# Angular Momentum Associated with Slow-Neutron Resonances*

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Available data on the statistical weight factor $g$ for $l=0$ neutron resonances are summarized and the reliability of each result is discussed. There are reliable measurements for seventeen resonances which lie between 0 and 50 kev. In many cases, the results were obtained in two or more independent experiments. In sixteen cases, the compound nucleus has the spin $J=I+\frac{1}{2}$, while in only one case the spin is $J=I-\frac{1}{2}$. No data are available for weak resonances. Despite the limited data, it is strongly indicated that the compound nucleus is formed preferentially in the $I+\frac{1}{2}$ spin state. At present there is no basis for deciding whether this result arises from a difference in the level spacing $D$, or a difference in the average neutron width $\bar{\Gamma}_n^0$ for the two possible spin states. Regardless of the source of this effect it is apparent that the frequent assumption—that the strength function $\bar{\Gamma}_n^0/D$ is the same for the two spin states—may not be correct.

## I. INTRODUCTION

When a neutron combines with a target nucleus, a compound nucleus is formed having the excitation energy $E_C=E_B+E$, where $E_B$ is the binding energy ($\sim$6 to 10 Mev), and $E$ is the kinetic energy of the neutron. For relatively heavy target nuclei, e.g., $A>50$, the compound nucleus will usually have a high density of quantum states at the energy of excitation. If $E_C$ happens to coincide with an excited state of appropriate spin and parity, the probability for forming the compound nucleus is large and a resonance occurs in the neutron cross section at the corresponding kinetic energy, $E$.

Let us limit our discussion to target nuclei having ground state spin $I\neq 0$ (i.e., not even-even), and to interactions for which the neutron orbital angular momentum is zero ($l=0$). Interactions in the "slow" neutron region ($E<\sim 10$ kev) are, for all practical purposes, exclusively $l=0$. (At 1 ev, the ratio of the strength of resonances having $l=1$ to those having $l=0$ is of the order of $10^{-6}$ because of the effect of the centrifugal barrier.) The states of the compound nucleus which can be produced by a $l=0$ neutron are those having spin $J=I+\frac{1}{2}$ or $J=I-\frac{1}{2}$, and parity $\pi$, where $I$ and $\pi$ are the spin and parity of the ground state of the target nucleus, respectively.

The Breit-Wigner single-level formula,[1] which expresses the variation of cross section as a function of energy in the region of resonance, contains the statistical factor, $g=(2J+1)/[(2I+1)(2S+1)]$, where the neutron spin $S=\frac{1}{2}$. For $l=0$ and $I\neq 0$, $g$ has two possible values:

$$g_+=\tfrac{1}{2}[1+1/(2I+1)]$$
and
$$g_-=\tfrac{1}{2}[1-1/(2I+1)], \tag{1}$$

corresponding respectively to states of $J=I+\frac{1}{2}$ and $J=I-\frac{1}{2}$.

The $g$-values have been measured for only a few resonances. Because of the lack of this information, it has been normal to assume[2] that the resonances are about evenly distributed between $g_+$ and $g_-$. This assumption implies two conditions: (1) That roughly the same number of states of $(I+\frac{1}{2}),\pi$ and $(I-\frac{1}{2}),\pi$ exist in the compound nucleus,[3] and (2) that the probability of compound nucleus formation, $\Gamma_n$, is independent of the relative orientation of the spins of the neutron and nucleus.

If the above assumption is true, the measured $g$-values for randomly selected neutron resonances should be about equally distributed between $g_+$ and $g_-$. Existing data, however, contradict this, because $g_+$ appears to be highly favored. The importance of this effect makes it worthwhile to examine the available results rather carefully.

## II. SUMMARY OF EXPERIMENTAL DATA ON $g$-VALUES

### A. $g$-Values of the First Resonance

In recent years it has been possible to assign the $g$-values for several low-energy neutron resonances. In each case, the data are for the lowest energy resonance appearing in the cross section. If the binding energy, $E_B$, is not correlated with the spectrum of excited states of the compound nucleus, then the lowest resonance in the cross section should be either $g_+$ or $g_-$ with about equal probability. Thus, a measurement of the $g$-value of the lowest resonance for many isotopes would represent a random sampling of $g$-values.

