have made closure approximations that are essentially equivalent to setting $a \cong 0$, and thus arrived at the result

$$\sigma^{(2)} \cong \sigma_0^{(2)},$$

which, as is apparent from the small value obtained for $(2+C)a/k$ in Table I, is very close to $\sigma^{(2)}$ of Eq. (23). Although these approximations lead to similar rough numerical results for electron-deuteron scattering where a/k is small and $C \cong -2$, the actual setting of $a \cong 0$ implies that $E_0 \cong Q^2\hbar^2/M$, which is an unreasonable equality for more tightly bound nuclei where E_0 is negative and much larger in magnitude than the deuteron binding energy. For nuclear systems such as triton or helium, factors like $a = Q^2\hbar^2/M - E_0$ are expected to become important and must be considered in any careful estimate of the dispersive contribution to the scattering cross section.

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High-Energy Phenomena in Nuclear Emulsions*

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Three interactions occurring in nuclear emulsions and caused by primaries of charge $\geqslant 2$ and energy $> 10^4$ Bev/nucleon are described. The electromagnetic cascade from one of these showers has been studied in detail. The mean free path for direct pair production (trident) has been measured in various intervals. The mean free path at energies < 10 Bev is shorter than the theoretical one calculated by Racah, but the small number of cases (7) at these energies allows large fluctuations. For the mean free path of the high-energy electrons whose energies measured by scattering have a lower limit of 6 Bev placed on them, we find a singificant disagreement with theory only if the average electron energy is as low as 10 Bev. We have shown that the average electron energy is more like 50–100 Bev, thus giving no disagreement with the theoretical cross section for direct pair production.

1. INTRODUCTION

THE study of high-energy interactions is important for two reasons. The first of these is the study of the flux and energy spectrum of the cosmic radiation itself. The magnetic field of the earth can be used to study the spectrum up to about 15 Bev. It is also possible to directly measure the energy of the primary particles by scattering up to about 10 Bev. Above 15 Bev the flux and energy spectrum must be obtained from the analysis of the nuclear interactions and the associated electromagnetic interactions of the cosmic radiation with the detector.

Another reason for studying high-energy interactions is to study the mechanism of the high-energy interaction itself. Analysis of high-energy interactions can provide information about nuclear forces and meson production. Through the mutual meson link they provide a beam of high-energy gamma rays for the study of electromagnetic processes. The term "high-energy phenomena" as used here refers to events in which the total energy of the intiating particle is greater than 10^4 Bev.

Only a few such high-energy events have been reported previously.[1–5] This is because with present-day balloon techniques exposures longer than twenty-four hours are difficult. For this reason it has not been possible to obtain from emulsions a reliable flux or charge spectrum of particles in this energy region. However, with the advent of large strip stacks and improved balloon techniques it should be possible in the near future to obtain reasonable statistics in a single exposure. In order to go beyond the range of modern accelerators,[6] we shall study events whose energy exceeds 10^4 Bev.

We have analyzed three such events,[7–9] all of which have been initiated by particles of charge greater than or equal to two. One of these events has been analyzed in great detail because of the long path length in emulsion of the secondary products of the interactions. Sections 2 and 3 of this paper present the data of the primary and secondary nuclear interactions. Section 4 presents the data on the electromagnetic cascade. Section 5 summarizes our conclusions.

2. PRIMARY NUCLEAR INTERACTIONS

The events were found either in a routine scan for heavy particles or under a rapid scan-by-eye technique developed to find high-energy events on 4 in.×10 in.

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strips. In this technique the plates are scanned with a jeweler's eyepiece for high-energy interactions. After an event has been found, the location is marked on the glass with a grease pencil. The large plate is then transferred to a scanning microscope to identify the event. The plates are then cut up and the event relocated and examined on a conventional microscope. A large fraction of the events located in this type of scan prove to be heavy particles. The ionization and orientation of the heavy particles found can be used to estimate the detection efficiency for interactions of a particular energy. Carbon particles at minimum ionization with a projected length of 1 cm can be detected with high efficiency by this method. Thus a high-energy interaction or "jet" with 36 minimum ionization tracks can be detected, provided the length in one emulsion is greater than 1 cm and the particles are still in a fairly tight cone.

Estimation of the Energies

In order to estimate the energy of jets, it is necessary to make certain assumptions about the nature of the interaction and the angular distribution of the mesons in the center-of-mass (c.m.) system of the colliding nucleons. We have assumed that the collisions are between pairs of nucleons of the colliding nuclei. Thus the analysis of the angular distribution will give the velocity of the individual nucleons of the incident particle. It is assumed that the angular distribution of the mesons in the c.m. system is symmetrical with respect to rotation of the collision axis through 180°, and that the mesons have relativistic velocities in the c.m. system. With these three assumptions, it can be shown[4] that the ratio of total to rest energy of the incident particle is given by:

$$\gamma_I = 2\cot\theta_f \cot\theta_{1-f} - 1, \tag{1}$$

where θ_f and θ_{1-f} are the angles which contain a fraction f and $1-f$ of the shower particles.

