Possible Interference Phenomena between Parity Doublets*

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Possible interference phenomena between parity doublets are discussed. In particular, the decay particle distributions of $\Sigma^{\pm}$ and Λ^0 produced in K^-+p capture and in π^-+p collision are analyzed. These distributions may have interference terms with odd parity. Experimental detection of such terms would constitute an unambiguous proof of the existence of parity doublets. It is further pointed out that such interference terms have a rather interesting time dependence.

TO explain the apparent equality of masses of $\tau^+(\equiv K_{\pi 3}{}^+)$ and $\theta^+(\equiv K_{\pi 2}{}^+)$ it has been proposed[1] that there is a symmetry of all strong interactions called parity conjugation. All strange particles with odd strangeness would then exist as parity doublets, i.e., two states of opposite parity.

We wish to point out in this note that for parity doublets with extremely close mass values (e.g., if case B of reference 1 prevails) there are possible experiments to determine unambiguously the parity doublet structure through the observation of interference phenomena between the two parities. These interferences characteristically produce in the observed quantities terms that have an odd parity. In Secs. I, II, and III, one such possible experiment—the K^- capture in hydrogen—is analyzed in detail. In Sec. IV a discussion of the general description of a beam of particles with a parity doublet structure is given. The remaining two sections, V and VI, are concerned with the production of such a beam and the angular distributions in its decay. In particular, the distribution of the decay pion of a hyperon produced in a collision between pions and nucleons is discussed in some detail (cases B and C of Sec. VI).

In this note parity is assumed to be absolutely conserved. The assumption of parity nonconservation in weak interactions has been discussed elsewhere.[2] It leads to angular distributions qualitatively different from those discussed in this note. A summary of these differences is given in the appendix.

I.

Let us consider the production and the subsequent decay of a Σ^- in the capture of K^- by protons at rest,

$$K^-+p\rightarrow\Sigma^-+\pi^+, \tag{1}$$

$$\Sigma^-\rightarrow n+\pi^-, \tag{2}$$

which has been observed in emulsions and in a bubble chamber. (Entirely similar considerations can also be applied to the Σ^+ and Λ^0 produced.) We want to discuss the distribution of the decay process with respect to the angle θ between the direction of motion of the Σ^- and that of the decay neutron in the rest system of the Σ^-. This distribution has been discussed by Treiman.[3]

We shall show that with the existence of parity doublet states for the Σ^-, odd powers of $\cos\theta$ may appear in this distribution as a result of interference between the two states of the parity doublet: $\Sigma_1{}^-$ and $\Sigma_2{}^-$.

To simplify the discussion let us first assume that the spin of the Σ^- is $\frac{1}{2}$. Also assume for the time being that the incoming K^- is a particle of definite parity, say θ^-. Take the z axis to be parallel to the direction of motion of Σ^-, and resolve all angular momenta along the z axis. The Σ^- travelling along the z axis can have an angular momentum $m=+\frac{1}{2}$ or $m=-\frac{1}{2}$ along the z axis. It is clear that there is no interference between these two states of different m. Each of them, however, is a mixture of $\Sigma_1{}^-$ and $\Sigma_2{}^-$. The wave function $\psi_{\frac{1}{2}}$ for Σ^- with $m=\frac{1}{2}$ at the time that it is produced (defined to be $t=0$) can be written as[4]

$$\psi_{\frac{1}{2}}=a\psi_{\frac{1}{2}}(\Sigma_1)+b\psi_{\frac{1}{2}}(\Sigma_2). \tag{3}$$

By a reflection with respect to a plane containing the z axis, one obtains the wave function $\psi_{-\frac{1}{2}}$ for the *produced* Σ^- with $m=-\frac{1}{2}$, at $t=0$:

$$\psi_{-\frac{1}{2}}=a\psi_{-\frac{1}{2}}(\Sigma_1)-b\psi_{-\frac{1}{2}}(\Sigma_2). \tag{4}$$

The minus sign comes from the fact that the Σ_1 and Σ_2 have opposite parities. At time t after the production of the Σ, the wave functions of Σ_1 and Σ_2 acquire, respectively, the factors $\exp(-\frac{1}{2}\lambda_1 t-im_1t)$ and $\exp(-\frac{1}{2}\lambda_2 t-im_2t)$, where $\lambda_1{}^{-1}$, $\lambda_2{}^{-1}$ are the lifetimes of the Σ_1 and Σ_2, and m_1 and m_2 are their masses. Thus if the decay occurs[5] at time t, the wave function for the

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[1] T. D. Lee and C. N. Yang, Phys. Rev. **102**, 290 (1956); M. Gell-Mann, *Proceeding of the 1956 Rochester Conference on High-Energy Physics* (Interscience Publishers, Inc., New York, to be published).

[2] T. D. Lee and C. N. Yang, Phys. Rev. **104**, 254 (1956).

