High-Energy Proton-Proton Scattering and the Lévy Potential*

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The predictions of Lévy's potential for high-energy proton-proton scattering have been examined. The potential was used in the form of Blatt and Kalos, and contains three adjustable parameters. Values of the parameters that fit the low-energy data as well as values that lead to approximately correct total cross sections at high energies have been tried. Phase shifts obtained by numerical integration at several energies were used to calculate the cross section and the polarization. Definite disagreement with experimental results was found. Features of Lévy's potential responsible for the disagreement are discussed.

I. INTRODUCTION

The success of a form of Lévy's potential[1] in fitting the low-energy nucleon-nucleon data except for the quadrupole moment[2] made it appear desirable to examine the predictions of the same potential over the energy region from 100 Mev to 440 Mev. The potential is given by

$$V/m_\pi c^2=\tfrac{1}{3}(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2)\left(\frac{g^2}{4\pi}\right)\left(\frac{m_\pi}{2M}\right)^2 \times\left[\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+S_{12}\left(1+\frac{3}{x}+\frac{3}{x^2}\right)\right]\frac{e^{-x}}{x} -\frac{6}{\pi}\left(\frac{G^2}{4\pi}\right)^2\left(\frac{m_\pi}{2M}\right)^2\frac{K_1(2x)}{x^2}. \quad (1)$$

Here M is the nucleon mass, m_π the pion mass, and $x=r/(\hbar/m_\pi c)$, where $\hbar/m_\pi c=1.40\times10^{-13}$ cm was used. Table I gives values $G^2/4\pi$, $g^2/4\pi$, and the core radius x_c which approximately fit the low-energy data.[2]

Table I. Blatt-Kalos parameters fitting low-energy nucleon-nucleon data.

Deisgnation \ Quantity	$G^2/4\pi$	$g^2/4\pi$	x_c
1 BK	13.232	7.108	0.42
2 BK	18.615	11.188	0.56

II. METHOD OF CALCULATION

Phase shifts were obtained from radial functions calculated by numerical integration. For uncoupled radial equations the modified Hartree procedure[3] was used with an interval of 0.01 for $x<1.5$ and 0.02 for $x>1.5$. A generalization of the above method[4] to coupled second-order linear differential equations was used for states coupled by the tensor force. If the coupled equations are

$$y''+\mathbf{z}y=0, \quad (2)$$

where y is a 1-by-n column matrix and $\mathbf{z}$ an n-by-n matrix, then, letting $\xi=(I+\mathbf{z}\Delta^2/12)y$, where I is a unit matrix and Δ the interval in the independent variable, one finds[5]

$$\xi(x+\Delta)=-\xi(x-\Delta)+[2I-\Delta^2\mathbf{z} +(\Delta^4/12)\mathbf{z}^2]\xi(x)+O(\Delta^6). \quad (3)$$

This expression reduces to the modified Hartree form for n uncoupled equations if $\mathbf{z}$ is diagonal. The integrations were performed with the aid of an IBM 602A machine.

The cross section and polarization were calculated from the phase shifts by using the amplitudes of Breit, Ehrman, and Hull.[6]

III. RESULTS

The isotopic triplet phase shifts for 1 BK and 2 BK are given in Tables II and III. Singlet phase shifts of orbital angular momentum L are denoted by K_L.

Table II. 1 BK phase shifts, in degrees.

Phase shift \ Energy Mev	100	240	300	440
K_0	23.8	−7.0	−16.6	−34.5
K_2	3.2	16.1	20.9	26.4
K_4		1.2	1.9	3.7
$\delta^P{}_0$	63.5	53.5	46.9	33.5
$\delta^P{}_1$	11.0	18.9	16.3	8.1
$\delta^\alpha{}_2$	30.6	34.1	30.5	22.0
ϵ_2	−2.5	−6.6	−9.3	−22.8
$\delta^\beta{}_2$	0.4	3.8	6.3	9.7
$\delta^F{}_3$	0.0	1.4	3.8	6.0
$\delta^F{}_4$	0.6	4.1	7.2	12.0
$\delta^H{}_4$		0.30	0.57	1.44
$\delta^H{}_5$		0.24	0.47	1.21
$\delta^H{}_6$		−0.03	−0.05	0.11

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[1] M. M. Lévy, Phys. Rev. **88**, 725 (1952).