An estimate of γ_s', the ratio of the total energy to the rest energy of the shower particles in the c.m. system can be obtained from the angular distribution. γ_s' is given by the relation:

$$\gamma_s' = (\cot\theta_f \cot\theta_{1-f} \sin^2\theta_{\max} + \cos^2\theta_{\max})^{\frac{1}{2}}, \tag{2}$$

where $\theta_{\max}$ is the maximum angle observed in the shower. For showers of this energy range, we can make the approximation:

$$\gamma_s' \approx \left(1 + \frac{\sin^2\theta_{\max}}{\sin\theta_f \sin\theta_{1-f}}\right)^{\frac{1}{2}}. \tag{3}$$

The use of the angular distribution to estimate the energy of an interaction involves explicit assumptions about the nature of the angular distribution of the shower particle in the c.m. system. It is not possible to measure directly the energies of the particles involved. However, in one of the interactions the path length of the shower particles in emulsion is so long that three secondary interactions are observed and the development of the electromagnetic cascade studied, each of these providing a check on the energy of the primary particle estimated in this way.

Description of the Interactions

Interaction A occurs in the glass backing of an emulsion. The shower enters the emulsion after traveling 2.4 cm in the glass. The shower then travels 4.7 cm in one emulsion before leaving that emulsion and entering a facing emulsion. It enters the second emulsion near the edge and leaves after traveling about 1 cm in that emulsion. Because of the distortion and excessive blackening which occurs near the edge of an emulsion, the shower has not been studied in the second emulsion except to obtain the angular distribution of the secondary interaction which occurs there. The other two secondary interactions of A occur in the first emulsion. Thirty-one charged particles with the proper orietation and at minimum ionization enter the emulsion and travel 4.7 cm without suffering a measurable deflection. Seven of these enter the emulsion in a region in which only half of the available solid angle is observable in the emulsion so that these seven particles must be given a weight of two. Thus, the total number of charged particles is 38±4. In addition to the charged

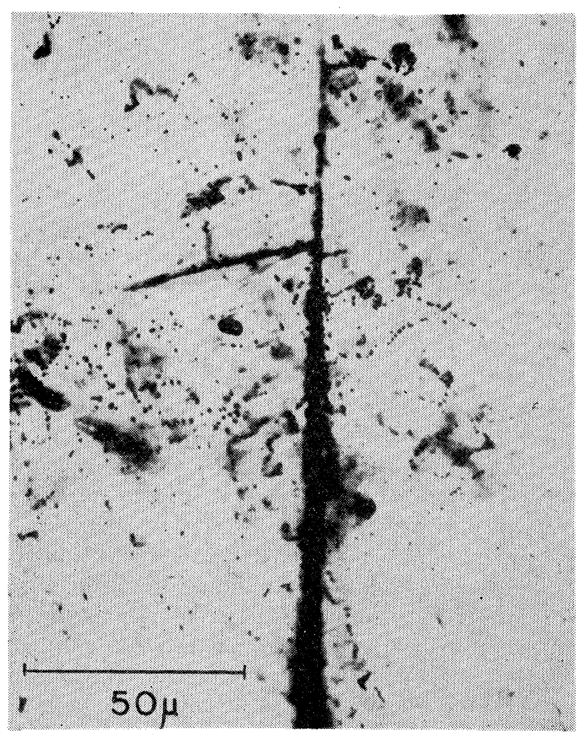

FIG. 1. Photomicrograph of an interaction of a $(1.4\pm0.7)\times10^{15}$ ev/nucleon nitrogen nucleus in which 110 singly charged shower particles and a boron fragment go on.

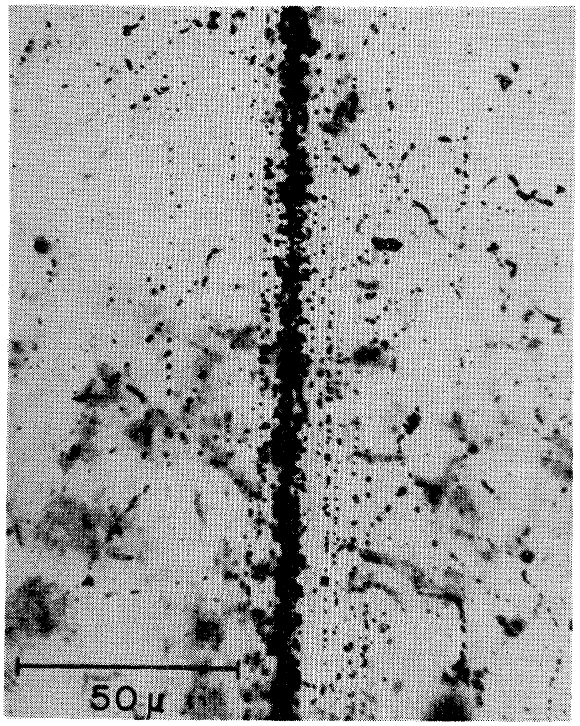

FIG. 2. The shower resulting from the interaction of Fig. 1, 4 mm from the point of interaction.

shower particles there is a track of a particle with a charge of 5 at the center of this shower. Since this is a fragment of the incident particle, it is possible to set 5 as a lower limit to the charge of the initiating particle. It is not possible to determine the charge of the incident particle because its entire path length before the interaction is in glass.

