20}$$

where θ' is the angle between $\mathbf{M}$ and the X axis.

These anisotropy fields can be added to the applied magnetic field to account for the effect of anisotropy. For the ferromagnetic resonance experiment, the angle between the direction of $\mathbf{M}$ and the direction of the applied field (the Z axis) is always small, so that if the direction of easy magnetization is in the Z direction θ will be small while if the direction of easy magnetization is also in the X direction θ' will be approximately $\pi/2$. Thus in the first instance $H_A=2K/M_0$, while in the second instance $H_A=-2K/M_0$. These added anisotropy fields will not affect the resonance line width, but will shift the position of the resonance peak.

Films of high-anisotropy materials, such as pure cobalt, could not be used in the reversal-time experiment. The reason for this is that the coercive field of the cobalt films was so high (>30 oe) that driving fields obtainable with the reversal time apparatus were not sufficient to reverse the direction of magnetization.

In reversal-time experiments, θ and θ' will change by π radians, and hence Eqs. (19) and (20) must be used for the anisotropy fields. The case in which the applied field is in the direction of easy magnetization (the square hysteresis loop direction) will be considered first. By using the fact that $\cos\theta=M_z/M_0$, Eq. (14) becomes

$$\dot{M}_z/(M_0^2-M_z^2)=(\gamma/\alpha_0M_0)(H_r+2KM_z/M_0^2), \tag{21}$$

when the anisotropy field of $2KM_z/M_0^2$ is added to the reversing field of H_r. For reasons which will appear in Sec. VII, the term $\alpha/(\alpha^2+1)$ has been changed to α_0.

Equation (21) can now be rearranged and integrated.

$$\int \frac{dM_z}{(M_0^2-M_z^2)(H_r+2KM_z/M_0^2)}=\int \frac{\gamma}{\alpha_0 M_0}dt.$$

In the reversal experiment, M_z starts at $-M_0$ and H_r has the opposite direction to the initial direction of M_z. For the case in which

$$|H_r|<2K|M_z|/M_0^2,$$

the limits on the integrals are from $M_z=-M_0$ at $t=-\infty$ to $M_z=-H_rM_0^2/2K$, at each point the integral has a singularity and the time again becomes infinite. For the case in which $|H_r|>2K|M_z|/M_0^2$ this singularity does not occur, and the limits are from $M_z=-M_0$ at $t=-\infty$ to $M_z=M_0$ at $t=+\infty$.

The first set of limits corresponds to a reversing field too small to overcome the anisotropy field, so that magnetization reversal does not take place. The second set corresponds to a reversing field large enough to overcome this anisotropy field and then produce magnetization reversal. A material which behaved this way would have a rectangular hysteresis loop in the easy direction of magnetization with a coercive force of $2K/M_0$. When the reversing field is applied in the plane of the film and in the difficult direction of magnetization the anisotropy field is $-2K/M_0$ rather than $+2K/M_0$; thus, the term $2KM_z/M_0^2$ in Eq. (21) changes sign. For $|H_r|<2K|M_z|/M_0^2$ the limits are $M_z=\pm H_rM_0^2/2K$ at $t=\pm\infty$ respectively. As H_r is increased, the amount of reversal increases until complete reversal takes place when $|H_r|\geqslant 2K|M_z|/M_0^2$ and the limits on M_z then become $\pm M_0$. A material which behaved in this way would not have a hysteresis loop in the difficult direction of magnetization, for the magnetization would then be a single-valued function of the reversing field.

It follows from the above equation that a reversing field strong enough to reverse the magnetization when applied along the easy axis will also reverse the magnetization when made alternately positive and negative along the hard axis.

The observed hysteresis loops of the evaporated films have some area in the difficult direction, and the field needed to drive the film to saturation in the difficult direction is always greater than the coercive force in the easy direction. Thus, the above calculation will not account for all the properties of the observed hysteresis loops. The field needed to drive the film to saturation in the difficult direction is a measure of the anisotropy field. It follows that in the easy direction the film is reversing in a field less than that required for reversal by domain rotation, for some films much less.