[2] J. M. Blatt and M. H. Kalos, Phys. Rev. **92**, 1563 (1953).

[3] L. Feinstein and M. Schwarzschild, Rev. Sci. Instr. **12**, 405 (1941).

[4] Suggested to the authors by Professor Breit.

[5] L. Goldfarb and D. Feldman, Phys. Rev. **88**, 1099 (1952) have given a scheme for numerical integration of coupled differential equations based on work of reference 3. They do not take into account the fourth power of the interval.

[6] Breit, Ehrman, and Hull, Phys. Rev. **97**, 1051 (1955); hereafter referred to as BEH.

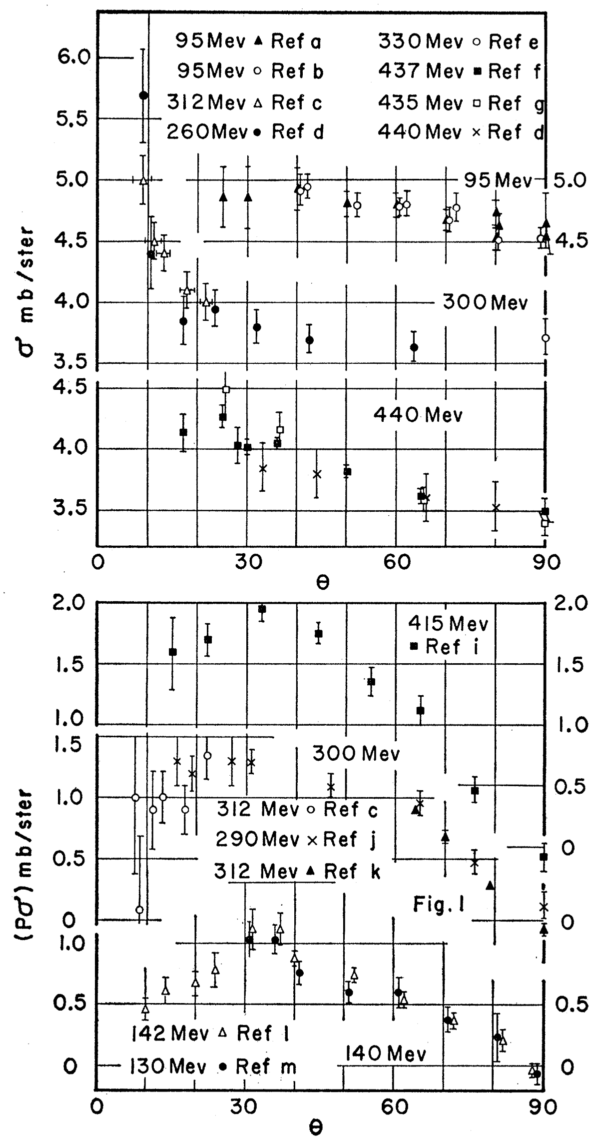

FIG. 1. Summary of high-energy p-p scattering data. The references are as follows: [a] Kruse, Teem, and Ramsey, Phys. Rev. **101**, 1079 (1956); [b] Kruse, Teem, and Ramsey, Phys. Rev. **94**, 1795 (1954); [c] Chamberlain, Pettengill, Segrè, and Wiegand, Phys. Rev. **95**, 1348 (1954); [d] J. D. Garrison, thesis, University of California Radiation Laboratory Report UCRL-2659, 1954 (unpublished); [e] Chamberlain, Pettengill, Segrè, and Wiegand, Phys. Rev. **93**, 1424 (1954); [f] Sutton, Fields, Fox, Mott, and Stallwood, Phys. Rev. **97**, 783 (1955); [g] Mott, Sutton, Fox, and