B is an interaction of a nitrogen primary with a nucleus of the emulsion. 110 charged particles traveling with minimum ionization originate in this interaction. There are three slow evaporation particles. The shower travels 2.9 cm and through three emulsions before leaving the stack. Figure 1 is a photomicrograph of *B* at the interaction. Figure 2 shows the core of the shower at a distance of 5 mm from the interaction. Figure 3 shows the shower 2.9 cm from the interaction. In this picture the shower particles have become separated from the core, and the track of the boron fragment is visible.

C is an interaction of an alpha particle with a nucleus of the glass backing of an emulsion. In this case the core travels 650 microns in glass before entering the emulsion. Twenty-three charged particles with minimum ionization emerge from the glass. After leaving this emulsion the shower travels through 2.2 radiation lengths of lead before entering another emulsion. In this distance the shower had multiplied to such an extent that it was impossible to count the number of particles. The data on the three interactions are given in Table I.

Interaction *B* is of interest because it is the most energetic event which we have observed to date. Because of the large number of shower particles in this interaction, it is possible to estimate the energy of the incident particle by using several different fractions of the shower particles. The vaues of γ_I obtained for the various values of f used are given in Table II. The agreement between the values of γ_I in Table II indicate that the assumptions about the angular distribution and velocities of the shower particles in the c.m. system are consistent with the results. The average of these values of γ_I and the measured charge give for the total energy of this particle $(1.4\pm0.7)\times10^{15}$ ev or 2.3×10^{3} ergs.

The integral angular distributions of the three events are given in Fig. 4. The solid curve is the shape of the integral angular distribution predicted by the Fermi theory[10] for grazing collisions. The broken curves are for isotropic and $\cos^4\theta'$ angular distributions in the c.m. system. The curves are plotted as a function of the normalized angle α. This is a convenient parameter to use in the analysis of high-energy interactions and was introduced by Bradt, Kaplon, and Peters.[11] The angular distributions are not isotropic in the c.m.

FIG. 3. The shower of Fig. 1, 2.9 cm from the point of interaction. The track of the boron fragment is visible at the core of the shower.

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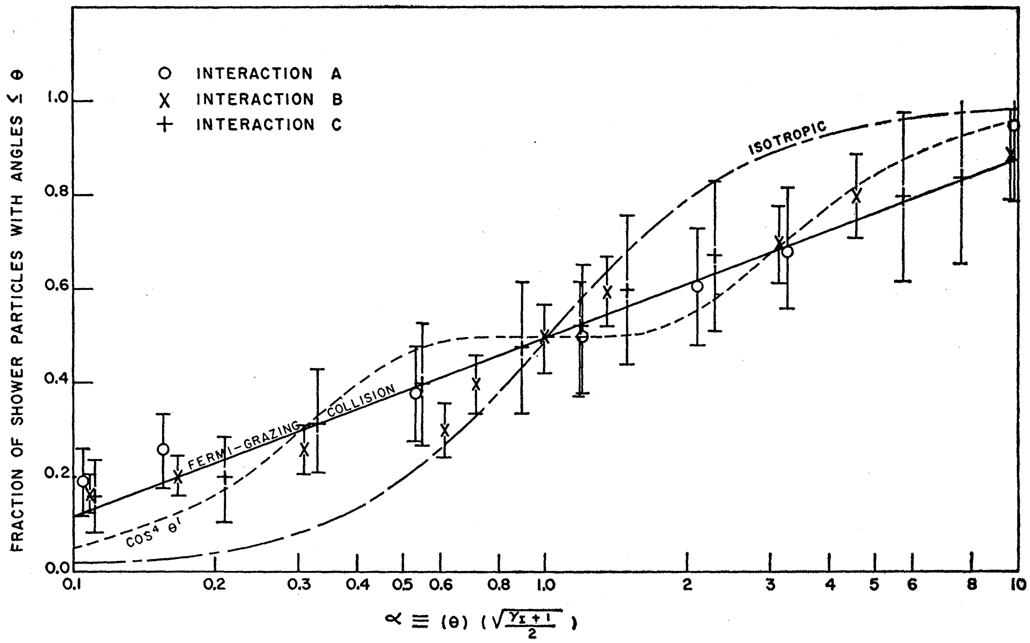

FIG. 4. Normalized integral angular distributions for the three interactions. The solid curve is the angular distribution predicted by the Fermi theory using an impact parameter corresponding to a grazing collision. The broken curves are the predicted angular distributions assuming that the mesons are produced isotropically or with a $\cos^4\theta'$ angular distribution in the c.m. system.

system. They agree best with the Fermi[10] theory for an impact parameter corresponding to a grazing collision.

The estimated energies and multiplicities for these three events are given in Table III. In collisions between aggregates of nucleons it is difficult to determine the multiplicity of meson production. In a given event it is not possible to determine exactly the number of nucleon-nucleon or meson-nucleon collisions which took place. Column three of Table III is an estimate of the number of collisions which took place. Since the particle which initiated event A entered the stack in the same glass plate in which it interacted, it is not possible to estimate the number of collisions which took place. The estimate of 2–6 collisions in B is based on the fact that the incident heavy was not completely broken up and that there were only three evaporation particles in the interaction. Since event C was a collision between an alpha particle and either a silicon or oxygen nucleus, it is estimated that at most there would have been four collisions. The observed multiplicities obtained from the estimated numbers of collisions is in agreement with the $\gamma_I^{\frac{1}{4}}$ dependence of multiplicity with energy observed at lower energies[4,12,13] and predicted by both Fermi[10] and Heisenberg[14] theories.