The measured $g$-values for low-energy resonances are summarized in Table I. Out of nine cases, $g_+$ was obtained for eight, $g_-$ for only one. (Note that this one $g_-$ is a doubtful assignment.)

Before accepting these results, it is necessary to

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[1] G. Breit and E. P. Wigner, Phys. Rev. 49, 519, 642 (1936).

[2] See, for example, Harvey, Hughes, Carter, and Pilcher, Phys. Rev. 99, 10 (1955), reference 10; Sailor, Foote, and Landon, Phys. Rev. 93, 1292 (1954).

[3] The densities of levels in the compound state are predicted theoretically to be a function of $J$. In an extreme case, it appears possible that the density of $J=1$ levels might be a factor of two or three times the density of $J=0$ levels. See, for example, C. Bloch, Phys. Rev. 93, 1094 (1954); and C. Critchfield and S. Oleksa, Phys. Rev. 82, 243 (1951).

examine the experiments for systematic errors and to attempt to appraise their general reliability. For purposes of rating the various results, let us define a *confidence factor* which would be unity if the assignment were absolutely certain, and would be 0.50 if it were completely uncertain, i.e., if the assignment were a pure guess. Thus, a confidence factor of 0.75 would indicate a three to one chance that the assignment is correct.

The results in Table I were obtained by two general methods: (1) by making transmission measurements on polarized nuclei with polarized neutrons (designated by $P$ in the Method column of the table); (2) by measuring the cross section for resonance scattering (designated by $S$). The two methods are quite independent and contain no mutual systematic errors so far as can be determined.

The polarization work, carried out at Oak Ridge National Laboratory,[4-6] used thermal neutrons which were monochromatic and polarized. It has been successfully applied to $In^{115}$ and $Sm^{149}$ in which the thermal cross section is dominated by a single resonance. The measurement on $Mn^{55}$ made by this group[6] yielded no $g$-value assignment because the thermal region is affected by two or more resonances.

The resonance scattering measurements have been made with four independent experimental arrangements: (1) "back-scattering" method[7] used at Chalk River; (2) "thick-sample" method[8] used at Columbia University; (3) "bright-line" method[9] used at Brookhaven National Laboratory; and (4) "thick-thin sample" method[10] used at Brookhaven. Results by the four methods have been in mutual agreement, whenever data have been repeated by more than one method.

Let us now examine each case specifically:

$Rh^{103}$: This resonance was measured by Brockhouse[7] using the "back-scattering" method. He states that the counting rates were statistically significant, but were low compared to background. For this reason, the result cannot be assigned a large confidence factor.

$Ag^{107}$: This is the only $I-\frac{1}{2}$ case reported in the table. Analysis of the data[11] was made difficult by the tail of the 5.19-ev $Ag^{109}$ resonance. In addition, the total cross section data of various groups have not yielded consistent parameters. This is due partly to the large Doppler broadening of the resonance and partly to insufficient instrument resolution in the total cross section work. As a result of these factors, Sheer and Moore have described their $g$-value assignment as "tentative." We, therefore, feel justified in assigning a low *confidence factor* to this result.

$Ag^{109}$: Highly detailed data by Wood,[12] and Sheer and Moore[11] are in excellent agreement and make the $g$-value assignment highly reliable.

$Cd^{113}$: Accurate data by Brockhouse[7] and less accurate data by Beeman[13] are in agreement. A high degree of confidence can be placed in this result.

$Te^{123}$: Independent measurements by Foote[14] and Heindl[15] agree. The data in both cases are highly detailed and comprehensive, so the result has high reliability.

$Au^{197}$: The 4.91-ev resonance in gold has been studied more carefully than any other neutron resonance. The scattering measurements were made by three methods, the "thick sample,"[8] the "bright-line,"[12] and the "thick-thin sample"[10] methods. Although Titman and Sheer[8] initially chose the $I-\frac{1}{2}$ value, a later improvement in the total cross section data caused them to change their assignment to $I+\frac{1}{2}$.[16] All results are now in excellent agreement and a maximum confidence factor appears justified.