Kane, Phys. Rev. **90**, 712 (1953); [h] Smith, McReynolds, and Snow, Phys. Rev. **97**, 1186 (1955); [i] Kane, Stallwood, Sutton, Fields, and Fox, Phys. Rev. **95**, 1694 (1954); [j] Chamberlain, Segrè, Tripp, Wiegand, and Ypsilantis, Phys. Rev. **93**, 1430 (1954); [k] Chamberlain, Donaldson, Segrè, Tripp, Wiegand, and Ypsilantis, Phys. Rev. **95**, 850 (1954); [l] Harwell data (private communication to G. Breit); [m] J. M. Dickson and D. C. Slater, Nature **173**, 946 (1954).

Uncoupled triplet phase shifts are written $\delta^L{}_J$. The coupled triplet states are parameterized in terms of real eigenphase shifts $\delta^\alpha{}_J$, $\delta^\beta{}_J$, and a real coupling

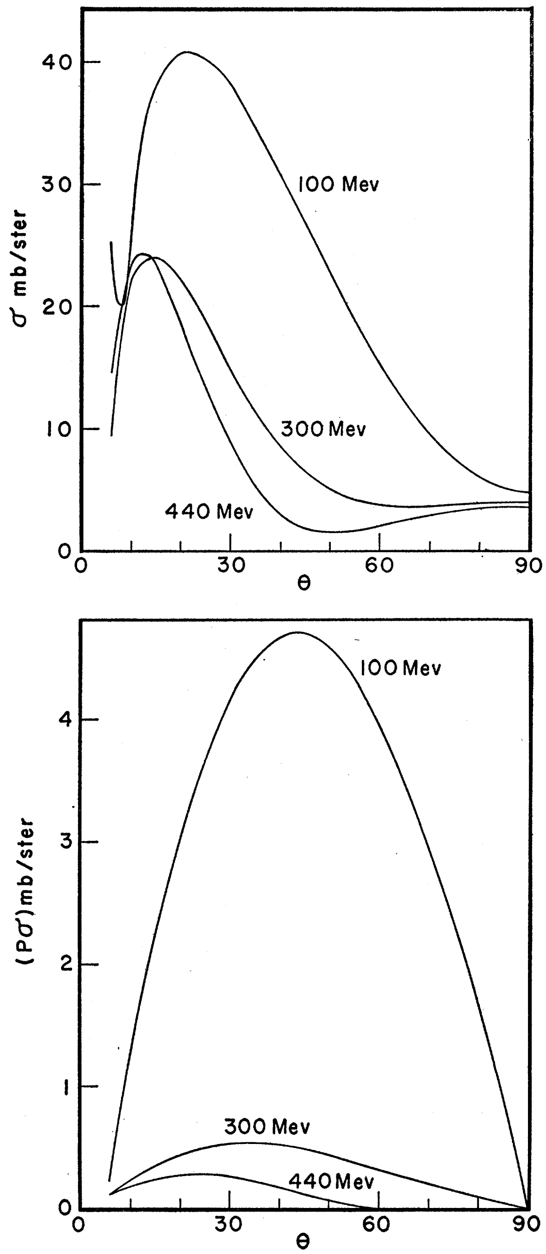

FIG. 2. Theoretical differential cross section and $P\sigma$ predicted by 1 BK, using the phase shifts of Table II. Values are plotted against the center-of-mass angle, θ, in degrees.

parameter ϵ_J,[7] the convention being that ϵ_J approaches zero at zero energy.

For the $^3F_4$ and $^3H_4$ states the coupling was neglected. Because of the large values of L, the effect of coupling is negligible in this case.