The average energy of the mesons in the c.m. system of events A and C as determined from the maximum emission angle is between that predicted by the Fermi theory and that predicted by the Heisenberg theory; the average energy of the mesons of B is in agreement with Fermi theory.

TABLE I. Summary of experimental data on three high-energy showers observed in emulsions. γ_c is the ratio of total to rest-mass energy of the incident primary in the center-of-mass system. γ' is the γ in the center-of-mass system for the secondary mesons.

Interaction	Charge of incident particle	Charge of outgoing fragment	No. of evaporation particles	No. of shower particles	γ_c	Energy of incident particle Bev/nucleon	γ'
A	≥ 5	5	?	38 ± 4	150 ± 35	4.5×10^4	22–11
B	7 ± 1	5 ± 1	3	110 ± 5	240 ± 70	1.1×10^5	110–50
C	2	$\leqslant 1$	?	25	120 ± 30	2.7×10^4	32–13

TABLE II. γ_I, the ratio of total energy to rest-mass energy of the incident particle, is given for different f values for interaction B.

f	θ_f milliradians	θ_{1-f} milliradians	γ_I
0.1	0.32	53	$1.2\pm0.4\times10^5$
0.2	0.70	19.5	$1.5\pm0.5\times10^5$
0.3	2.60	13.4	$0.6\pm2\ \times10^5$
0.4	3.00	5.8	$1.1\pm0.3\times10^5$
0.5	4.2	4.2	$1.1\pm0.3\times10^5$

3. SECONDARY NUCLEAR INTERACTIONS

There are three secondary interactions of mesons in event A. The data for the three interactions are given

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[13] Tchang-Fong Hoang, J. phys. radium 14, 395 (1953).
[14] W. Heisenberg, Z. Physik 133, 65 (1952).

TABLE III. Multiplicity of charged mesons produced in high-energy interactions in emulsion stacks.

Event	Estimated energy Bev/nucleon	Estimated number of encounters	Multiplicity of charged mesons Observed	Predicted by Fermi	Predicted by Heisenberg
A	4.5×10^4	≥ 1	≤ 38	16	20
B	1.1×10^5	$2-6$	$18-55$	21	25
C	2.7×10^4	$1-4$	$6-25$	15	18

in Table IV. Two values are given for the estimated energy. The first of these is determined from the analysis of the angular distribution of the secondary interactions; the second is based on the average energy of the mesons in the c.m. system as determined from the maximum angle of emission of the shower particles in the parent interaction. The energies of two of the secondaries lies within the range predicted from the angular distribution of the parent interaction. The energy of the third is much lower. However, judging by the large number of evaporation particles, there has been more than one collision in this event.

The angular distributions of $A1$ and $A2$ are consistent with that predicted by Fermi theory for a median impact parameter; $A3$ is consistent with an isotropic distribution in the c.m. system. There is no evidence in any of these three events for the asymmetric angular distributions of the type reported by Lal and coworkers,[3] in which the mesons in the secondary interactions were emitted predominantly in the backward direction in the c.m. system.

In principle, one should be able to use the secondary interactions in this event to study the variation of multiplicity with impact parameter. All three of the secondary interactions are initiated by particles emitted in the forward cone in the c.m. system. However, in this event it is not possible to obtain any reliable information because the numbers of evaporation particles in the three secondary events indicates the possibility of more than one collision in at least one of the events.

4. ELECTROMAGNETIC SHOWER

In addition to the 31 charged particles observed in interaction A, there are 14 primary pairs produced by γ rays from the decay of the π^0 mesons. Along each of these pairs an electron shower develops. We have

TABLE IV. Summary of experimental data for the secondary interactions in Interaction A.

Event	No. of evaporation particles	No. of charged shower particles	Total energy of shower particle in Bev from angular distribution of secondary	Total energy of shower particle in Bev from angular distribution of primary interaction
$A1$	3	7	620	$460-230$
$A2$	4	13	270	$460-230$
$A3$	7	20	30	$460-230$

studied this soft shower in detail over a length of 5 cm. The flatness of the tracks (some as long as 5 cm in a 400-micron emulsion), and the presence of the accompanying boron core have allowed us to make relative scattering measurements which for the fortuitously oriented electrons are quite reliable. Our preliminary report on this shower[8,9,15] stressed the rapid shower development and the high trident cross section (direct pair production by an electron). Other workers[16–22] have investigated this problem with conflicting results. Block and King[16] have calculated the fraction of bremsstrahlung pairs which will materialize so close to the track of the parent electron as to be mistaken for a trident (so-called pseudotridents). They have also measured the trident cross section[17] between 0.1 and 10 Bev and find it in agreement with the theoretical value of the cross section as calculated by Bhabha,[23] Nishina,[24] and Racah.[25] Koshiba and Kaplon,[18,19] Debenedetti *et al.*[20] and Lohrmann[21] have measured the trident cross section and find an anomalously large cross section at high energies. Leonard,[22]

TABLE V. Comparison of the observed cascade development with the results of a Monte Carlo calculation. The first row represents the number of additional electrons, Ne, of energy greater than 100 Mev within 0.5 radiation length of the initiating pair origin. Row A is the observed frequency in our 14 showers for the various values of Ne (the number of electrons >100 Mev). Rows B, C, and D are the results of a Monte Carlo calculation on 35 showers assuming an initial photon energy of 1000, 100, and 10 Bev, respectively (Tables A-2-a, A-2-b, and A-2-c of Koshiba and Kaplon[19]).

