Rotational and Intrinsic Levels in Tm^{169} and Lu^{175}†*

E. N. HATCH, F. BOEHM, P. MARMIER,‡ AND J. W. M. DUMOND
California Institute of Technology, Pasadena, California
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Nuclear levels in Tm^{169} excited by electron capture of Yb^{169}, and levels in Lu^{175} excited by both beta decay of Yb^{175} and electron capture of Hf^{175} have been studied by using the curved-crystal gamma-ray spectrometer and the ring-focusing beta-ray spectrometer, as well as a semicircular beta-ray spectrometer for low energies. From the precision energies and the multipolarity determinations, the levels in Tm^{169} have the following energies in kev, and spin and parity assignments: A (ground state) (1/2+), B 8.42 (3/2+), C 118.20 (5/2+), D 138.95 (7/2+), E 316.19 (7/2+), F 379.31 (7/2−), G 472.91 (9/2−). Levels A, B, C, and D are members of a rotational band whose characteristic constants are given. Levels E and F are interpreted as particle excitations and level G as a rotational level based on the state F. The Lu^{175} excited states have the following energies in kev, spins, and parities: A (ground state) (7/2+), B 113.81 (9/2+), C 251.46 (11/2+), D 343.40 (5/2+), E 396.31 (9/2−), F 432.76 (7/2+), G 504.7 (1/2+). A, B, and C form a rotational band for which the characteristic constants are given. Some features of the levels and transition probabilities are discussed and compared with the unified model. A brief survey of second-order rotational energy constants and of intrinsic excitation levels is given.

INTRODUCTION

THE unified nuclear model as first proposed by Bohr and Mottelson[1] has proved to be very successful in explaining nuclear levels and transition probabilities in strongly deformed nuclei. It is well known that the relative spacings of rotational energy levels predicted by the Bohr-Mottelson model are verified by experiment to better than one part in one hundred. Furthermore, recent analysis by Mottelson and Nilsson[2] provides good indication that the strong-coupling unified model of a spheroidal nucleus can predict successfully the ground-state configuration of the odd particle. This analysis is based on calculations by Nilsson[3] of independent-particle energy levels and wave functions for a spheroidal potential. The unified model, therefore, can be considered a good guide in predicting not only collective excitation states, but also, at least qualitatively, intrinsic states whose excitation we might expect in a nuclear decay process.

The energy of a rotational state with spin I belonging to a band with quantum number K can be written[4] as

$$\begin{aligned} E_K(I) = {} & E_K^{(0)} \\ & + E_K^{(1)}[I(I+1)+\delta_{K,1/2}a(-1)^{I+1/2}(I+1/2)] \\ & + E_K^{(2)}[I(I+1)+\delta_{K,1/2}a(-1)^{I+1/2}(I+1/2)]^2, \end{aligned} \quad (1)$$

where $E_K^{(0)}$ is a constant, $E_K^{(1)}$ is the rotational splitting term $\hbar^2/2\mathcal{J}$, $\mathcal{J}$ being the moment of inertia associated with the rotational motion, and $E_K^{(2)}$ is a small, usually negative second-order correction term. The decoupling term containing the decoupling parameter a is different from zero only for $K=1/2$.

The significance of the second-order term is becoming increasingly evident. This term, as Bohr and Mottelson have shown,[1] is characteristic of the rotation-vibration interaction and thus of the change in deformation of the spheroidal nucleus. A similar second-order term with positive sign may occur, as Kerman[5] has pointed out, as a result of the interplay of collective motion and single particle motion. The second-order term may provide, therefore, a tool in studying rotation-particle coupling.

It has been observed that the odd-A nuclei of the rare earth elements show pronounced rotational spectra in addition to low-lying intrinsic structure. These nuclei appear to be most appropriate for studies of nuclear excitation using precision spectroscopic methods. Therefore it has seemed worthwhile to reinvestigate with these methods two strongly deformed odd-A nuclei, Tm^{169} and Lu^{175}.[6] There have been several previous investigations of both Tm^{169} [7–10] and Lu^{175}.[9,11–14]

Tm^{169} has a spin 1/2 ground state and is expected

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‡ Present address: Physikalisches Institut der Eidgenössischen Technischen Hochschule, Zurich, Switzerland.
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[2] B. R. Mottelson and S. G. Nilsson, Phys. Rev. **99**, 1615 (1955).
[3] S. G. Nilsson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **29**, No. 16 (1955).
[4] K is defined as the projection of the nuclear spin I on the nuclear symmetry axis.
[5] A. K. Kerman, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **30**, No. 15 (1956).
[6] Preliminary results have been reported previously by Hatch, Marmier, Boehm, and DuMond, Bull. Am. Phys. Soc. Ser. II, **1**, 170 (1956), and by Boehm, Hatch, Marmier, and DuMond, Bull. Am. Phys. Soc. Ser. II, **1**, 170 (1956).
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TABLE I. Data for transitions in Tm^{169}.