Figure 1 summarizes the experimental data with which all calculated results are to be compared. Since the convenient quantity theoretically is the polarization

[7] F. Rohrlich and J. Eisenstein, Phys. Rev. **75**, 705 (1949); J. M. Blatt and L. C. Beidenharn, Phys. Rev. **86**, 399 (1952).

multiplied by the differential cross section rather than the polarization itself, all double-scattering data have been presented as $P\sigma(\theta)$. In some cases this necessitated converting published data on asymmetries or polarization into the form desired. For example, the double-scattering data at 415 Mev[8] were combined with single-scattering data from the same laboratory[9] at 437 Mev to obtain the points plotted in Fig. 1. A curve drawn by eye through the differential cross section points of Fig. 1 for 437 Mev supplied values of $\sigma(\theta)$ at the desired angles. The errors indicated are those of the double-scattering data only, since these are relatively larger than the single-scattering errors. For this case there are single-scattering data at the closer energy 419 Mev[10] which indicate a smaller cross section at smaller angles than for 437 Mev. The change in $P\sigma(\theta)$ is about 15%, but in view of the comparisons to be made later this difference is negligible even if it is experimentally significant.

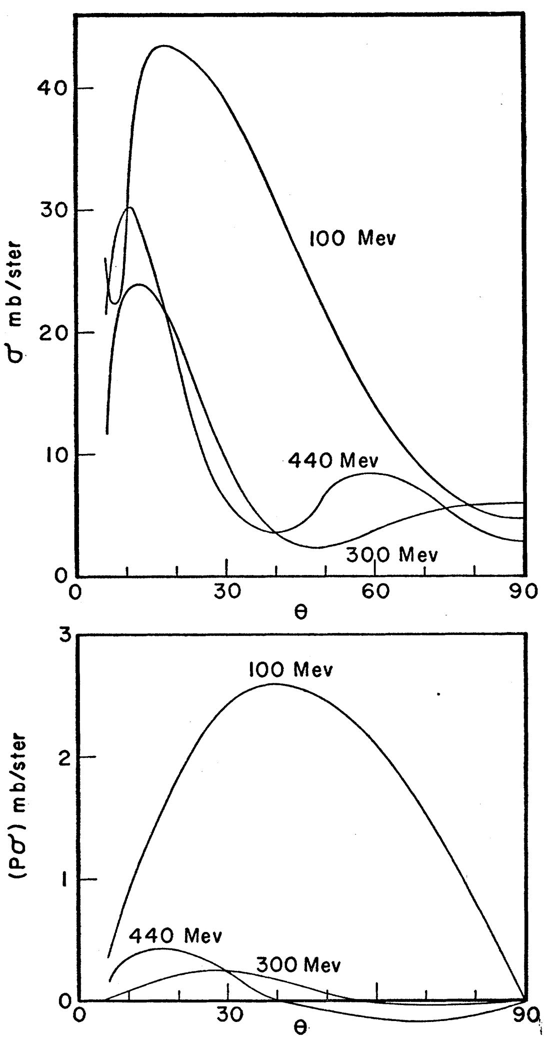

FIG. 3. Theoretical differential cross section and $P\sigma$ predicted by 2 BK, using the phase shifts of Table III. Values are plotted against the center-of-mass angle, θ, in degrees.

The intention in presenting the data has been to provide a general idea of the size and angular distribution of σ and $P\sigma$, and for this reason not all available data have been included.

The differential cross section $\sigma(\theta)_{p-p}$ and $P\sigma(\theta)_{p-p}$ obtained from the given phase shifts are shown in Figs. 2 and 3.

In addition to the cases that fit low-energy data, other values of the parameters have been tried. The case most systematically investigated, denoted hereafter by 2 BK, M, is obtained by reducing the central force in 2 BK to the value $G^2/4\pi=15.694$. Phase shifts for six energies are given in Table IV.

Figure 4 shows the cross section and $P\sigma(\theta)$ for 2 BK, M.

TABLE III. 2 BK phase shifts, in degrees.