[3] S. B. Treiman, Phys. Rev. **101**, 1216 (1956).

[4] The amplitudes a and b refer respectively, to the reactions $\theta^-+p\rightarrow\Sigma_1{}^-+\pi^+$, and $\theta^-+p\rightarrow\Sigma_2{}^-+\pi^+$. The interference term between the parity doublets Σ_1 and Σ_2 can exist only if both a and b are nonvanishing.

[5] The meanings of $A(t)$ and $B(t)$ are intuitively evident. Their use can be rigorously justified by using the Weisskopf-Wigner treatment of time-dependent Schrödinger equation [Z. Physik **63**, 54 (1930); **65**, 18 (1930)].

decay neutron is

$$\psi_{\frac{1}{2}}(\Sigma_1)\to A(t)\begin{pmatrix}1\\0\end{pmatrix},\quad \psi_{\frac{1}{2}}(\Sigma_2)\to B(t)\begin{pmatrix}\cos\theta\\ \sin\theta e^{+i\varphi}\end{pmatrix};$$
$$\psi_{-\frac{1}{2}}(\Sigma_1)\to A(t)\begin{pmatrix}0\\1\end{pmatrix},\quad \psi_{-\frac{1}{2}}(\Sigma_2)\to B(t)\begin{pmatrix}\sin\theta e^{-i\varphi}\\ -\cos\theta\end{pmatrix},\tag{5}$$

where

$$A(t)=A_0e^{-im_1t-\frac{1}{2}\lambda_1t},$$

and

$$B(t)=B_0e^{-im_2t-\frac{1}{2}\lambda_2t}.\tag{6}$$

Here θ and φ are the spherical coordinates of the neutron. Its wave function is, therefore, for $m=\frac{1}{2}$:

$$\begin{pmatrix}aA+bB\cos\theta\\ bB\sin\theta e^{i\varphi}\end{pmatrix},\tag{7a}$$

and for $m=-\frac{1}{2}$:

$$\begin{pmatrix}-bB\sin\theta e^{-i\varphi}\\ aA+bB\cos\theta\end{pmatrix}.\tag{7b}$$

Squaring and adding, one obtains the angular distribution of the decay neutron. It is proportional to

$$W_{\frac{1}{2}}(\theta)=f+g\cos\theta,\tag{8}$$

where

$$f=|aA|^2+|bB|^2,$$

and

$$g=2\,\mathrm{Re}[aAb^*B^*].\tag{9}$$

The odd term $\cos\theta$ gives rise to a forward-backward asymmetry. Its presence would constitute an unambiguous proof of the existence of the parity doublet structure of the Σ^-.

From (6) one concludes that the even part of the angular distribution varies with time as (constant) $e^{-\lambda_1t}$ + (constant) $e^{-\lambda_2t}$. The odd part of the distribution varies with time as $e^{-\frac{1}{2}(\lambda_1+\lambda_2)t}\cos[(m_1-m_2)t+\text{constant}]$. The presence of similar mixed time constant terms has been discussed by Treiman and Sachs[6] in another connection.

The foregoing considerations should be applied only if the mass difference $|m_1-m_2|$ is less than the energy uncertainty in the K^-+p system. For mass differences $|m_1-m_2|\lesssim 10^{-5}$ ev this is certainly correct. In the case of large mass differences, these two parity doublets would undergo γ transition; e.g., $\Sigma_1\to\Sigma_2+\gamma$. Such cases will not be discussed here.

II.

The actual situation is more involved because the captured K^- may exist in two parities; i.e., one must consider also the capture of τ^- by p. Altogether there are four processes:

$$\theta^-+p\to\begin{cases}\Sigma_1^-+\pi^+, & (10)\\ \Sigma_2^-+\pi^+, & (11)\end{cases}$$

and

$$\tau^-+p\to\begin{cases}\Sigma_2^-+\pi^+, & (10')\\ \Sigma_1^-+\pi^+. & (11')\end{cases}$$

The processes (10) and (11) were studied in the above discussion. Process (10′) is related to process (10) by the operation of parity conjugation C_P. [We adopt the convention $C_P|\theta^-\rangle=|\tau^-\rangle$ and $C_P|\tau^-\rangle=|\theta^-\rangle$, and similarly for Σ_1 and Σ_2.] The two processes therefore have equal amplitudes. The same holds true for (11′) and (11). The capture of τ^- therefore produces the parity-conjugate wave functions for Σ at $t=0$; i.e., the parity-conjugate states of Eqs. (3) and (4):

$$\psi_{\frac{1}{2}}'=a\psi_{\frac{1}{2}}(\Sigma_2)+b\psi_{\frac{1}{2}}(\Sigma_1),\tag{3'}$$

and

$$\psi_{-\frac{1}{2}}'=a\psi_{-\frac{1}{2}}(\Sigma_2)-b\psi_{-\frac{1}{2}}(\Sigma_1).\tag{4'}$$