Initial and final energy levels	γ-ray energy (kev)	γ-ray relative intensity	Internal conversion coefficients[a] $\alpha_{K\mathrm{exp}}$	$\alpha_{K\mathrm{theo}}$	α_{LI}	α_{LII}	α_{LIII}	Decay[b] fraction percent	Multipolarity
BA	8.42±0.05	[c]	···	···	···	···	···	(95)	(*M*1)
DC	20.75±0.05	[c]	···	···	$\alpha_{LI}:\alpha_{LII}:\alpha_{LIII}=20:2.5:1$			(13)	(*M*1)
FE	63.12±0.01	65	[d]	···	0.10[e]	0.02	0.03	90	*E*1
GF	93.60±0.04	4.4	2.4	3.2[f]	0.42	···	···	11	*M*1(+*E*2)
CB	109.78±0.02	22	2.1	2.0	0.34	···	···	54	*M*1[g]
CA	118.20±0.03	2.6	0.66	0.70	···	[h]	[h]	5	*E*2
DB	130.53±0.03	15	0.54	0.54	···	0.31	0.27	23	*E*2
ED	177.24±0.05	31	0.51	0.52	0.09	···	···	35	*M*1[i]
EC	197.97±0.06	51	0.40	0.38	0.07	···	···	51	*M*1[i]
FD	240.4 ±0.7	1	[j]	···	···	···	···	1	(*E*1)
FC	261.0 ±0.5	8	[k]	···	···	···	···	6	(*E*1)
EB	307.7 ±0.5	18	0.04	0.05	···	$\alpha_L=0.01$		13	*E*2

[a] All conversion coefficients except those entered in the column headed $\alpha_{K\mathrm{theo}}$ are experimental values. The theoretical α_K were obtained from tables computed by Sliv.[22]
[b] Each decay fraction is proportional to $(1+\alpha_{\mathrm{total}})$ times the corresponding γ-ray intensity and is normalized so that the sum of the decay fractions into the ground state is 100%. α_{total} was obtained by adding the appropriate conversion coefficients. If an experimental coefficient was not available, the corresponding theoretical value was used in α_{total}.
[c] These transitions were observed only with the semicircular β-ray spectrometer.
[d] *K*-conversion line was observed at 3.8 kev, but its intensity was not obtained because of large uncertainty in window absorption of semicircular spectrometer detector.
[e] Theoretical values given by Rose *et al.*[23] for *E*1 transition at 63.1 kev: $\alpha_{LI}=0.09$, $\alpha_{LII}=0.03$, and $\alpha_{LIII}=0.04$.
[f] $\alpha_{K\mathrm{theo}}$ is the Sliv[22] value for *M*1 transition.
[g] Upper limit of *E*2 admixture is 3%.
[h] *L*-lines observed but not resolved from other conversion lines.
[i] Upper limit of *E*2 admixture is 20%.
[j] Transition observed with curved-crystal spectrometer only.
[k] *K*-line observable but of low intensity.

to exhibit the anomalous rotational spectrum as predicted by Eq. (1) for $K=1/2$. While these investigations were under way Johannson[8] and Cork[9] have published decay schemes of this nucleus and have pointed out some of the principal features of the nuclear spectrum. The present work on Tm^{169} will furnish principally the rotational parameters of the ground state band and an assignment of higher intrinsic levels. With respect to Lu^{175} this investigation will give, apart from precision energies, a level assignment similar to the one proposed by Chase and Wilets[15] with one additional level not reported earlier.

Following neutron capture in natural Yb, Yb^{169} and Yb^{175} are formed having half-lives of 31 days and 4.2 days, respectively. Yb^{169} decays by electron capture to excited states in Tm^{169}. Beta decay of Yb^{175} excites levels in Lu^{175}. On the other hand, levels in Lu^{175} are reached by electron capture of Hf^{175}, which is produced by neutron capture in Hf^{174}. These three decay processes have been studied in the present investigation.

INSTRUMENTS AND SOURCES

The two-meter curved-crystal gamma-ray diffraction spectrometer has been employed for the energy determination of the observed gamma-ray transitions in Tm^{169} and Lu^{175} (except where noted in Tables I and II). This instrument and its operation have been described earlier.[16] For the precision determination of gamma-ray energies the resolution of this spectrometer was $\Delta E/E=0.3\times10^{-4}\ E$, where E is the gamma-ray energy in kev and ΔE is the width of the line profile at half-maximum.

In conjunction with the gamma-ray spectrometer, two beta-ray spectrometers were used: the homogeneous-field ring-focusing spectrometer[17] and a semicircular spectrometer.

The ring-focusing spectrometer in some cases has been operated with a beta-gamma coincidence arrangement of the well-known fast-slow type. The fast coincidences were obtained in a Garwin-type[18] coincidence mixer with 2×10^{-8} second resolving time and with previous fast-pulse amplification and pulse shaping. Variable delay lines could be inserted in either channel. For coincidence measurements the gamma-ray detector was a NaI crystal and the electron detector a 0.5-mm stilbene crystal, 2 inches in diameter, mounted on a Lucite light pipe 13 inches long. Since the magnetic field decreases rapidly outside the ellipsoidal coils of the spectrometer, a thin μ-metal shielding was sufficient to insure negligible residual magnetic field at the position of the photomultiplier. Pulse height discrimination and slow triple coincidence mixing was of conventional type. The coincidence output counting rate was usually recorded together with the counting rate of the beta channel on a continuous chart recorder. The resolution of the beta-ray spectrometer was 0.3% for most of the standard runs and 1% for the coincidence runs.