Phase shift \ Energy Mev	100	300	440
K_0	6.2	−42.6	−65.4
K_2	5.7	26.4	24.9
K_4		3.6	7.1
$\delta^{P}{}_0$	60.1	28.1	8.4
$\delta^{P}{}_1$	14.8	2.5	−12.4
$\delta^{\alpha}{}_2$	32.5	19.1	20.9
ϵ_2	−3.4	−32.4	−71.2
$\delta^{\beta}{}_2$	1.3	7.0	−4.3
$\delta^{F}{}_3$	0.3	5.8	13.9
$\delta^{F}{}_4$	1.2	11.6	21.6
$\delta^{H}{}_4$		1.1	2.8
$\delta^{H}{}_5$		0.1	0.6
$\delta^{H}{}_6$		0.9	2.4

IV. DISCUSSION

For the cases that fit the low-energy data, the calculated total cross sections at higher energies considerably exceed the experimental values. The differential cross sections show a strong forward peak in contrast with the nearly isotropic scattering found experimentally. Both effects can be traced to the very large attraction of the central potential just outside the core, which causes most of the phase shifts with $L>0$ to be large and positive. The large values of these phase shifts are responsible for the high total cross sections. The small-angle scattering is contributed mostly by the amplitudes α_2, α_5, and α_S of BEH, since the other amplitudes vanish at $\theta=0°$. In these amplitudes all the Q_{LJ}'s of BEH enter with a positive sign, and all the Legendre polynomials occurring in each α have the same sign at small angles. For this reason all positive phase shifts in each α produce a constructive interference at small angles, leading to a large forward

[8] Kane, Stallwood, Sutton, Fields, and Fox, Phys. Rev. **95**, 1694 (1954).

[9] Sutton, Fields, Fox, Mott, and Stallwood, Phys. Rev. **97**, 783 (1955).

[10] Marshall, Marshall, and Nedzel, Phys. Rev. **98**, 1513 (1955).

scattering. The theoretical polarization, obtained by dividing $P\sigma$ by σ, predicted by 1 BK and 2 BK is uniformly too small in absolute value. This is due to the nature of the noncentral force in Lévy's potential.

For a tensor force $P\sigma$ is in lowest order quadratic in the coefficient of S_{12} in the Hamiltonian so far as effects of phase shifts with the same L but different J are concerned. The corresponding effect is linear for an $\mathbf{L}\cdot\mathbf{S}$ force. This fact is related to but is not quite the same as that noted by Wolfenstein[11] regarding the vanishing of S_{12} and $\mathbf{L}\cdot\mathbf{S}$ effects on $P\sigma$ in first Born approximation. It is also related to the observation of Goldfarb and Feldman[12] regarding the greater effectiveness of the $\mathbf{L}\cdot\mathbf{S}$ force in producing polarization.

In addition to the small magnitude, the calculated polarization shows a change in sign between $\theta=0°$ and $\theta=90°$, with the zero moving to smaller angles as the energy increases. This is due to the increasingly large coupling of $^3P_2$ and $^3F_2$ states that takes place with increasing energy.

The modified potential 2 BK, M leads to a total cross section in fair agreement with experiment from

TABLE IV. 2 BK, M phase shifts, in degrees.

Phase shift \ E_{Mev}	100	170	240	300	370	440
K_0	−13.3	−30.0	−44.3	−55.0	−66.3	−76.7
K_2	3.3	8.6	12.9	14.8	14.8	13.2
$\delta^P{}_0$	36.2	32.3	23.4	15.0	5.8	−2.9
$\delta^P{}_1$	2.7	0.1	−5.4	−10.8	−18.4	−24.1
$\delta^\alpha{}_2$	14.7	14.2	10.8	10.0	11.6	13.7
ϵ_2	−5.8	−14.4	−34.0	−55.6	−69.0	−74.7
$\delta^\beta{}_2$	0.8	1.0	0.3	−2.8	−8.9	−15.8
$\delta^F{}_3$	−0.1	0.6	1.5	2.7	3.7	4.8
$\delta^F{}_4$	0.7	1.5	4.9	7.8	10.0	12.4

100 to 440 Mev, and the forward scattering peak is reduced due to the reduction in the attractive force. However, the differential cross section is far from isotropic, and the polarization is too small in magnitude and shows a change in sign between 0° and 90° at the higher energies in disagreement with experiment. The comments made about the polarization predicted by 1 BK and 2 BK also apply to the modified potential 2 BK, M.