$Sm^{149}$: This resonance has been studied both by scattering[7] and polarization[4] and the results obtained by these very different methods agree.

$Ta^{181}$: The data in this case are limited to "bright-line" measurements by Wood and Foote.[17] The total cross section parameters are less accurate than for $Au^{197}$ and $Ag^{109}$ because of the larger Doppler broadening of the resonance. The $g$-value determination was also

TABLE I. Measured $g$-values for the lowest-energy resonance in various isotopes. The spin of the target nucleus is $I$, the energy of the resonance is $E_0$, $g$ is the statistical weight factor defined in Eq. (1), and $J$ is the spin of the compound state. The method of measurement is designated as $S$ for neutron resonance scattering, or $P$ for polarization. The confidence factor was estimated by the author and is defined in the text.

| Target isotope | $I$ | $E_0$ (ev) | $g$ | $J$ | Method | Confidence factor | Reference |
|---|---|---|---|---|---|---|---|
| $Rh^{103}$ | 1/2 | 1.26 | 3/4 | $I+\frac{1}{2}$ | $S$ | 0.75 | a |
| $Ag^{107}$ | 1/2 | 16.6 | 1/4 | $I-\frac{1}{2}$ | $S$ | 0.55 | b |
| $Ag^{109}$ | 1/2 | 5.19 | 3/4 | $I+\frac{1}{2}$ | $S$ | 0.90 | b, c |
| $Cd^{113}$ | 1/2 | 0.18 | 3/4 | $I+\frac{1}{2}$ | $S$ | 0.95 | a, d |
| $Te^{123}$ | 1/2 | 2.23 | 3/4 | $I+\frac{1}{2}$ | $S$ | 0.90 | e, f |
| $Au^{197}$ | 3/2 | 4.91 | 5/8 | $I+\frac{1}{2}$ | $S$ | 0.95 | c, g, h |
| $Sm^{149}$ | 7/2 | 0.094 | 9/16 | $I+\frac{1}{2}$ | $S, P$ | 0.95 | a, i |
| $Ta^{181}$ | 7/2 | 4.28 | 9/16 | $I+\frac{1}{2}$ | $S$ | 0.85 | j |
| $In^{115}$ | 9/2 | 1.45 | 11/20 | $I+\frac{1}{2}$ | $S, P$ | 0.95 | a, k, l |

a See reference 7.
b See reference 11.
c See reference 12.
d See reference 13.
e See reference 14.
f See reference 15.
g See references 8 and 16.
h See reference 10.
i See reference 4.
j See reference 17.
k See reference 5.
l See reference 18.

[4] Roberts, Bernstein, Dabbs, and Stanford, Phys. Rev. **95**, 105 (1954).
[5] Dabbs, Roberts, and Bernstein, Phys. Rev. **98**, 1512 (1955).
[6] Bernstein, Roberts, Stanford, Dabbs, and Stephenson, Phys. Rev. **94**, 1243 (1954); J. W. T. Dabbs and L. D. Roberts, Phys. Rev. **95**, 970 (1954).
[7] B. N. Brockhouse, Can. J. Phys. **31**, 432 (1953); Brockhouse, Hurst, and Bloom, Phys. Rev. **83**, 840 (1951); B. N. Brockhouse and D. G. Hurst, Phys. Rev. **83**, 841 (1951).
[8] J. Titman and C. Sheer, Phys. Rev. **83**, 746 (1951).
[9] L. B. Borst, Phys. Rev. **90**, 859 (1953).
[10] H. L. Foote, Jr. and J. Moore (unpublished); Phys. Rev. **98**, 1161 (1955).
[11] C. Sheer and J. Moore, Phys. Rev. **98**, 565 (1955).
[12] R. E. Wood, Phys. Rev. **95**, 644 (1954); dissertation, University of Utah, 1955 (unpublished).
[13] W. W. Beeman, Phys. Rev. **72**, 986 (1947).
[14] H. L. Foote, Jr., dissertation, University of Utah, 1954 (unpublished); Phys. Rev. **94**, 790 (1954).
[15] C. J. Heindl, dissertation, Columbia University, 1956 (unpublished).
[16] See footnote 41 in reference 11.
[17] R. E. Wood and H. L. Foote, Jr. (unpublished).