Since the ring-focusing spectrometer is limited at the

[15] D. M. Chase and L. Wilets, Phys. Rev. **101**, 1038 (1956).
[16] Muller, Hoyt, Klein, and DuMond, Phys. Rev. **88**, 775 (1952).
[17] DuMond, Kohl, Bogart, Muller, and Wilts, Office of Naval Research Special Technical Report, No. 16, March, 1952, (unpublished).
[18] R. L. Garwin, Rev. Sci. Instr. *24*, 618 (1953).

TABLE II. Data for transitions in Lu^{175}.

Initial and final energy levels	γ-ray energy (kev)	γ-ray relative intensity	Experimental internal conversion coefficients α_K	α_L	Decay[a] fraction percent	Multipolarity
			Lu^{175} transitions following β decay from Yb^{175}			
BA	113.81±0.02	31	1.6	($\alpha_{LI}=0.39$, $\alpha_{LII}=0.10$, $\alpha_{LIII}=0.13$)	50	80% $M1$+20% $E2$[b]
CB	137.65±0.05	2.2	1.0	[c]	2.5	$M1+E2$
EC	144.85±0.03	5.9	0.11	...	2.9	$E1$
CA	251.3 ±0.5	3.8	[d]	...	2.0	($E2$)
EB	282.57±0.13	62	0.030	0.0037	28	98% $E1$+2% $M2$
EA	396.1 ±0.3	100	0.067	0.0085	48	80% $E1$+20% $M2$
			Lu^{175} transitions following electron capture from Hf^{175}			
FD	89.36±0.01	40	2.2	$\alpha_{LI,LII}=0.43$, $\alpha_{LIII}=0.022$	13	97% $M1$+3% $E2$
BA	113.81±0.05	3.6	∼2	$\alpha_{LI,LII}:\alpha_{LIII}$ ∼6	1.1	90% $M1$+10% $E2$[b]
GD	161.3 ±0.2	0.3	0.5	0.2	0.04	($E2$)
DB	229.6 ±0.6	7.3	0.11	0.05	0.7	$E2$
FB	318.9 ±0.6	[e]	...	...	...	($M1$)
DA	343.40±0.08	1000	0.105	0.019	97	$M1$[f]
FA	433.0 ±0.5	16	0.061	0.0095	1.5	$M1$[f]

[a] Decay fractions are computed and normalized as in Table I.
[b] Discrepancy in mixing percentages is within experimental errors.
[c] *L*-conversion lines were observed but not resolved from other lines.
[d] Transition observed with curved-crystal spectrometer only.
[e] Low-intensity transition observed with ring-focusing spectrometer only.
[f] Up to 25% $E2$ admixture cannot be excluded.

low-energy side to electron energies of about 25 kev owing to the proton resonance field measuring device, and also owing to the detector cut-off, a low-energy semicircular beta-ray spectrometer has been employed. This 180° spectrometer has a radius of 12 cm. The field is produced by two iron-free coils in near Helmholtz conditions. The gradient of the field near the extreme trajectories provides a second-order focusing. Field current stabilization is obtained to two parts in ten thousand with a rotating mechanical unit and electronic servo system. Source and Geiger-counter window dimensions for a standard resolution of 0.8% are 0.1 cm×4 cm. With a 10-μg/cm^2 Formvar counter window, transmission is assured down to 2-kev electron energy.

The radioactive sources were obtained by irradiating Yb_2O_3 and $Hf^{174}O_2$ with neutrons at high flux in the Materials Testing Reactor at Arco, Idaho. The Yb_2O_3 was specified to be 99.8% pure. The enriched $Hf^{174}O_2$ (10% Hf^{174}) was loaned by the Stable Isotope Division of the Oak Ridge National Laboratory.

The curved-crystal spectrometer required line sources of about one curie strength. The source material was enclosed before irradiation in quartz capillaries of 0.007-inch inner diameter and 2-cm length.

Sources for both beta-ray spectrometers were prepared by vacuum evaporation of the radioactive material onto a thin mica sheet from which the actual spectrometer sources were punched. The ring-focusing spectrometer required at 0.3% resolution a 1.2-mm diameter disk source. The semicircular spectrometer requires strip sources of the dimensions given above. The radioactive deposit on the mica is always invisible, in the worst case several light wavelengths thick. Such sources give rise to line profiles free from "tails" down to lowest energies.

RESULTS

The precision energies of gamma-ray transitions in Tm^{169} and Lu^{175} were obtained by matching the profiles of the gamma lines resulting from reflection from the opposite sides of the (310) planes of the curved quartz diffracting crystal. The well-known wavelength[19] of the Ta $K_{\alpha1}$ x-ray emitted from the radioactive Hf source was used as standard for all measurements. To improve resolution of the instrument at higher energies, the quartz (110) planes, which exhibit strong reflection in third and fifth order, were employed in one case (343.40-kev line of Lu^{175}).

The relative intensities of the gamma-ray transitions were determined by comparing areas of the photopeaks due to gamma rays reflected at the Bragg angle. Corrections for self-absorption in the source, reflectivity[20] of the curved quartz crystal, and the efficiency of the NaI scintillaton detector were applied.

The spectrum from a radioactive Yb source from 48 to 400 kev as recorded on a chart recorder from the output of the spectrometer is shown in Fig. 1. The Yb source emitting this spectrum consisted of 50 mg of

[19] Y. Cauchois and H. Hulubei, *Longueurs D'Onde des Émissions X et des Discontinuités d'Absorption X* (Hermann and C[ie], Paris, 1947).