It appears that Lévy's potential is inadequate to account for the high-energy proton-proton scattering data. The work of Gelernter and Marshak[13] leads to the same conclusion. Jastrow[14] has made other fits to low-energy data employing different core radii for singlet and triplet potentials. He allows an effective difference between the second and fourth order potential strengths, although without systematically investigating the effect of the difference. The present work does not altogether exclude the possibility of such a phenomenologic potential, but in view of the utter failure of the Blatt-Kalos version, the two-core radii modification appears somewhat unlikely as well. It may be of interest to observe that the Brueckner-Watson[15] potential has not yielded more than qualitative fits to intermediate-energy data. Similarly the work of Gammel and Thaler[16] employing a potential calculated by Gartenhaus[17] from Chew's phenomenologic approach has given results about as discouraging as those presented here.

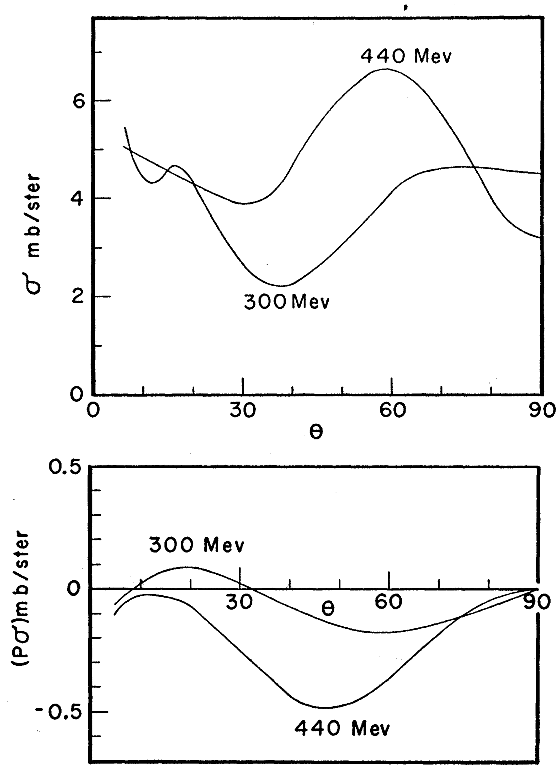

FIG. 4. Theoretical differential cross section and $P\sigma$ predicted by 2 BK, M, using the phase shifts of Table IV. Values are plotted against the center-of-mass angle, θ, in degrees.

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[11] L. Wolfenstein, Phys. Rev. **82**, 308 (1951); **76**, 541 (1949).
[12] L. Goldfarb and D. Feldman, reference 5. The fact noted here is implied, however, in a statement made more recently in L. Wolfenstein, Bull. Am. Phys. Soc. **1**, 36 (1956), paper IA 4.
[13] H. Gelernter and R. E. Marshak, Bull. Am. Phys. Soc. Ser. II, **1**, 37 (1956). The large phase shifts for higher angular momenta reported in this paper are not in agreement with present results. It is understood, however, that calculational errors bearing on this discrepancy have been found in their work by these authors.
[14] R. Jastrow, Phys. Rev. **91**, 749 (1953).
[15] K. Brueckner and K. Watson, Phys. Rev. **92**, 1023 (1953).
[16] J. L. Gammel and R. M. Thaler (private communication).
[17] S. Gartenhaus, Phys. Rev. **100**, 900 (1955).