If the original K particle is a *coherent* mixture of θ^- and τ^- with amplitudes α_θ and α_τ, the wave functions for Σ become:

$$\alpha_\theta\psi_{\frac{1}{2}}+\alpha_\tau\psi_{\frac{1}{2}}'=(\alpha_\theta a+\alpha_\tau b)\psi_{\frac{1}{2}}(\Sigma_1)+(\alpha_\theta b+\alpha_\tau a)\psi_{\frac{1}{2}}(\Sigma_2),\tag{12}$$

and

$$\alpha_\theta\psi_{-\frac{1}{2}}+\alpha_\tau\psi_{-\frac{1}{2}}'=(\alpha_\theta a-\alpha_\tau b)\psi_{-\frac{1}{2}}(\Sigma_1)+(-\alpha_\theta b+\alpha_\tau a)\psi_{-\frac{1}{2}}(\Sigma_2).\tag{13}$$

In the subsequent decay process, Σ_1 and Σ_2 may interfere (but not the $m=\frac{1}{2}$ and $m=-\frac{1}{2}$ states, as remarked before). It is evident from (12) and (13) that a summation over $m=\pm\frac{1}{2}$ results in the cancellation of all interference terms $\alpha_\theta\alpha_\tau$ between θ and τ. The particles θ and τ therefore behave in this process always as *incoherent* states.

From (3′) and (4′) the angular distribution $W(\theta)$ for the case of τ^- capture can be readily written down. One then obtains for the general case of the capture of a coherent mixture of θ^- and τ^- with amplitudes α_θ and α_τ the angular distribution:

$$W_{\frac{1}{2}}(\theta)=\mathfrak{f}+\mathfrak{g}\cos\theta,\tag{14}$$

where

$$\mathfrak{f}=|\alpha_\theta|^2(|aA|^2+|bB|^2)+|\alpha_\tau|^2(|bA|^2+|aB|^2),\tag{15}$$

$$\mathfrak{g}=2\,\mathrm{Re}\{AB^*(|\alpha_\theta|^2ab^*+|\alpha_\tau|^2ba^*)\},\tag{16}$$

where A and B are functions of time given by Eq. (6).

The constants $|\alpha_\theta|$, $|\alpha_\tau|$, a, b, A_0, and B_0 that go into formulas (14) and (15) all have direct physical meanings. The constants A_0 and B_0 satisfy the following conditions:

$$|A_0|^2=\lambda_1,\tag{17}$$

$$|B_0|^2=\lambda_2,\tag{18}$$

[6] S. B. Treiman and R. G. Sachs (to be published).

and

$$\frac{B_0}{A_0}=\pm\left(\frac{\lambda_2}{\lambda_1}\right)^{\frac{1}{2}}e^{i\delta(\Sigma_2)-i\delta(\Sigma_1)},\tag{19}$$

where $\delta(\Sigma_2)$ and $\delta(\Sigma_1)$ are, respectively, the phase shifts in the π^-+n system in the spin-parity states resulting from the decay of Σ_2 and Σ_1. Equations (17) and (18) express the conservation of probability. (Other decay modes of Σ^- are neglected.) Equation (19) follows[7] from the principle of invariance under time reversal.

By measuring the time and angular dependence of $W(\theta)$ one could thus, through Eqs. (14)–(19), in principle not only establish the possible existence of the parity doublet structure for Σ, but possibly also its spin[3] (see the next section), the lifetimes for Σ_1 and Σ_2, their mass difference, the mixing proportions $|\alpha_\theta|$ and $|\alpha_\tau|$ of the captured K^-, and the magnitudes and relative phase of the amplitudes a and b of Σ_1^- and Σ_2^- produced in the capture of θ^-.

III.

The only assumption made in deriving Eqs. (14)–(16) is that the spin of the Σ is $\frac{1}{2}$, and that the p and the K^- are unpolarized. If the spin of the Σ is not $\frac{1}{2}$, similar considerations lead to an angular distribution containing a part odd in $\cos\theta$, coming from the interference of Σ_1 and Σ_2. The experiment could thus still serve to test the parity doublet structure of the Σ, but the angular dependence of the even and odd parts would not in general be uniquely determined by the spin of the Σ.