[20] Curved crystal reflectivity was measured [E. N. Hatch, Ph.D. thesis, California Institute of Technology, 1956 (unpublished)] over the range 84 kev to 340 kev and was found to be consistent with a $1/E^2$ energy dependence, in agreement with earlier work by Lind [Lind, West, and DuMond, Phys. Rev. **77**, 475 (1950)].

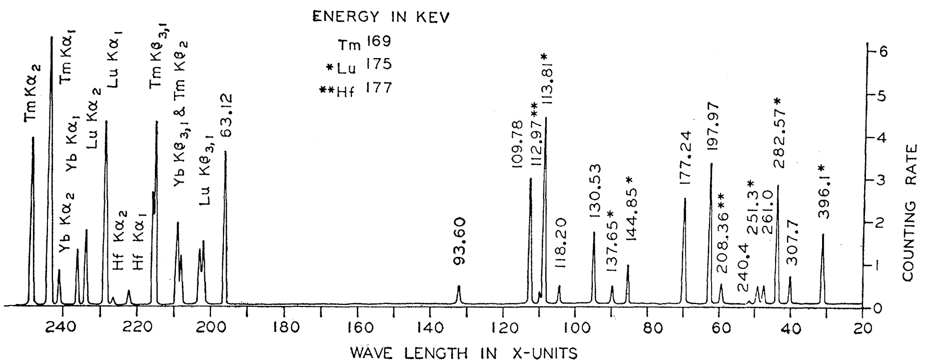

FIG. 1. Gamma-ray and x-ray spectrum from a neutron-irridiated Yb source from 48 to 400 kev as observed with the curved-crystal spectrometer. The gamma lines which are labeled with their energies in kev correspond to transitions in Tm^{169}, Lu^{175}, and Hf^{177}.

radioactive Yb_2O_3 and had an intensity of approximately one curie. This source provided an energy resolution, $\Delta E/E$, of about twice that which resulted with the source used for the precision energy measurements. The lines in the spectrum shown in Fig. 1 are due to the de-excitation in Tm^{169}, Lu^{175}, and Hf^{177}. The last decay is evidenced by the Hf K_α x-rays and the Hf^{177} 112.97 and 208.36 kev gamma lines. Hf^{177} is formed by beta decay of Lu^{177}, which itself is a decay product of Yb^{177}.

In Fig. 2 (A) is shown the Tm^{169} beta-ray spectrum from 2 to 12.5 kev as recorded on a chart recorder from the output of the semicircular spectrometer. The unlabeled peaks are principally L-Auger lines. To identify the Tm L-Auger lines in this spectrum, an independent study of the Lu L-Auger spectrum emitted by a Hf^{175} source was undertaken, since in the Lu^{175} spectrum no conversion electrons at low energies are present. Figure 2(B) represents a typical chart run of the Lu^{175} L-Auger spectrum.

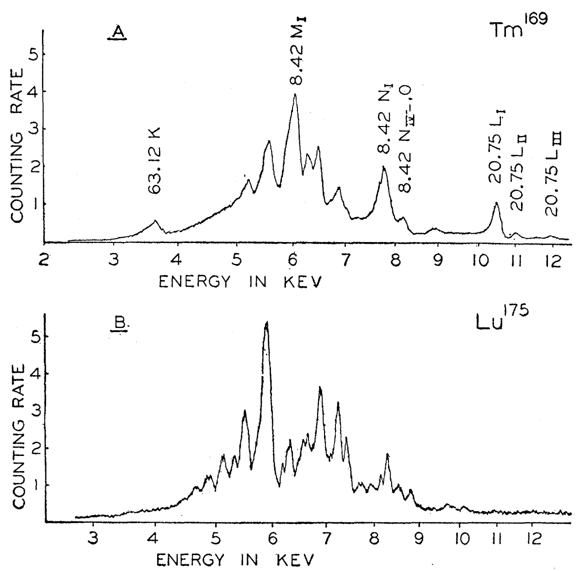

FIG. 2 (A) Conversion line and L-Auger spectrum of Tm^{169} from 2 to 12.5 kev observed with the semicircular beta-ray spectrometer. The momentum resolution is 1%. (B) For comparison a pure Lu L-Auger spectrum following Hf^{175} electron capture is shown. Here the momentum resolution is 0.7%. The group of lines between 5 and 6 kev is due mainly to Auger conversion of the $L_{\alpha 1}$ x-rays, those around 7 kev mainly to $L\beta_1$ and $L\beta_2$ x-rays, and the higher group to $L\gamma_1$ x-rays. The abscissa values of curves A and B are slightly shifted in order to achieve an approximate correspondance between Auger lines of both elements.

From the beta-ray spectra obtained with the ring-focusing spectrometer, relative intensities of the conversion lines resulted from comparing the peak heights after correcting for absorption in the detector window.

The combined results of the gamma- and beta-ray measurements are presented in Tables I and II and in Figs. 3 and 4. The experimental conversion coefficients in Table I resulted from dividing the intensity of each conversion line by the corresponding gamma-ray intensity. A normalization factor for transitions following the Yb^{169} and Yb^{175} decays was established by assuming that the Tm^{169} transition DB had the theoretical K-conversion coefficient of a pure $E2$ transition. This assumption was based on the experimental conversion ratio $L_I:L_{II}:L_{III}$, from which it is evident that transition DB is $E2$ with less than 5% $M1$ admixture. Similarly, the conversion coefficients for transitions following the Hf^{175} decay have been determined by using the 133.02-kev $E2$ transition[21] in Ta^{181} as normalization standard. Ta^{181} was present in the Hf source as a product of the beta decay of Hf^{181}.