TABLE II. $g$-value assignments for $l=0$ resonances in the energy region $E_0<50$ kev. The method of determining $J$ is indicated by $R$ when resonance was resolved and peak cross section could be used, $A$ when area of the transmission was analyzed for two or more samples, and $F$ when curve fitting was used.

| Target isotope | $I$ | $E_0$ (kev) | $g$ | $J$ | Method | Confidence factor | Reference |
|---|---|---|---|---|---|---|---|
| $Na^{23}$ | 3/2 | 2.9 | 5/8 | $I+\frac{1}{2}$ | $A$ | 0.85 | a |
| $Al^{27}$ | 5/2 | 36 | 7/12 | $I+\frac{1}{2}$ | $A$ | 0.80 | b |
| $V^{51}$ | 7/2 | 4.1 | 9/16 | $I+\frac{1}{2}$ | $A$ | 0.75 | c |
| $V^{51}$ | 7/2 | 6.6 | 9/16 | $I+\frac{1}{2}$ | $A$ | 0.75 | c |
| $V^{51}$ | 7/2 | 11.5 | 9/16 | $I+\frac{1}{2}$ | $A$ | 0.75 | c |
| $Mn^{55}$ | 5/2 | 0.337 | 5/12 | $I-\frac{1}{2}$ | $F, A$ | 0.85 | d |
| $Mn^{55}$ | 5/2 | 1.08 | 7/12 | $I+\frac{1}{2}$ | $F, A$ | 0.85 | d |
| $Mn^{55}$ | 5/2 | 2.36 | 7/12 | $I+\frac{1}{2}$ | $F, A$ | 0.90 | c, d |
| $Tl^{203}$ | 1/2 | 0.24 | 3/4 | $I+\frac{1}{2}$ | $R$ | 0.75 | e |

[a] Hibdon, Muehlhause, Selove, and Woolf, Phys. Rev. **77**, 730 (1950).
[b] Toller, Newson, and Merzbacher, Phys. Rev. **99**, 1625 (1955), and H. W. Newson and W. Haeberli (private communication).
[c] H. Marshak and H. W. Newson (unpublished); some of these results are quoted in Phys. Rev. **98**, 1162 (1955). The confidence factors are those of H. Marshak (private communication).
[d] See reference 23.
[e] A. Stolovy and J. A. Harvey (to be published); see page 1336 of J. S. Levin and D. J. Hughes, Phys. Rev. **101**, 1328 (1956).

more difficult than for the other two because of ths higher spin of $Ta^{181}$. The results are reliable, but lese so than for $Au^{197}$ and $Ag^{109}$.

$In^{115}$: A disagreement exists between (a) the older data by Brockhouse[7] and by Borst[9] and (b) more recent results by Dabbs *et al.*,[5] and by Moore.[18] However, the assignment of Brockhouse[7] was based on erroneous total cross-section parameters.[19] The improved parameters of Landon[20] would tend to bring this into agreement with Dabbs and Moore. The data of Borst[9] were the first taken with the "bright-line" method, and since they did not have the advantage of many later refinements, they are very inadequate. The highly refined results of Moore[18] and the polarization work[5] make the assignment of $I+\frac{1}{2}$ very reliable.

It is evident from the above discussion that all results except $Rh^{103}$ and $Ag^{107}$ are dependable and must be considered significant. It should be mentioned that for $Ag^{109}$, $Te^{123}$, $Au^{197}$, $Ta^{181}$, and $In^{115}$ the observed $g$-value agrees quite accurately with the value calculated from Eq. (1). Values are not available to the author for the other cases. Also for $Ag^{109}$, $Au^{197}$, and $In^{115}$ the interference "minimum" indicates that the potential scattering for the $I-\frac{1}{2}$ state is of the expected magnitude, namely $\sigma_{p(I-\frac{1}{2})}\approx 4\pi g_- R^2$.