A reversal time T' will now be defined as the time required for M_z to go from $-M_0/\sqrt{2}$ to $+M_0/\sqrt{2}$. From Eq. (21), T' will be given by the equation

$$T'=\frac{\alpha_0}{\gamma[(2K/M_0)^2-H_r^2]}\times\left\{-1.76H_r+\frac{2K}{M_0}\ln\left[\frac{(\sqrt{2}H_r+2K/M_0)}{(\sqrt{2}H_r-2K/M_0)}\right]\right\}. \quad (22)$$

For $K=0$, this equation reduces to the equation

$$T'=1.76\alpha_0/\gamma H_r, \quad (23)$$

which is the same as Eq. (17), since $\alpha_0=\alpha/(\alpha^2+1)$. For $H_r\gg 2K/M_0$, the result will be approximately the same.

For H_r only slightly larger than $2K/M_0$, the above definition of T' will be about the same as the previous definition of the reversal time T, the half-width of the M_z pulse.

But it is shown above that for $H_r<2K/M_0$, $1/T$ is zero, while $1/T'$ as calculated from Eq. (22) may be greater than zero. The reason for this difference is that in the reversal time experiment, M_z starts at approximately $-M_0$ rather than at the $-M_0/\sqrt{2}$ limit of the integral of Eq. (21). When $H_r<2K/M_0$, the magnetization reversal cannot get started against the restraining force of the anisotropy field. If reversal started at $M_z=-M_0/\sqrt{2}$, then Eq. (22) would still apply for $H_r<2K/M_0$, but with M_z starting at about $-M_0$, Eq. (22) applies only for fields large enough for reversal to take place at all. Thus, a graph of $1/T'$ as a function of H should drop abruptly to zero at $H=2K/M_0$.

VII. COMPARISON OF REVERSAL-TIME THEORY AND EXPERIMENT

In this section a comparison between the measured reversal times of thin films and reversal times calculated by the use of the preceding theory will be presented. In making this comparison evaporated films of three quite different alloy compositions were used.

A film of 80% Ni and 20% Fe showed no observable uniaxial anisotropy, and therefore the measured reversal times of this film as a function of driving field were compared with reversal times calculated by the use of Eq. (23). The other two films both contained cobalt, and both showed a uniaxial anisotropy. Therefore the reversal times of these films were compared with results calculated by the use of Eq. (22).

The magnetization reversal measurements on the thin films of (80% Ni, 20% Fe) show that the reversal time of this film is approximately inversely proportional to the reversal field, H, as indicated by Eqs. (17) and (23). Also Eq. (16) is the equation of a curve that has the same shape as the experimentally observed reversal pulse, and therefore this equation describes the shape of the reversal pulse qualitatively. But, if a value for the damping constant, α, is taken from ferromagnetic resonance experiments and substituted into

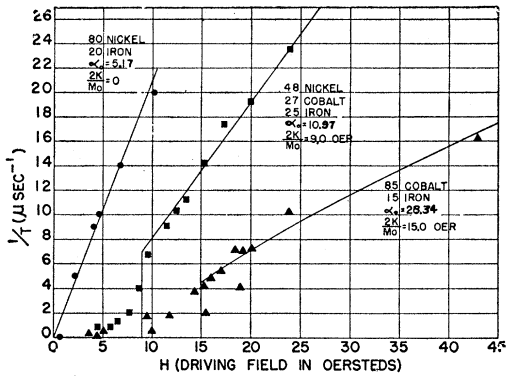

FIG. 4. Inverse switching time as a function of reversing field for (80% Ni, 20% Fe), (48% Ni, 27% Co, 25% Fe), and (85% Co, 15% Fe) films.

Eq. (17), the resulting calculated reversal time, T, does not agree with the measured value. For example, the measured reversal time of this evaporated film of (80% Ni, 20% Fe) is 0.95×10^{-7} second when H is 5.0 oersteds. If these values for T and H are substituted in Eq. (17) and the equation is then solved for α, it is found that α has the complex value of $0.097\pm0.997j$. But if α is determined from ferromagnetic resonance measurements on the film, it is found that it has the real value 0.0195.