By comparing the experimental K- and L-conversion coefficients with the corresponding theoretical coefficients, the multipolarities of the transitions were obtained. Theoretical K-shell conversion coefficients were taken from tables by Sliv,[22] while the theoretical L-subshell conversion coefficients were interpolated from the tables of Rose *et al.*[23]

It has been recently pointed out by Wapstra and Nijgh[24] that some experimental K-shell conversion coefficients for $M1$ transitions in nuclei with mass

[21] F. Boehm and P. Marmier, Phys. Rev. **103**, 342 (1956).
[22] L. Sliv (privately circulated tables).
[23] Rose, Goertzel, and Swift (privately circulated tables).
[24] A. H. Wapstra and G. J. Nijgh, Nuclear Phys. **1**, No. 4, 245 (1956).

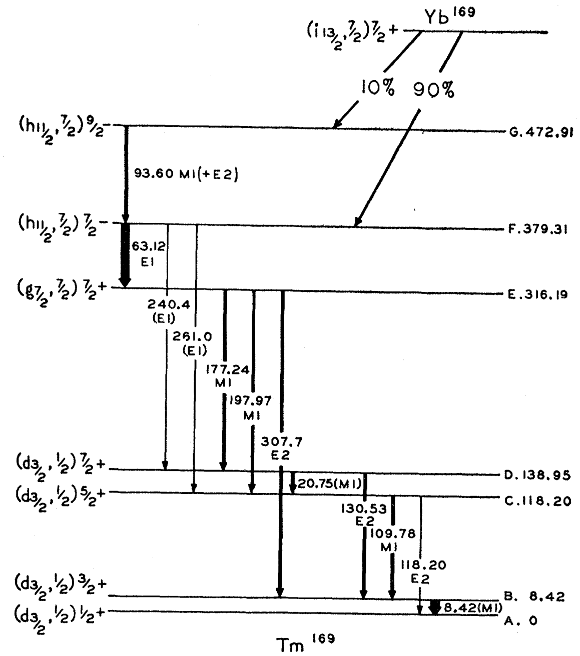

FIG. 3. Level scheme proposed for Tm^{169}. The energies are given in kev. The numbers characterizing each energy level are: the configuration in the spherical limit,[2] the K quantum number, and the nuclear spin and parity.

numbers $A \sim 200$ are about 40% lower than the corresponding theoretical coefficients of Rose *et al.*[23] Sliv[22] has computed K-shell conversion coefficients for elements $65<Z<95$, including the effect of the finite extension of the nucleus with the result that $M1$ coefficients are substantially lower than the earlier values by Rose. The experimental K-shell conversion coefficients for the Tm^{169} 109.78-, 177.24-, and 197.97-kev transitions are in good agreement with the theoretical $M1$ vaues of Sliv and are about 15% lower than corresponding coefficients computed by Rose *et al.*

The Tm^{169} 63.12-kev transition was assigned $E1$ multipolarity on the basis of its L-subshell conversion (Table I). This assignment is not in agreement with the $M1+E2$ assignment suggested by Cork *et al.*[9] Conversion peaks of the Tm^{169} 240.4- and 261.0-kev gamma rays were found to be very weak. By using gamma-ray intensity values (Table I), it was possible to set an upper limit for the K-conversion coefficients, from which it follows that the $E1$ multipolarities are most likely for these transitions.

Johannson[8] has reported a transition in Tm^{169} at 194 kev. There is no evidence for this transition from the present measurements.

Some of the genetic relationships between transitions in Tm^{169} were checked with the described beta-gamma coincidence apparatus. The results of these measurements, which will not be reported in detail, support the level scheme given in Fig. 3.

In Lu^{175} the multipole assignments given in Table II are essentially in agreement with those proposed by Mize *et al.*,[14] in particular with respect to the 282.57- and 396.1-kev transitions, which Cork *et al.*[9] have suggested a different assignment. Conversion electrons due to the 161.3-kev gamma ray have been found to be in coincidence with the 343.40-kev transition. This result means that the 161.3-kev gamma ray feeds either the level D or the level F (Fig. 4). However, no crossover transition of 161.3+89.36 kev has been