The question now arises as to whether or not the cases in Table I represent a *random* sampling. It should be noted that the nuclei are not confined to some particular class, but represent a variety of types including both even-odd and odd-even. The criteria for choosing an isotope for measurement consist primarily of the following factors: (1) availability of the material, (2) the practicality of fabricating a sample, (3) the existence of a low-energy resonance in the cross section which is strong enough to give a measurable effect. The third criterion undoubtedly discriminates against isotopes having an unusually weak lowest energy resonance, e.g., $U^{235}$. It tends also to favor $g_+$ resonances because, when all other factors are equal, the $g_+$ resonances are stronger by the ratio $g_+/g_-$. A $g$-value assignment has never been made for a very weak resonance. We must conclude, therefore, that the sampling in Table I has favored moderately strong and strong resonances.

[18] J. Moore, dissertation, Columbia University, 1956 (unpublished); Phys. Rev. **99**, 610 (1955).
[19] V. L. Sailor (unpublished).
[20] H. H. Landon and V. L. Sailor, Phys. Rev. **98**, 1267 (1955).

## B. Experimental Results at Higher Energy

A few $g$-value assignments have been made for $l=0$ resonances at higher energy in the lighter elements. These are all cases in which capture is negligible, so the resonance can be treated as purely scattering. The maximum cross section for these resonances reaches a value which is approximately $\sigma_0=4\pi\lambda\!\!\!^{-}{}_0^2 g$. Usually, the instrument resolution lowers the $\sigma_0$ appreciably so that some analysis is necessary. Occasionally, when resolution broadening is not excessive, the observed cross section will definitely exceed the $I-\frac{1}{2}$ value, making the $g$-value assignment relatively easy. Available results up to 50 kev are listed in Table II. Eight out of nine cases gave $I+\frac{1}{2}$ for the compound state.[21,22]

The values for $Mn^{55}$ deserve special attention. The data of Bollinger *et al.*[23] were obtained with the new fast chopper at the Argonne National Laboratory. Four resonances were present (the fourth at $E_0=7.3$ kev), of which three were fitted to a multilevel formula. The fit is reasonably convincing although not perfect. It appears that the assignments must be given a high confidence factor. Thus, at the present time, the $Mn^{55}$ resonance at $E_0=0.337$ kev is the only resonance below 50 kev which rather definitely appears to require the $I-\frac{1}{2}$ assignment.

Above 50 kev, available results seem about equally divided between $g_+$ and $g_-$ (see Table III).[24] These data have been included primarily for comparison purposes, and will not be discussed.

## III. CONCLUSIONS

The measurements of $g$-values for neutron resonances are limited in number; however, the reliability of

[21] *Note added in proof.*—The 2.9-kev resonance in $Na^{23}$ has recently been remeasured on the Harwell high-resolution time-of-flight spectrometer. The spin assigned is $J=I-\frac{1}{2}$ with a confidence factor of 0.95 in disagreement with the older result listed in Table II (J. E. Lynn, private communication).
[22] *Note added in proof.*—The 134-ev resonance in $Co^{59}$ is reported to have a spin of $J=I+\frac{1}{2}$ (J. E. Lynn and E. R. Rae, private communication). This result is also reported by E. R. Rae in the *Proceedings of the 1954 Glasgow Conference on Nuclear and Meson Physics* (Pergamon Press, London and New York, 1955), pp. 71 ff.
[23] Bollinger, Dahlberg, Palmer, and Thomas, Phys. Rev. **100**, 126 (1955).
[24] *Note added in proof.*—H. W. Newson (private communication) reports the following new assignments: The resonance in $Al^{27}$ at 36 kev is $I+\frac{1}{2}$ with a *confidence factor* of 0.80; the resonance in $Mg^{25}$ at 81 kev is $I-\frac{1}{2}$ and at 101 kev is $I-\frac{1}{2}$ each with a *confidence factor* of 0.60.

individual measurements is quite good in most cases. Unfortunately, no measurements have been possible for very weak resonances; and, at low energy (i.e., below 100 ev), there are no isotopes for which more than one $g$-value has been determined.[25] Despite these limitations the measurements, taken as a group, command attention. Of the eighteen resonances[26] in Tables I and II below 50 kev, sixteen are $g_+$, and only one appears definitely to be $g_-$. One is very uncertain but was assigned to $g_-$. The probability of obtaining such an overwhelming number of $g_+$ cases in a random sampling would be very small if $g_+$ and $g_-$ were equally probable.