If, however, the capture is from a state with total angular momentum $\frac{1}{2}$, the even and odd parts of the angular distribution would be dependent only on the spin J of the Σ:

$$W_J(\theta)=\mathfrak{f}F_J(\theta)+\mathfrak{g}G_J(\theta).\tag{20}$$

The coefficients $\mathfrak{f}$ and $\mathfrak{g}$ are the same as in (15) and (16); their time dependence can be obtained by using (6). The functions F_J and G_J are given by

$$F_J(\theta)=(J+\tfrac{1}{2})^2[P_{J-\frac{1}{2}}(x)]^2+\sin^2\theta[P_{J-\frac{1}{2}}'(x)]^2,\tag{21}$$

and

$$G_J(\theta)=(J+\tfrac{1}{2})^2P_{J-\frac{1}{2}}(x)P_{J+\frac{1}{2}}(x)-\sin^2\theta P_{J-\frac{1}{2}}'(x)P_{J+\frac{1}{2}}'(x),\quad(x=\cos\theta).\tag{22}$$

Here $P_l(x)$ is the Legendre polynomial:

$$P_l(x)=\frac{1}{2^l l!}\frac{d^l}{dx^l}(x^2-1)^l,$$

and

$$P_l'(x)=(d/dx)P_l.$$

The functions $F_J(\theta)$ have been tabulated by Adair[8] and Treiman.[3] We tabulate them here, together with $G_J(\theta)$,

$$\begin{aligned}
F_{1/2}&=1, & G_{1/2}&=\cos\theta,\\
F_{3/2}&=1+3\cos^2\theta, & G_{3/2}&=9\cos^3\theta-5\cos\theta,\\
F_{5/2}&=(9/4)(5\cos^4\theta-2\cos^2\theta+1) & G_{5/2}&=(9/4)(25\cos^5\theta-26\cos^3\theta+5\cos\theta).
\end{aligned}$$

It is seen that the functions G_J are large near $\theta=0$ or π, so that the asymmetry is large in the forward-backward direction along the direction of motion of the Σ.

IV.

In this section we shall discuss the general problem of the polarization of a beam of particles of spin $\frac{1}{2}$ with a parity doublet structure. A wave function describing such a particle has four components:

$$\psi=\begin{bmatrix}a_\uparrow\\a_\downarrow\\b_\uparrow\\b_\downarrow\end{bmatrix}.$$

The upper two components represent the amplitudes of the odd-parity particle with up and down z-component of spin, while the lower two components represent the amplitudes of the even-parity particle. The density matrix[9] D for a coherent collection of such particles is defined to be $\Sigma\psi\psi^\dagger$ and is a 4×4 Hermitian matrix († means Hermitian conjugate). It is convenient to write this matrix in the following form:

$$D=\begin{pmatrix}D_1 & \mathfrak{D}\\ \mathfrak{D}^\dagger & D_2\end{pmatrix},\tag{23}$$

where D_1 and D_2 are 2×2 Hermitian matrices representing, respectively, the density matrices of the even and the odd parity particles, and $\mathfrak{D}$ is a 2×2 matrix which characterizes the interference between the two particles. The matrix $\mathfrak{D}$ will be called the mixed density matrix. It can be split into the form

$$\begin{aligned}\mathfrak{D}&=\mathfrak{D}_r+i\mathfrak{D}_i,\\ \mathfrak{D}^\dagger&=\mathfrak{D}_r-i\mathfrak{D}_i,\end{aligned}\tag{24}$$

where $\mathfrak{D}_r$ and $\mathfrak{D}_i$ are both 2×2 Hermitian matrices. To characterize the matrices D_1, D_2, $\mathfrak{D}_r$, and $\mathfrak{D}_i$ it is most convenient to represent them by four real vectors $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathfrak{P}_r$, $\mathfrak{P}_i$, and four real intensities I_1, I_2, $\mathfrak{I}_r$, and $\mathfrak{I}_i$:

$$\begin{aligned}D_1&=\mathbf{P}_1\cdot\boldsymbol{\sigma}+I_1,\\ D_2&=\mathbf{P}_2\cdot\boldsymbol{\sigma}+I_2,\\ \mathfrak{D}_r&=\mathfrak{P}_r\cdot\boldsymbol{\sigma}+\mathfrak{I}_r,\\ \mathfrak{D}_i&=\mathfrak{P}_i\cdot\boldsymbol{\sigma}+\mathfrak{I}_i,\end{aligned}\tag{25}$$

where $\boldsymbol{\sigma}$ represents the Pauli spin matrices.

[7] The decay interactions is treated as a perturbation. See, e.g., K. M. Watson, Phys. Rev. **95**, 228 (1954); K. Aizu, *Proceedings of the International Conference on Theoretical Physics*, Kyoto and Tokyo, Japan, 1953 (Science Council of Japan, Tokyo, 1954).

[8] R. K. Adair, Phys. Rev. **100**, 1540 (1955).

[9] N. Mott and H. Massey, *Theory of Atomic Collision* (Oxford University Press, Oxford, 1949), second edition, Chap. 4. For other references, see e.g., L. Wolfenstein, *Annual Review of Nuclear Science* (to be published).