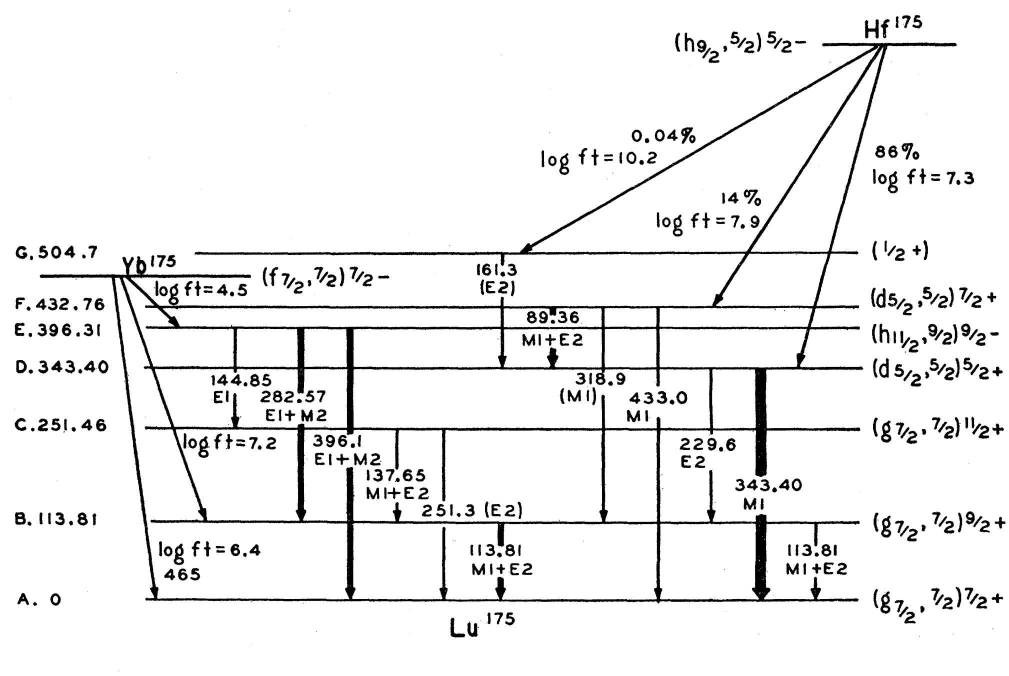

FIG. 4. Level scheme proposed for Lu^{175}.

found; therefore more weight is given to the former alternative. From the conversion properties, the 161.3-kev line probably has $E2$ character. If one assumes a 900-kev disintegration energy,[14,25] the electron-capture branches initiating from the $5/2-$Hf^{175} ground state to levels D and F have log ft values of 7.3 and 7.9, respectively. Thus these transitions can be classified as first forbidden. Because of the very low intensity of the 161.3-kev line, the electron-capture process feeding this line must have a very large log ft value, such as 10.2, and therefore is likely to be of the ($\Delta I=2$, yes) type ending on either a $1/2+$ or a $9/2+$ spin level. A $1/2+$ state G, as indicated on Fig. 4, explains best the observed features, particularly the absence of transitions to states with a spin higher than 5/2.

Levels in Tm^{169}

The level scheme for Tm^{169} obtained from the present measurements is shown in Fig. 3. Based on the measured ground-state spin of 1/2,[26] the assignment of spins and parities to levels B through F from the observed transition multipolarities is unique. The 93.60-kev transition was placed so as to decay into level F to agree with the delay-coincidence measurements by Johansson.[8] The spin of level G is not uniquely determined by multipolarity considerations.

Using previous data[7,10] Mottelson and Nilsson[27] have interpreted levels B, C, and D as belonging to the rotational band from the $K=1/2$ ground state of Tm^{169}. Level E has been assigned spin and parity $7/2+$ from the present data and is interpreted as being an individual particle excitation. From the established $E1$ multipolarity of the 63.12-kev transition, level F could be assigned spin and parity of either $5/2-$, $7/2-$, or $9/2-$. Since transition FB was not observed, and since pure $M2$ multipolarity for the 261.0-kev transition could be definitely excluded, level F was assigned $7/2-$ and interpreted as a particle excitation. The interpretation of level G as the first rotational state based on level F rather than as an individual particle state is consistent with the experimental data and will be discussed below.

From the gamma-ray decay fractions (Table I) it was established that 10% of the electron capture leads to Tm^{169} level G, while 90% decays to level F. According to the classification of odd-A nuclear ground states by Mottelson and Nilsson,[2] the Yb^{169} ground state is predicted to have spin $7/2+$. Thus the electron-capture decay from Yb^{169} to levels G and F would be first forbidden, in accord with a log ft of about 7 and 8, which one obtains assuming a total decay energy of 1.1 Mev.[25]

If the second-order term in Eq. (1) is neglected and the constants $E_{1/2}^{(0)}$, $E_{1/2}^{(1)}$, and a are determined using the measured gamma energies of states B and C, level D is predicted at an energy of $E_{1/2}(7/2)=137.85$ kev. Taking into account the second-order term and using the energies of levels B, C, and D, the following constants are obtained: $a=-0.7680$, $E_{1/2}^{(0)}=-18.247$ kev, $E_{1/2}^{(1)}=11.969$ kev, $E_{1/2}^{(2)}=+0.03389$ kev. The resulting value for the decoupling parameter a is in agreement with the calculation by Mottelson and Nilsson.[27] It should be noted that a positive second-order term is required to fit the measured energies. Similarly, a second-order term with positive sign has been found in the rotational band of W^{183}.[5]

Using the above parameters a 9/2 rotational level is predicted at 351.65 kev and an 11/2 level at 387.37 kev. Transitions to or from these levels have not been observed. Since these higher spin levels are not populated, the effect of K-mixing cannot be verified as has been done in the analysis of W^{183} by Kerman.[5] As mentioned above, level F appears to be a rotational level. This assumption is based on the apparent $M1+E2$ character of the 93.60-kev transition and on the observed branching ratio of the electron capture to levels F and G. To first approximation a splitting constant $E_{7/2}^{(1)}=10.4$ kev is obtained. This relatively small value for $E_{7/2}^{(1)}$ might indicate that a larger moment of inertia is associated with the rotational band based on level F compared with that of the ground-state band.†

For each of the observed intrinsic levels in Tm^{169} there is a corresponding individual particle level available from the curves by Mottelson and Nilsson.[2] The identification of the intrinsic levels, which is based on the observed spins and parities, is given in Fig. 3.