N_e	0	1 and 2	3 and 4	5 and 6	7 and 8	9–10
A	0.43	0.29	0.07	0.07	0.14	0
B	0.28	0.28	0.17	0.14	0.03	0.09
C	0.40	0.29	0.26	0.03	0.03	0
D	0.71	0.20	0.09	0.0	0	0

using 440-Mev electrons, has found the measured cross section to be in agreement with the theoretical value at this energy. Since we have never reported our results in detail, we felt that we should re-examine our data and report our results here.

We have followed all of the electrons in the electromagnetic cascade and have found 28 pairs whose origins were unresolvable from the electron track which was being followed. These pairs are either tridents or pseudotridents. Our limits of resolution are $0.2\,\mu$ in the

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[16] M. M. Block and D. T. King, Phys. Rev. **95**, 171 (1954).
[17] Block, King, and Wada, Phys. Rev. **96**, 1627 (1954).
[18] M. Koshiba and M. F. Kaplon, Phys. Rev. **97**, 193 (1955).
[19] M. Koshiba and M. F. Kaplon, Phys. Rev. **100**, 327 (1955).
[20] Debenedetti, Garelli, Talone, Vigone, and Wataghin, Nuovo cimento **1**, 226 (1956).
[21] E. Lohrmann, Nuovo cimento **4**, 820 (1956).
[22] Stanley L. Leonard, Bull. Am. Phys. Soc. Ser. II, **1**, 167 (1956).
[23] H. J. Bhabha, Proc. Roy. Soc. (London) **A152**, 559 (1935).
[24] Nishina, Tomonaga, and Kobayasi, Sci. Papers Inst. Phys. Chem. Research Tokyo **27**, 137 (1935).
[25] G. Racah, Nuovo cimento **14**, 93 (1937).

plane of the emulsion and 0.4 μ in depth. The energies of the electrons followed, as measured by scattering, ranged from 30 Mev to $\geqslant$45 Bev. A total path length of 219 cm was followed.

In order to compare the observed trident cross section with the theoretical, it is necessary to determine the average energy of the electrons followed and also to correct the number of pairs observed for the number of pseudotridents. It is particularly important to accurately estimate the energy. If the energy is underestimated, it will produce two effects, both of which tend to make the observed cross section appear to be larger than the theoretical. If one underestimates the energy, the theoretical cross section used for comparison will be too small, and the correction for pseudotridents will be too small, giving too large an experimental cross section.

The electrons which were followed are either primary pairs from the decay of π^0's into two γ rays or are secondary pairs produced either by bremsstrahlung or by direct pair production. The energies of the primary pairs cannot be measured directly by scattering because of their high energy. We have estimated their energy as follows:

i. We have assumed that the π^0 mesons are produced with the same energy as the charged shower particles, and the energy is shared equally between the four electrons produced. This gives an estimate of about 100 Bev as the energy of each electron from a primary pair.

ii. We have assumed that existing shower theory is correct, and the electromagnetic cascades observed on the primary pairs have been compared to the Monte Carlo calculations of Kaplon.[19] Table V gives the results of the comparison for our 14 showers. Although the statistics are poor, the results are more consistent with the assumption that the initiating photon energies are of the order of 100 Bev rather than 10 Bev.

iii. We have estimated the energy of the primary pairs by measuring their spatial separation as a function of the distance from the origin. Lohrmann[26] has calculated the theoretical expression for taking into account bremsstrahlung and multiple scattering. The observed separation of the electrons varies as the $\frac{3}{2}$ power of the distance from the origin showing that the separation is due to scattering. The data on the primary pairs are tabulated in Table VI. k, the photon energy, tabulated in the last column of Table VI is calculated from Lohrmann's formula, assuming equipartition of energy between the electrons of the pair, and therefore should be regarded as a lower limit to the energy of

TABLE VI. Summary of the data on the primary pairs. The second column gives the distance in cm from the nuclear interaction to the origin of the pair. The third column gives the angle which the origin of the pair makes with the shower axis. Columns 4, 5, and 6 give the energies of each electron and of the photon based on scattering measurements. Column 7 gives the energy of the photon calculated from the spatial separation of the pair using Lohrmann's formula and assuming equipartition of energy.