These sixteen real parameters must satisfy the condition that D is a positive Hermitian matrix. This is equivalent to the following conditions:

$$I_1 \geqq |\mathbf{P}_1|, \quad I_2 \geqq |\mathbf{P}_2|, \quad 2I_1I_2 \geq \mathfrak{I}\mathfrak{I}^* + \mathfrak{P}\cdot\mathfrak{P}^*,$$
$$I_1(I_2^2-\mathbf{P}_2^2)+I_2(I_1^2-\mathbf{P}_1^2) \geqq (I_1+I_2)(\mathfrak{I}\mathfrak{I}^*+\mathfrak{P}\cdot\mathfrak{P}^*)$$
$$-(\mathbf{P}_1+\mathbf{P}_2)\cdot(\mathfrak{I}\mathfrak{P}^*+\mathfrak{I}^*\mathfrak{P})-i(\mathbf{P}_1-\mathbf{P}_2)\cdot(\mathfrak{P}\times\mathfrak{P}^*), \quad (26)$$

and $\det D \geqq 0$. We have here used the notations,

$$\mathfrak{I}=\mathfrak{I}_r+i\mathfrak{I}_i, \quad \mathfrak{P}=\mathfrak{P}_r+i\mathfrak{P}_i. \quad (27)$$

The physical meanings of I_1 and I_2 are quite clear: they represent the intensities of these two kinds of particles. The vectors $\mathbf{P}_1/I_1$ and $\mathbf{P}_2/I_2$ are the usual polarization vectors for the two particles. $\mathfrak{P}_r$ and $\mathfrak{P}_i$ characterize the polarization of the interference effects between the two particles.

Under a space inversion, one has

$$D_1 \rightarrow +D_1,$$
$$D_2 \rightarrow +D_2,$$

while

$$\mathfrak{D} \rightarrow -\mathfrak{D}.$$

Thus I_1, I_2 are scalars; $\mathbf{P}_1$, $\mathbf{P}_2$ are pseudovectors; $\mathfrak{I}_r$, $\mathfrak{I}_i$ are pseudoscalars, and $\mathfrak{P}_r$, $\mathfrak{P}_i$ are vectors.

In general, because of the decay of these parity doublets and because of their possible mass difference the density matrix would vary with time at a rate given by

$$D_1(t)=D_1(0)e^{-\lambda_1 t},$$
$$D_2(t)=D_2(0)e^{-\lambda_2 t}, \quad (28)$$

and

$$\mathfrak{D}(t)=\mathfrak{D}(0)e^{-\frac{1}{2}(\lambda_1+\lambda_2)t-i(m_1-m_2)t},$$

where λ_1^{-1} and λ_2^{-1} are, respectively, the lifetimes of the two particles with opposite parities and m_1-m_2 is their mass difference. The state of a collection of such particles is completely specified by its density matrix. All probabilities that can be measured must be linearly dependent on the matrix elements of the density matrix.

The foregoing discussions can be applied to particles of any spin. In particular, for zero-spin particles, the density matrix is specified by the (real) intensities I_1 and I_2, and the mixed (complex) intensity $\mathfrak{I}$,

$$D=\begin{pmatrix} I_1 & \mathfrak{I} \\ \mathfrak{I}^* & I_2 \end{pmatrix}. \quad (29)$$

The inequalities

$$I_1 \geq 0, \quad I_2 \geq 0, \quad \text{and} \quad I_2I_2 \geq |\mathfrak{I}|^2 \quad (30)$$

are equivalent to the condition that D is positive. Just as before, I_1 and I_2 are scalars, $\mathfrak{I}$ is a (complex) pseudoscalar.

V.

We shall now consider the decay of a spin-$\frac{1}{2}$ particle with a parity doublet structure described by the density matrix $D(t)$ given by Eqs. (23)–(25). If in the decay the momentum $\mathbf{k}$ of a single decay particle in the rest system of the decaying particle is observed, the distribution of this momentum $\mathbf{k}$ is given by the probability function:

$$W(\mathbf{k})=\xi+\boldsymbol{\eta}\cdot\mathbf{k}, \quad (31)$$

where

$$\xi=(\text{const})I_1+(\text{const})I_2,$$

and

$$\boldsymbol{\eta}=(\text{const})\mathfrak{P}_r+(\text{const})\mathfrak{P}_i.$$

The constants in these equations are real quantities that may depend on k^2, but are independent of time. This follows from the fact that W is linear in the elements of D and is a scalar.[10]

The quantities I_1, I_2, $\mathfrak{P}_r$, and $\mathfrak{P}_i$ are functions of time, as implied by (28). If $\mathfrak{P}_r$ and $\mathfrak{P}_i$ are not parallel, the vector $\boldsymbol{\eta}$ [which describes the average direction of $\mathbf{k}$] will rotate with time with a frequency equal to $(1/2\pi)(m_1-m_2)$.