One striking feature of the level scheme in Fig. 3 is the lack of observation of allowed electron-capture decay to level E. From conventional beta-decay selection rules this transition would be expected to compete strongly with the less energetic first-forbidden decay to levels G and F. A possible explanation is perhaps found in the selection rules given by Alaga[28] for beta transitions in strongly deformed nuclei. According to these selection rules electron capture from the assumed $7/2+$ ground state of Yb^{169} to level E would be classed as a hindered transition and would have a reduced transition probability over the unhindered decay to levels G and F. A similar explanation may also account for the lack of an observed cross-over transition GE, since the same type of selection rules should apply to gamma-ray transitions.

If it is assumed that transitions FE, FD, and FC are pure $E1$ transitions, the experimental branching ratio of the reduced transition probabilities can be obtained. This ratio, $B(63.12):B(240.4):B(261.0)=3.2$

[25] K. Way and M. Wood, Phys. Rev. **94**, 119 (1953).
[26] K. H. Lindenberger and A. Steudel, Naturwiss. **42**, 41 (1955).
[27] B. R. Mottelson and S. G. Nilsson, Z. Physik **141**, 217 (1955).

† *Note added in proof.*—Recent lifetime studies on Tm^{169} levels have suggested that level G could be interpreted as an intrinsic excitation state rather than as a rotational level [Mihelich, Ward, and Jacob (private communication)].
[28] G. Alaga, Phys. Rev. **100**, 432 (1955).

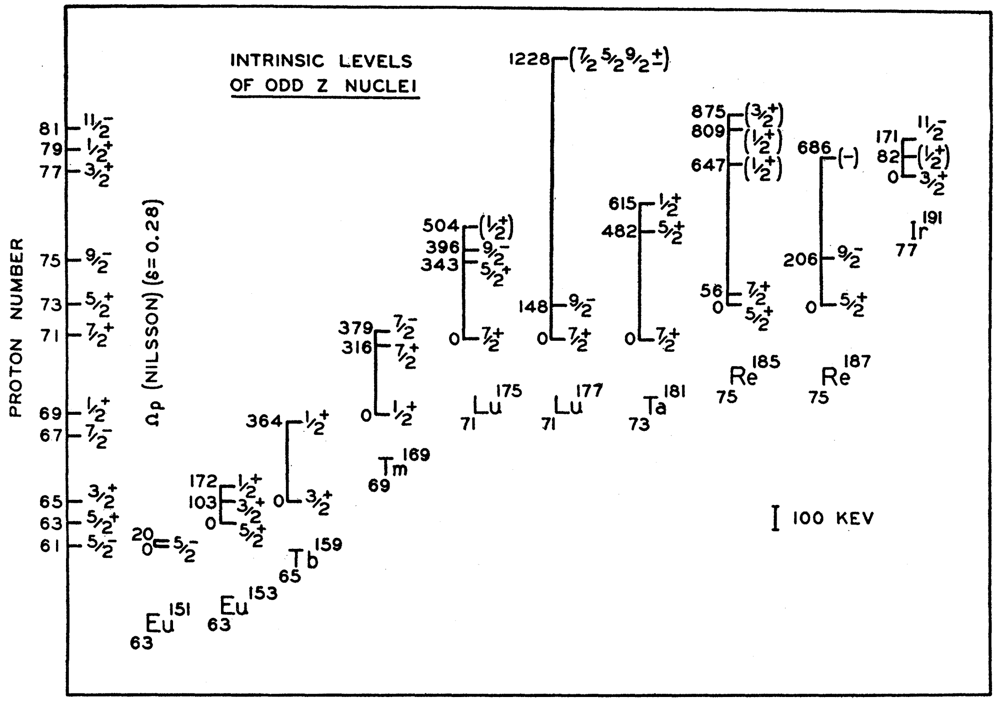

FIG. 5. Intrinsic levels in some odd-Z nuclei. The levels are compared with the calculations of Nilsson assuming a deformation parameter δ of 0.28 (see text).

$\times 10^3$:1:6, indicates that the 240.4- and the 261.0-kev transitions are greatly suppressed compared with the 63.12-kev transition. This suppression can be explained by K-forbiddenness. The fact that the $\Delta K=3$ transitions occur at all probably indicates K-admixture in the bands. In the same way the long half-life of level E (0.7 μsec)[29] can be attributed to K-forbiddenness, as Johannson[8] has indicated.

Levels in Lu^{175}

The level schemes obtained from the investigations of Lu^{175} are shown in Fig. 4. The essential features of the decay from both Yb^{175} and Hf^{175} have been given previously by Mize *et al.*,[14] and have been discussed by Chase and Wilets.[15]

If the second-order term in Eq. (1) is neglected, the formula predicts the energy of level C to be $E(11/2) = 252.91$ kev which compares closely with the measured energy of 251.46 kev. Including the second-order term, one can calculate the following constants: $E_{7/2}^{(0)} = -201.74$ kev, $E_{7/2}^{(1)} = 12.913$ kev, and $E_{7/2}^{(2)} = -0.006595$ kev. In addition, a level at $E_{7/2}(13/2) = 412.10$ kev is predicted. No gamma transitions to or from this level have been observed. It is interesting to note that if the same values for $E_K^{(1)}$ and $E_K^{(2)}$ as were obtained for the ground-state band are applied to the $K=5/2$ band, the calculated energy of transition FD is 89.26 kev, which is within 0.1 kev of the experimental value. This close agreement possibly indicates that the moment of inertia of the $K=5/2$ band is very close to that of the ground-state band.