Pair	Distance from origin cm c	Angle with core radians	E_1 Bev	E_2 Bev	E_1+E_2 Bev	k Bev
9	1.17	0.011	$\geqslant$9	2.3 $\pm$0.5	$\geqslant$11	16
50	1.77	0.045	$\geqslant$1.5	$\geqslant$1.5	$\geqslant$3	20
5	1.93	0.0036	$\geqslant$13	$\geqslant$13	$\geqslant$26	75
25	1.93	0.0045	5$\pm$1	$\geqslant$0.6	$\geqslant$5.6	8
36	2.59	0.0080	$\geqslant$5	$\geqslant$2.5	$\geqslant$8	28
8	2.60	0.011	$\geqslant$10	$\geqslant$10	$\geqslant$20	14
39	2.64	0.0003	$\geqslant$22	0.26$\pm$0.03	$\geqslant$22	6
15	2.93	0.0022	$\geqslant$17	$\geqslant$7	$\geqslant$24	200
37	3.19	0.0017	$\geqslant$9	2.8 $\pm$0.4	$\geqslant$12	36
51	3.54	0.0022	$\geqslant$5	$\geqslant$5	$\geqslant$10	40
53	4.59	0.0009	>18	>18	>36	200
56	4.87	0.012	1.8$\pm$0.3	2.4 $\pm$0.4	4.2$\pm$0.5	19
59	4.98	0.00028				1440
58	5.85	0.00026				820

TABLE VII. Summary of data on the secondary pairs produced in this event. The table is divided into three parts according to where the pair occurs. The left side gives the data on unresolved pairs, the right side the data on resolved pairs. Column 1 gives the parent electron. Column 2 and 3 give energy estimates of the parent from scattering and spatial separation respectively. Column 4 gives the distance in radiation lengths from the origin of parent to origin of pair. Column 5 gives the energy of the created pair if one assumes that the two lower energy particles are the pair. Column 6 gives $(1-f)$ for each pair with the energy of the curve used to read f shown in parentheses. Column 7 and 8 on the right-hand side give the corresponding distance and energy for the resolved pairs.

Parent	E_e Bev	$k/2$ Bev	R rad. lengths	k (pair) Bev	$1-f$	R rad. lengths	k (pair) Bev
			0–0.5 radiation lengths				
5.1	$\geqslant$13	38	0.286	1.2	0.05(100)	0.462	3.2
5.14[a]	1–2	2	0.019	0.7	0.05(5)	0.327	0.11
8.1	$\geqslant$10	7	0.062	$\geqslant$6.8	0.00(100)	0.442	0.20
8.2	$\geqslant$10	7	0.377	2.3	0.10(100)	0.462	0.11
8.4	$\geqslant$12	3	0.175	1.5	0.10(50)	0.462	0.24
8.7	$\geqslant$7	0.8	0.306	$\geqslant$2.8	0.65(10)	0.338	0.65
8.39	$\geqslant$11	1.2	0.356	$\geqslant$5	0.70(10)	0.301	0.74
8.35	$\geqslant$6	10	0.026	0.4	0.10(50)		
9.1	$\geqslant$9	8	0.223	0.21	0.15(100)		
15.11	$\geqslant$6	11	0.005	2.2	0.01(10)		
37.2	$\geqslant$9	18	0.279	7.3	0.05(100)		
39.8	$\geqslant$5	1.6	0.207	0.48	0.57(10)		
50.1[b]	$\geqslant$1.5	10	0.150	0.42	0.65(5)		
50.2[b]	$\geqslant$1.5	10	0.102	0.97	0.50(5)		
51.9[c]	0.5	0.16	0.043	0.03	0.38(5)		
58.1	$\geqslant$40	410	0.093	$\geqslant$6	0.00(100)		
58.2	$\geqslant$40	410	0.134	5	0.00(100)		
59.2	$\geqslant$50	720	0.231	$\geqslant$20	0.00(100)		
58.2.3[b]	5	1	0.260	0.30	0.73(5)		
			0.5–1 radiation lengths				
5.1.2[a]	1–2	0.21	0.720	0.025	0.88(5)	20 resolved	
8.1.2	$\geqslant$4	46	0.812	0.65	0.80(10)	pairs produced	
8.2.1[b]	5	7	1.000	0.05	0.89(5)		
15.2	$\geqslant$7	100	0.618	0.7	0.25(100)		
39.2	$\geqslant$22	3	0.780	2.0	0.32(100)		
51.2	$\geqslant$5	20	1.000	1.0	0.55(50)		
53.1	$\geqslant$20	100	0.520	0.79	0.20(100)		
			>1.0 radiation lengths				
15.1	$\geqslant$14	100	1.102	0.18	0.42(100)	27 resolved	
15.2.1	$\geqslant$6	100	1.205	0.05	0.46(100)	pairs produced	

[a] Trident occurs on an electron in the 1–2 Bev energy range.
[b] Trident occurs on an electron in the 2–6 Bev energy range.
[c] Trident occurs on an electron in the 0.5–1.0 Bev energy range.

[26] E. Lohrmann, Nuovo cimento 2, 1029 (1955).

TABLE VIII. Correction for pseudotridents and mean free path for trident production computed by method 1. The radiation length in emulsion is 2.9 cm.

Energy region Bev (measured by scattering)	Path length followed (cm of emulsion)	No. of unresolved pairs	True tridents from method 1	$\lambda_{\text{method 1}}$ (cm of emulsion)
0–0.5	71.2	0	0	...
0.5–1	14.8	1	0.38	39±39
1–2	24.8	2	0.93	26_{-13}^{+0}
2–6	38.3	4	2.77	14_{-5}^{+13}
>6	70.3	21	4.60	15±7

the photon. This is particularly true in the case of pairs 9, 29, and 37 where scattering on the individual members of the pair indicates that one is much lower in energy than the other.