It may be instructive, as an example, to construct explicitly the density matrix for the case of a Σ^- produced in K^-+p capture discussed in Secs. I and II. Consider the general case in which the incoming K^- is a coherent mixture of θ and τ with amplitudes α_θ and α_τ, respectively. By using Eqs. (3), (3′), (4), (4′) one can readily verify that at the time of production (i.e., $t=0$) the density matrix $D(0)$ is described by the parameters[11]:

$$\begin{aligned}
I_1(0) &\equiv I_1(t)|_{t=0}=|\alpha_\theta|^2|a|^2+|\alpha_\tau|^2|b|^2,\\
I_2(0) &= |\alpha_\theta|^2|b|^2+|\alpha_\tau|^2|a|^2,\\
\mathfrak{I}(0) &= \alpha_\theta\alpha_\tau^*|a|^2+\alpha_\tau\alpha_\theta^*|b|^2,\\
\mathbf{P}_1(0) &= \mathrm{Re}[\alpha_\theta\alpha_\tau^*ab^*](\mathbf{k}_\Sigma/k_\Sigma),\\
\mathbf{P}_2(0) &= \mathrm{Re}[\alpha_\theta\alpha_\tau^*ba^*](\mathbf{k}_\Sigma/k_\Sigma),\\
\mathfrak{P}(0) &= [|\alpha_\theta|^2ab^*+|\alpha_\tau|^2ba^*](\mathbf{k}_\Sigma/k_\Sigma),
\end{aligned} \quad (32)$$

where $\mathbf{k}_\Sigma$ is the momentum of the Σ. At a later time these parameters would vary with time according to Eq. (28).

Assume that at time t the Σ^- decays into $n+\pi^-$. One can show that, by using Eqs. (5) and (6), the angular distribution $W(\theta)$ of the decay neutron is related linearly to $D(t)$ by

$$W=\mathrm{Trace}\left[(A_0 \quad B_0\boldsymbol{\sigma}\cdot\mathbf{e}_n)D(t)\begin{pmatrix} A_0^* \\ B_0^*\boldsymbol{\sigma}\cdot\mathbf{e}_n\end{pmatrix}\right], \quad (33)$$

where A_0 and B_0 are defined in Eq. (6) and $\mathbf{e}_n$ is a unit vector along the direction of motion of the neutron. This gives the angular distribution (14) derived previ-

[10] By the same reasoning, one can show that in a similar case involving the decay of a spin-0 particle with a parity doublet structure the corresponding distribution function is of the form: $W(\mathbf{k})=\alpha I_1+\beta I_2$, where I_1, I_2 are defined by Eq. (29). Here α and β are functions of k^2.

[11] It is of interest to notice that under an inversion, $\alpha_\theta \rightarrow \alpha_\theta$ while $\alpha_\tau \rightarrow -\alpha_\tau$. Therefore, $\mathfrak{P}$ is a (complex) vector while $\mathbf{P}_1$ and $\mathbf{P}_2$ are (real) pseudovectors. Similarly, $\mathfrak{I}$ is a (complex) pseudoscalar while I_1 and I_2 are (real) scalars.

ously. It is of the general form (31) as expected. In this case $\mathfrak{P}_r$ and $\mathfrak{P}_i$ are both parallel to the direction of motion of the Σ^-. The asymmetry is thus at all times a forward-backward asymmetry.

VI.

The density matrix of a particle produced in the strong interaction between pions and nucleons must display certain symmetries resulting from the invariance of strong interactions under parity conjugation C_P and under space rotations and inversion. We discuss here a few special cases:

A. In the collision of two nucleons or a nucleon and a pion:

$$A+B\rightarrow K^+ + \text{others},$$

consider the K^+ beam produced in a particular direction. Assume the K^+ to have spin zero. After summation over the states of the "other" particles the density matrix $D(t)$ of K^+, defined by Eq. (29), cannot have mixed intensities. This follows from the fact that the mixed intensity $\mathfrak{J}$ is a pseudoscalar. It therefore cannot be formed out of the two observed momenta in the process, the incoming momentum, and the momentum of K^+. Furthermore, the invariance of the production process under C_P requires that at the time of production $(t=0)$

$$[D(t),C_P]_{t=0}=0. \tag{34}$$

Thus, $D(t)$ is given by

$$D(t)=\begin{pmatrix} Ie^{-\lambda_1 t} & 0 \\ 0 & Ie^{-\lambda_2 t}\end{pmatrix}.$$

The K^+ beam is therefore an *incoherent mixture of even and odd-parity states with intensities that decrease as simple exponentials:* $Ie^{-\lambda_1 t}$ and $Ie^{-\lambda_2 t}$, where λ_1^{-1} and λ_2^{-1} are the lifetimes of these two parity states.