The difficulty of the complex damping constant can be removed by replacing the damping term of the Landau-Lifshitz equation of motion, Eq. (1), by a different damping term suggested by Gilbert.[17] The new equation of motion becomes

$$\frac{d\mathbf{M}}{dt}=\gamma(\mathbf{M}\times\mathbf{H})-\frac{\alpha_0}{M}\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right). \quad (24)$$

Equation (24) can be changed into a form similar to the Landau-Lifshitz equation by taking the cross product of $\mathbf{M}$ and the terms of Eq. (24). Then

$$\mathbf{M}\times\frac{d\mathbf{M}}{dt}=\gamma[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})]-\frac{\alpha_0}{M}\left[\mathbf{M}\times\left(\mathbf{M}\times\frac{d\mathbf{M}}{dt}\right)\right]. \quad (25)$$

The last term is equal to

$$-(\alpha_0/M)[(\mathbf{M}\cdot d\mathbf{M}/dt)\mathbf{M}-M^2(d\mathbf{M}/dt)]$$

by a vector identity, and $\mathbf{M}\cdot(d\mathbf{M}/dt)$ is zero because the magnitude of $\mathbf{M}$ does not change; therefore, the damping term becomes $\alpha_0 M(d\mathbf{M}/dt)$. Equations (24) and (25) can be solved to give

$$\frac{d\mathbf{M}}{dt}=\frac{\gamma}{(1+\alpha_0^2)}(\mathbf{M}\times\mathbf{H})-\frac{\gamma\alpha_0}{M(1+\alpha_0^2)}[\mathbf{M}\times(\mathbf{M}\times\mathbf{H})]. \quad (26)$$

[17] T. Gilbert, Armour Research Foundation, Report No. 11, January 25, 1955 (unpublished).

Comparison of Eq. (26) with the Landau-Lifshitz equation shows that the two equations are the same except that γ has been replaced by $\gamma/(1+\alpha_0^2)$ and α has been replaced by α_0. For cases in which the damping constant α_0 is small, as in the microwave resonance experiment, Eq. (26) reduces to the Landau-Lifshitz equation, which is therefore valid for use in the resonance calculation.

If γ and α in Eq. (17) are replaced by $\gamma/(\alpha_0^2+1)$ and α_0 respectively, the equation becomes

$$T=1.76\alpha_0/\gamma H_0.$$

It was in anticipation of this result that the term $\alpha/(\alpha^2+1)$ was changed to α_0 in Sec. VI.

For the three film compositions investigated, the values of α_0 were chosen which would best fit the experimental reversal time data. For the films with a uniaxial anisotropy, a value for $2K/M_0$ had to be chosen also. For the films of 80% Ni and 20% Fe, $2K/M_0$ was assumed to be zero.

Although the values of α_0 and K/M_0 were chosen to give the best fit to the experimental data, the choice of a value for K/M_0 is not completely arbitrary, for it is shown in Sec. VI that the width of the hysteresis loop in the hard direction is $4K/M_0$. This width usually cannot be measured very precisely because the ends of the loop are curved and not well defined. Nevertheless, the width of the hysteresis loops of the films of the two alloys containing cobalt were found to be approximately equal to twice the value of $2K/M_0$ chosen to fit the reversal-time data.

The values of α_0 were determined graphically. Reciprocals of the measured reversal times as a function of the reversing field for the three alloys were plotted as shown in Fig. 4. A value for α_0 was then found which when substituted into Eq. (23) gave a curve which appeared to fit best the experimental points for the 80% Ni, 20% Fe alloy. Similarly, values for α_0 and $2K/M_0$ were found which would fit the experimental data for the (48% Ni, 30% Fe, 22% Co) alloy and the (85% Co, 15% Fe) alloy. The three resulting curves also are shown in Fig. 4.

For the (80% Ni, 20% Fe) and the (48% Ni, 30% Fe, 22% Co) alloys, the curves fit the experimental data quite well. The data for the (85% Co, 15% Fe) alloy spread quite a bit and do not fit the curve as well.

Upon comparison of the values of the damping constants calculated from reversal time measurements with those calculated from the ferromagnetic resonance experiments, and empirical proportionality, $\alpha_f=C\alpha_0$ is seen to hold for the three diverse alloys under consideration. The values of α_f, α_0, and C for these alloys are shown in Table II.

Thus, although C is an empirical constant, it appears that this constant can be used for a number of different alloys, and that the resonance line width is proportional to the reversal time.

Three tests are not enough to prove the general applicability of the theory, but they are sufficient evidence to show that for such alloys of iron, cobalt, and nickel, the theory is capable of predicting switching time from ferromagnetic resonance data.

The (85% Co, 15% Fe) alloy was the only composition found that had a very broad resonance line and a coercive force small enough that reversal time measurements could be made on it.

The physical reason that the damping constants for magnetization reversal and for ferromagnetic resonance are not the same is not known. However, it is necessary to take into consideration the arbitrary way in which the damping constant is introduced into the Landau-Lifshitz equation. One has no *a priori* physical reason to expect identical values of α from the two calculations. However, it is physically plausible that α should, in this way, be proportional to H, corresponding to a field independent relaxation time. The experimental results thus far seem to point in this direction.