The average energy of the photons from Table VI is 200 Bev and is consistent with the estimate of the average energy obtained from the Monte Carlo calculation. Thus the estimate of the energy of the shower particles from the electromagnetic cascade is in agreement with the estimate from the charged shower particles. We therefore believe that the high-energy electrons whose energy is too great to be measured by scattering do have energies of the order of 100 Bev. 21 of the 28 observed tridents occur on electrons in this energy range. Table VII summarizes the data on the secondary pairs of this event. The ratio of unresolved pairs to resolved pairs is 19/7 in the first half a radiation length, 7/20 in the second half, and 2/27 in the third half a radiation length. This shows that a large fraction of the unresolved pairs in the first half a radiation length are pseudotridents.

If P is the total number of pairs produced at the end of R radiation lengths, f the fraction of the bremsstrahlung pairs which give rise to pseudotridents, and T' the number of unresolvable pairs produced in R radiation lengths, then T, the number of real tridents produced in R, is:

$$T=(T'-fP)/(1-f). \tag{4}$$

TABLE IX. Correction for pseudotridents and mean free path calculation by method 2. f is the fraction of bremsstrahlung pairs occurring within 0.2μ of its parent electron; P is the total number of pairs observed; T' the number of pairs that are unresolved from the track of the parent electron; and T is the corrected number of true tridents [calculated by Eq. (4)]. λ is the mean free path for trident production. Total path length observed was 70.3 cm.

Assumed primary energy Bev	f	P	T'	T	λ (cm of emulsion)
		0.5 radiation length			
100	0.82	21	14	0	...
50	0.61	21	14	3.1	44±26
10	0.28	21	14	11.2	6±2
		1.0 radiation length			
100	0.60	46	19	0	...
50	0.43	46	19	0	...
10	0.19	46	19	12.7	5.5±1.5

We have used for f the values from Kaplon and Koshiba.[19] We believe that $0.3\,\mu$ is more nearly correct than $0.2\,\mu$ for our resolution since, owing to shrinkage, the resolution in depth is only $0.4\,\mu$ while in the plane of the emulsion it is $0.2\,\mu$. However, this does not change their f values by more than 10–15%. We have corrected for pseudotridents in two different ways. (1) We assigned a probability of being a trident, $1-f$, to each individual unresolved pair. This probability is listed in column 6 of Table VII. This method gives us an upper limit on the number of true tridents. (2) We used Eq. (4) and computed the probable number of true tridents at the end of 0.5 and 1.0 radiation lengths.

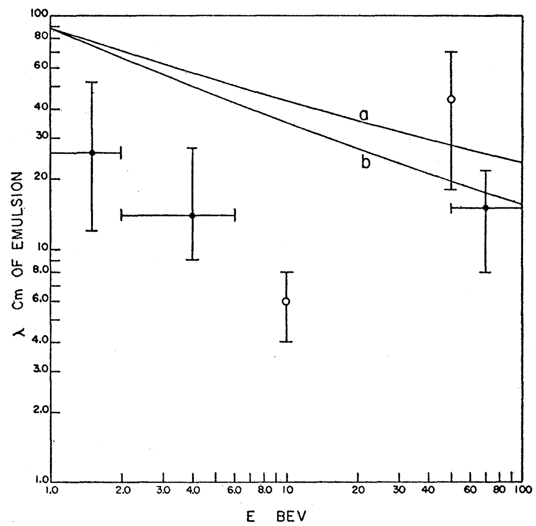

FIG. 5. Comparison of theoretical and experimental mean free path for trident production. Curves a and b are Racah's values for screened and unscreened cross sections respectively. The solid circles are the experimental points from Table VIII. The open circles are the points from Table IX at 0.5 radiation length, assuming that the average energy of the electrons is 10 Bev or 50 Bev.

Table VIII gives the results obtained by method 1; Table IX gives the results obtained by method 2. We have not used the seven tridents occurring on electrons of energy <6 Bev since we used the path length of the high-energy electrons (70 cm) to compute the mean free path in Table IX. We see that if the average energy of these electrons is 100 Bev, all of the unresolved pairs can be accounted for by bremsstrahlung pairs. Only if the energy of the electrons is as low as 10 Bev do we obtain a mean free path which is in serious disagreement with the theoretical mean free path.

Figure 5 shows the trident mean free path as a function of energy. Block *et al.*[17] have shown that Racah's[25] and Bhabha's[23] results are consistent. We have plotted in Fig. 5 the values of Racah taken from Block *et al.*[17] Because of the poor statistics we do not feel that the two low-energy points are in disagreement with the theoretical mean free path. The high-energy point is in serious disagreement with theory only if the average energy of the electrons is 10 Bev. We feel that the average energy of the electrons cannot be this low but is more like 50 to 100 Bev.

5. CONCLUSIONS

There exist in the cosmic radiation heavy primaries of energies greater than 10^{14} ev per nucleon. The flux of such particles as determined from these three interactions and an estimate of the volume of emulsion exposed and scanned in our laboratory is of the order of 10^{-4} particle/m^2 sec steradian.