B. The same consideration can be applied to a beam of hyperons, e.g., Λ^0, produced in

$$A+B\rightarrow \Lambda^0 + \text{others}. \tag{35}$$

Assume that the Λ^0 has spin $\frac{1}{2}$. After summation over the states of the "other" particles the parameters defined in Eq. (25) must satisfy[12] the following conditions at the time of production $(t=0)$:

$$\begin{aligned} &I_1=I_2,\\ &\mathbf{P}_1=\mathbf{P}_2=(\text{constant})\ (\mathbf{k}_{\text{in}}\times\mathbf{k}_\Lambda)\\ &\mathfrak{J}=0,\\ &\mathfrak{P}_i=0,\\ &\mathfrak{P}_r=(\text{constant})\ \mathbf{k}_{\text{in}}+(\text{constant})\ \mathbf{k}_\Lambda.\end{aligned} \tag{36}$$

Here $\mathbf{k}_{\text{in}}$ and $\mathbf{k}_\Lambda$ are, respectively, the momenta of the incoming particle and the Λ^0 in the center-of-mass system of the process (35). At a later time they vary in accordance with Eq. (28). Thus the ordinary polarization vectors $\mathbf{P}_1$ and $\mathbf{P}_2$ remain perpendicular to the production plane, while the mixed polarization vector $\mathfrak{P}_r$ and $\mathfrak{P}_i$ are always parallel to each other and remain in a fixed direction $\boldsymbol{\eta}$ in the production plane independent of the time.

The direction $\boldsymbol{\eta}$ has a direct physical meaning in the decay of a Λ^0. According to Eq. (31), the momentum $\mathbf{k}_\pi$ of the decay pion in the rest system of Λ^0 has an angular distribution:

$$W(\mathbf{k}_\pi)=\xi+\boldsymbol{\eta}\cdot\mathbf{k}_\pi. \tag{31}$$

The direction $\boldsymbol{\eta}$ is thus the average direction of $\mathbf{k}_\pi$. If the distribution (31) is indeed experimentally found with a nonvanishing $\boldsymbol{\eta}$, one would have a proof of the existence of a parity doublet structure for Λ^0.

The exact direction of $\boldsymbol{\eta}$ in the production plane depends on the detailed dynamics of the production process. It is in general a function of the production angle and energy of the Λ^0 beam. This makes it quite difficult (at least at the present time) to check experimentally the validity of Eq. (31). It should be remarked that the direction of $\boldsymbol{\eta}$ is determined by the strengths of the various angular momentum states participating in the production process. E.g., in the reaction

$$\pi^+ + n\rightarrow\Lambda^0+K^+, \tag{37}$$

if the initial state has a definite total angular momentum J and a definite parity and if the spins of K^+ and Λ^0 are 0 and $\frac{1}{2}$, then one can show that at the time of production $(t=0)$

$$\begin{aligned} &I_1=I_2,\\ &\mathfrak{P}_r=I_1(\mathbf{k}_\Lambda/|\mathbf{k}_\Lambda|),\\ &\mathfrak{J}=0,\end{aligned}$$

and

$$\mathbf{P}_1=\mathbf{P}_2=\mathfrak{P}_i=0. \tag{38}$$

Thus

$$\boldsymbol{\eta}/\!\!/\mathbf{k}_\Lambda/|k_\Lambda|.$$

The asymmetry in the distribution (31) for the decay pion is in this case a forward-backward asymmetry with respect to the direction of motion of the Λ^0.

TABLE I. Symmetry and possible asymmetries in the angular distribution of the decay pion from $\Sigma^\pm$ or Λ^0.

$\Sigma^\pm$, Λ^0 produced in	Assumption (1): Parity doublet with small mass difference	Assumption (2): Parity nonconservation in decay[a]
(K^-+p) at rest	Forward-backward asymmetry[b]	Forward-backward symmetry[b]
(pion+nucleon) or (nucleon+nucleon)	Asym. with respect to ρ.[c] Up-down symmetry[d]	Sym. with respect to ρ.[c] Up-down asymmetry[d]

[a] See reference 2.
[b] Forward and backward refer to directions parallel and antiparallel to the direction of motion of the hyperon.
[c] ρ is a rotation through 180° around the normal to the production plane. (See reference 2.)
[d] Up and down here refer to the production plane. The direction of $\mathbf{k}_{\text{in}}\times\mathbf{k}_{\text{hyperon}}$ (right-handed convention) is defined to be up. (See reference 2.)

[12] All of these conditions except $\mathfrak{P}_i=0$, $I_1=I_2$, $\mathbf{P}_1=\mathbf{P}_2$ follow from the transformation properties of I, P, etc., under rotation and inversion. The conditions $\mathfrak{P}_i=0$, $I_1=I_2$, $\mathbf{P}_1=\mathbf{P}_2$ follow from the fact that in this case the density matrix commutes with C_P at the time of production.

TABLE II. Properties of ξ and $\boldsymbol{\eta}$ defined in Eq. (A.1).