Considering levels A through F in Fig. 4, the following gamma transitions have not been observed in the present measurements: FE, ED, DC, and FC. The $E1$ transition rate between F and E would be expected to be reduced because of K forbiddenness. An $M2$ transition between levels E and D would be very weak due to the competition of lower multipole transitions to the $K=7/2$ band. The absence of the other transitions can be accounted for in a similar way. The fact that the 318.9- and 433.0-kev transitions are less intense than the 89.36-kev transition may be due to the former two transitions being hindered by selection rules of the Alaga[28] type.

The intrinsic levels A, D, and E in Fig. 4 have been assigned to states from the Mottelson and Nilsson[2] curves by Chase and Wilets.[15] Level G with proposed spin and parity $1/2^+$ can be interpreted as a $(d_{3/2},1/2)$ particle excitation. A low-lying first rotational state $(d_{3/2},1/2)3/2$, like the ones found in Tm^{169} and Ta^{181},[21] has not been observed.

CONCLUSION

In Table III are compiled second-order correction terms, $E_K^{(2)}$, for some odd-A isotopes. In compiling these terms no perturbation effect, such as rotation-particle coupling,[5] has been considered. Table III, therefore, gives some indication of cases in which the effect of rotation-particle coupling is appreciable.

[29] S. DeBenedetti and F. K. McGowan, Phys. Rev. **74**, 728 (1948); E. W. Fuller, Proc. Phys. Soc. (London) **A63**, 1044 (1950); A. W. Sunyar and J. W. Mihelich, Phys. Rev. **81**, 300 (1951).

TABLE III. Coefficients $E_K^{(2)}$ for second-order terms of some odd-A nuclei.[a]

Nucleus	K	$E_K^{(2)}$ (kev)
Lu^{175}	7/2+	−0.006595
Hf^{177}	7/2−	−0.006157[b]
Ta^{181}	7/2+	−0.003 (±0.004)[c]
W^{183}	3/2−	+0.05131[d]
W^{183}	1/2−	+0.00318[d]
Tm^{169}	1/2+	+0.03389

[a] For computation of these coefficients, energies of the first and second rotational levels were used, except in the case of nuclei with spin 1/2 ground states where the third rotational level also had to be considered.
[b] P. Marmier and F. Boehm, Phys. Rev. **97**, 103 (1955).
[c] See reference 21. The energy of the second rotational state was taken from Coulomb excitation data[30] and therefore has large uncertainty.
[d] See reference 5 and Murray, Boehm, Marmier, and DuMond, Phys. Rev. **97**, 1007 (1955).

Lu^{175}, Hf^{177}, and Ta^{181} show negative values for $E_K^{(2)}$ of about the same magnitude. In all three cases the lowest excited intrinsic level is situated higher than 300 kev above the ground state. The negative sign indicates the preponderance of a rotation-vibration type of interaction. The rotational spectrum of Tm^{169} and two bands in W^{183} indicate a positive second-order term. In these cases rotation-particle coupling is presumably strong.

It is interesting to note some regularities in the scheme of intrinsic levels studied here and reported earlier by other workers.[30] We have compiled in Fig. 5 the intrinsic levels of some odd-Z nuclei in the rare earth region. The levels are compared with the calculations of Nilsson[3] assuming a deformation parameter δ of 0.28. The display of the calculated levels in the ordinate is qualitative only and does not represent an energy scale. As Mottelson and Nilsson[2] have shown, experimentally observed groundstates of the nuclei presented in Fig. 5 follow remarkably well the predicted odd proton states. The Mottelson and Nilsson[2] assignment for the groundstates has been assumed in Fig. 5. The following points are worth noting: The $(g_{7/2},7/2)$ state, which is a groundstate in Lu^{175}, Lu^{177}, and Ta^{181} appears also as an excited state in Tm^{169} and Re^{185}. The $(d_{3/2},1/2)$ configuration, which is the ground state of Tm^{169}, appears also as an excited state in Tb^{159} and possibly in Lu^{175} and Ta^{181}. The $(d_{5/2},5/2)$ configurations occur in Lu^{175} and Ta^{181} at similar excitation energies and also form the ground states of the Re isotopes. The 9/2− levels occur in Lu^{175}, Lu^{177}, and Re^{187} as excited states, but do not appear as ground states, as Mottelson and Nilsson[2] have pointed out. Similar regularities in iridium and gold isotopes have been discussed by Mihelich and de-Shalit[31] on the basis of the spherical shell model.

[30] Hollander, Perlman, and Seaborg, Revs. Modern Phys. **25**, 469 (1953); *Nuclear Data Cards*, edited by C. L. McGinnis (National Research Council, Washington, D. C.).

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[31] J. W. Mihelich and A. de-Shalit, Phys. Rev. **93**, 135 (1954).