Figure 4 shows that magnetization reversal still takes place at a field less than the calculated value of 9 oersteds ($H_0=2K/M_0$) for the (48% Ni, 30% Fe, 22% Co) alloy. This slow reversal at a field too low for domain rotation against the anisotropy field may be due to wall motion.

Kittel[18] has derived an equation for wall motion for cases in which eddy currents may be neglected. His result is

$$v=4(\gamma^2M_0^2+\lambda^2)AH_0/\lambda M_0\sigma_0,$$

where v = the wall velocity, γ = the gyromagnetic ratio, M_0 = the saturation magnetization, λ = the damping constant, A = the exchange factor,[19] H_0 = the driving field, σ_0 = the surface energy of the wall at rest, and λ can be determined from ferromagnetic resonance experiments.

If the proper values of the above terms for an alloy of (48% Ni, 30% Fe, 22% Co) are substituted into the above equation, the result is $v\approx10^3$ cm second^{-1} oersted^{-1}. If the path of the wall motion were known, the reversal time for fields less than $2K/M_0$ possibly could be calculated by using this velocity, but unfortunately the path is not known. If it is assumed that a domain wall normal to the film surface moves across the film, a distance of about one cm, with the above speed, the resulting calculated reversal time is much too long compared to the measured reversal time. If it is assumed that a wall parallel to the film surface moves through the film, a distance of 10^{-5} to 10^{-4} cm, the resulting calculated reversal time is much too short. Thus, neither of these paths seems to be correct.

[18] C. Kittel, Phys. Rev. **80**, 918 (1950).
[19] C. Kittel, Revs. Modern Phys. **21**, 541 (1949).

TABLE II. Proportionality of phenomenological damping constants from resonance and reversal experiments.

Composition	α_f	α_0	C
80% Ni, 20% Fe	0.0195	5.17	3.77×10^{-3}
48% Ni, 30% Fe, 22% Co	0.0292	10.97	2.66×10^{-3}
15% Fe, 85% Co	0.1102	28.34	3.89×10^{-3}

In obtaining reversal times from ferromagnetic resonance line widths, the applied reversing field should be considerably larger than $2K/M_0$, for under these conditions magnetization reversal takes place by domain rotation, and there is only one reversal time parameter, the angular velocity damping term. For lower fields where wall motion may take place, reversal time will depend on both wall velocity and the path of the wall motion.

VIII. SUMMARY

The theory that has been presented accounts for the effects of the reversing field and anisotropy on the reversing time, gives a qualitative description of the shape of the reversal pulse, and accounts for the experimental fact that reversal time is not dependent on film thickness. The theory also agrees with the results of the crossed-coil experiments.[1] These successes show that the theory of magnetization reversal by domain rotation, with the angular velocity limited by some sort of viscous damping, is a useful one. But the physical nature of the damping term is not known. It was demonstrated experimentally that the damping term is proportional to the ferromagnetic resonance damping term, but the damping constants for reversal time and ferromagnetic resonance are not the same. This difference is one of the most interesting results of this investigation. The authors feel that more work should be devoted to a study of this difference.

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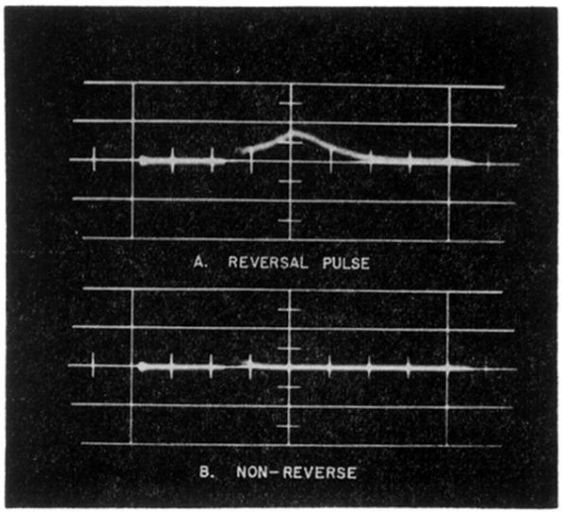

FIG. 3. Reversal signal for 10 000 A (85% cobalt, 15% iron) film.
(a) Reversal signal; (b) nonreversal condition.