$\Sigma^{\pm}$, Λ^0 produced in	Assumption (1): Parity doublet with small mass difference	Assumption (2): Parity nonconservation in decay
(K^-+p) at rest	1. $\boldsymbol{\eta} /\!/ \mathbf{k}_{\text{hyperon}}$ 2. [a] $\xi=(\text{const})e^{-\lambda_1 t}+(\text{const})e^{-\lambda_2 t}$ $\boldsymbol{\eta}\propto\exp[-\frac{1}{2}(\lambda_1+\lambda_2)t-i(m_1-m_2)t]$	1. $\boldsymbol{\eta}=0$ 2. [a] $\xi=(\text{const})e^{-\lambda t}$
(pion+nucleon) or (nucleon+nucleon)	1. [b] $\boldsymbol{\eta}=\eta_1\mathbf{k}_{\text{in}}+\eta_2\mathbf{k}_{\text{hyperon}}$ 2. time dependence of ξ and $\boldsymbol{\eta}$ same as above 3. [c] $\eta_1=\sum_{n=0}^{2L} A_n\cos^n\alpha$ $\eta_2=\sum_{n=0}^{2L-1} B_n\cos^n\alpha$ $\xi=\sum_{n=0}^{2L} C_n\cos^n\alpha$	1. [b] $\boldsymbol{\eta}=\eta_0(\mathbf{k}_{\text{in}}\times\mathbf{k}_{\text{hyperon}})$ 2. ξ and $\boldsymbol{\eta}$ both $\propto e^{-\lambda t}$ 3. [c] $\eta_0=\sum_{n=0}^{2L-1} A_n'\cos^n\alpha$ $\xi=\sum_{n=0}^{2L} C_n'\cos^n\alpha$

[a] In the case of parity doublets, λ_1^{-1}, λ_2^{-1} and m_1, m_2 refer to the lifetimes and masses of the two doublets. In the case of parity nonconservation λ^{-1} is the lifetime of the hyperon.
[b] $\mathbf{k}_{\text{in}}$ and $\mathbf{k}_{\text{hyperon}}$ are the momenta of the incoming particle and the outgoing hyperon measured in the c.m. system of the production process.
[c] α denotes the angle between $\mathbf{k}_{\text{in}}$ and $\mathbf{k}_{\text{hyperon}}$, and L is the maximum orbital angular momentum of the outgoing hyperon in the production process.

C. Finally, let us study cases where one does not sum over all the states of the other particles. To be specific, we consider the reaction

$$\pi^+ + n \rightarrow \Lambda^0 + \tau^+, \tag{39}$$

where the K meson is observed to be $\tau^+ \equiv K_{\pi 3}^+$. The density matrix of the Λ^0 in this case does not commute[12] with C_P at the time of production. Assume the Λ^0 spin to be $\frac{1}{2}$. Invariance with respect to space rotation and inversion requires that:

(i) $\mathfrak{J}=0$,

(ii) $\mathbf{P}_1$ and $\mathbf{P}_2$ are both parallel to $\mathbf{k}_{\text{in}}\times\mathbf{k}_\Lambda$,

and

(iii) $\mathfrak{P}_r$ and $\mathfrak{P}_i$ must both lie in the plane of production (i.e., the plane containing $\mathbf{k}_{\text{in}}$ and $\mathbf{k}_\Lambda$). Unlike the previous case (B), $\mathfrak{P}_r$ and $\mathfrak{P}_i$ are now, in general, not parallel to each other. This has some curious consequences as remarked before in Sec. V. If at time t, Λ^0 decays into $p+\pi^-$, the angular distribution W of the decay pion would still be given by (31). The direction of $\boldsymbol{\eta}$ now is a time dependent mixture of $\mathfrak{P}_r$ and $\mathfrak{P}_i$ and would rotate in the production plane with the frequency $(1/2\pi)(m_1-m_2)$.

The generalization of the considerations of these three sections to particles of higher spin than $\frac{1}{2}$ is straightforward but complicated.

APPENDIX

We summarize in Table I the various symmetries and possible asymmetries in the angular distribution of the decay pion from Σ^-, Σ^+, or Λ^0 produced in various processes under two different assumptions: (1) Each of these hyperons can exist in two degenerate states of opposite parities, with parity rigorously conserved in their decays. (2) Each of these hyperons can exist only in a single state of a definite parity (defined by the strong interactions). Parity is not conserved[2] in the decay processes.

If the hyperon has spin $\frac{1}{2}$, the distribution of the decay pion momentum $\mathbf{k}_\pi$ in the rest system of the hyperon is given by the weight function [see Eq. (31) and reference 2]:

$$W(\mathbf{k}_\pi)=\xi+\boldsymbol{\eta}\cdot\mathbf{k}_\pi, \tag{A.1}$$

where ξ and $\boldsymbol{\eta}$ are independent of $\mathbf{k}_\pi$. They have qualitatively different properties under different assumptions which are summarized in Table II.