The mean free path for trident production measured in such high-energy showers is extremely sensitive to the energy of the electrons. Koshiba and Kaplon[19] have outlined the experimental difficulties involved in measuring such high energies. As Block and King[16] have pointed out, the correction for pseudotridents is particularly important at these high energies, especially within the first radiation length of the origin of the parent pair. Our results based on the 28 unresolved pairs found in this single high-energy event are in agreement with theory unless the average energy of the high-energy electrons is as low as 10 Bev. We have shown that the average energy of the electrons is of the order of 50 to 100 Bev rather than 10 Bev, thus giving no disagreement between theory and experiment.

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Scattering of K^+ Particles*

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An experiment has been performed to investigate the relative probability of scattering of $K_{\pi 2}$ and τ mesons. If, as is thought on the basis of the analysis of Dalitz and others, these particles differ in spin-parity properties, they may be expected to have different scattering cross sections, and the relative proportions of τ and $K_{\pi 2}$ mesons in a beam of scattered K particles would differ from their relative proportions in a beam of directly-produced particles. In the present experiment, a measurement was made of the composition of a K^+-particle beam in which it is estimated that more than $\sim 75\%$ of the particles had undergone scattering. Comparison with other results on the composition of beams consisting predominantly of directly-produced particles indicates that the relative numbers of τ and $K_{\pi 2}$ mesons are unchanged, within $\sim 30\%$. The results are therefore consistent with the assumption that the τ and $K_{\pi 2}$ are the same particle.

INTRODUCTION

As has been shown by Dalitz,[1] the energy distribution and angular correlation of the three charged pion secondaries of τ mesons provide a means of determining the spin and parity of the τ. There exists now a large amount of data which have been analyzed to obtain this information,[2] and the results indicate that the spin-parity properties of the τ meson differ from those of the θ or $K_{\pi 2}$ meson, which decays into two pions.[3–7] Most of the available data are

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[1] R. H. Dalitz, Phys. Rev. **94**, 1046 (1954); Proceedings of the Fifth Annual Conference on High-Energy Physics (Interscience Publishers, Inc., New York, 1955).

[2] Report of the Pisa Conference on Elementary Particles, 1955, Nuovo cimento (to be published); Feld, Odian, Ritson, and Wattenberg, Phys. Rev. **100**, 1539 (1955); R. Haddock, Phys. Rev. **100**, 1803(A) (1955); Bhowmik, Evans, Van Heerden, and Prowse, Nuovo cimento **3**, 574 (1956); Orear, Harris, and Taylor, Phys. Rev. **102**, 1676 (1956); Biswas, Ceccarelli-Fabrichesi, Ceccarelli, Gottstein, Varshneya, and Waloschek, Nuovo cimento **3**, 825 (1956); Heckman, Smith, and Barkas, University of California Radiation Laboratory Report UCRL-3291, February 22, 1956 (unpublished); See also Proceedings of the Sixth Annual Conference on High Energy Physics, (to be published).

[3] The experimentally established decay schemes, $\theta^0 \to \pi^+ + \pi^- + 214$ Mev[4] and $K_{\pi 2}{}^+ \to \pi^+ + \pi^0 + 219$ Mev,[5–7] indicate that since both θ^0 and $K_{\pi 2}{}^+$ decay into two pions, with Q values which agree within the statistical errors, it is reasonable to regard the θ^0 as the neutral counterpart of the $K_{\pi 2}$, and to discuss them as a single particle.

[4] Thompson, Burwell, Cohn, Huggett, and Karzmark, Phys. Rev. **95**, 661 (1954).

FIG. 1. Photomicrograph of an interaction of a $(1.4\pm 0.7)\times 10^{15}$ ev/nucleon nitrogen nucleus in which 110 singly charged shower particles and a boron fragment go on.

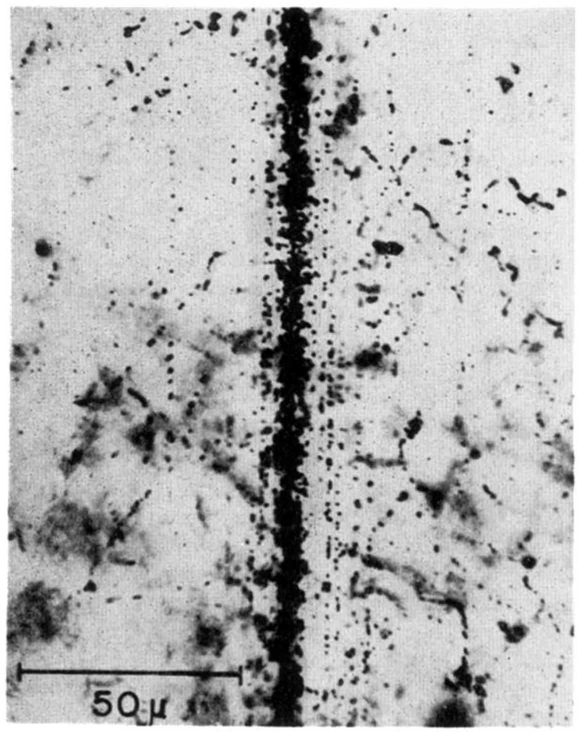

FIG. 2. The shower resulting from the interaction of Fig. 1, 4 mm from the point of interaction.

FIG. 3. The shower of Fig. 1, 2.9 cm from the point of interaction. The track of the boron fragment is visible at the core of the shower.