In view of the limited data, the conclusions must be tentative. The results indicate that resonances having average or greater than average strength are predominantly $g_+$. These results could be attributed to a difference in the average level spacing[27] $D$, or to a difference in the average neutron width $\bar{\Gamma}_n^0$ for the two possible values of $J$. At the present time there is no definite evidence to indicate how either $D$ or $\bar{\Gamma}_n^0$ should depend on $J$. It would appear that the strength function, $\bar{\Gamma}_n^0/D$ could be quite different for the two possible spin states, being substantially larger for the $J=I+\frac{1}{2}$ state.

More comprehensive data, particularly on the weaker resonances, are badly needed. Until such data become available it appears imprudent to assume that the observed resonances are equally distributed between the two spin states or that $\bar{\Gamma}_n^0/D$ is the same for the two spin states.

The author wishes to thank H. W. Newson, W. Haeberli, and H. Marshak for permission to quote their results on $Al^{27}$, $V^{51}$, and $Mn^{55}$ prior to publication.

TABLE III. $g$-value assignments for $l=0$ resonances lying above 50 kev. See Table II for key to symbols.

| Target isotope | $I$ | $E_0$ (kev) | $g$ | $J$ | Method | Confidence factor | Reference |
|---|---|---|---|---|---|---|---|
| $Na^{23}$ | 3/2 | 297 | 3/8 | $I-\frac{1}{2}$ | | 0.75 | a |
| $Na^{23}$ | 3/2 | 542 | 3/8 | $I-\frac{1}{2}$ | | 0.55 | a |
| $Al^{27}$ | 5/2 | 90 | 7/12 | $I+\frac{1}{2}$ | $R$ | 0.90 | b |
| $V^{51}$ | 7/2 | 69.1 | 9/16 | $I+\frac{1}{2}$ | $R$ | 0.75 | c |
| $V^{51}$ | 7/2 | 87.1 | 7/16 | $I-\frac{1}{2}$ | $R$ | 0.75 | c |

[a] P. H. Stelson and W. M. Preston, Phys. Rev. **88**, 1354 (1952).
[b] See reference b, Table II.
[c] See reference c, Table II.

[25] *Note added in proof.*—Ten resonances in silver have been studied by E. R. Rae with the Harwell time-of-flight spectrometer, and have been reported by him at the recent International Conference on Nuclear Reactions, Amsterdam, July 2–7, 1956. He reports six cases of spin $I+\frac{1}{2}$, two cases of $I-\frac{1}{2}$, and two not assigned. The $I+\frac{1}{2}$ resonances are in $Ag^{109}$ at 5.2, 30.7, 42, and 71 ev and in $Ag^{107}$ at 41, and 51.8 ev. The $I-\frac{1}{2}$ resonances are in $Ag^{109}$ at 56 and 88 ev. The spins of the resonances in $Ag^{107}$ at 16.3 and 45 ev could not be assigned. Rae points out that these results are not inconsistent with the weighting factor $(2J+1)$ which appears in the level density formula proposed by Bethe in 1937 [H. A. Bethe, Revs. Modern Phys. **9**, 69 (1939), see Eq. 302]. The presence of such a factor in the level density could possibly account also for the results listed in Tables I and II. More data are needed before a definite decision can be made.

[26] *Note added in proof.*—An additional $g_-$ case should appear in Table I. This is the "negative energy" resonance in $Hg^{199}$ at $E_0\approx -2$ ev, which is probably $J=I-\frac{1}{2}$. The absence of the ground-state transition in the capture gamma-ray spectrum following thermal neutron capture indicates that the $Hg^{200}$ compound nucleus is formed in a $J=0$, odd parity state. This assignment was made by B. B. Kinsey and G. A. Bartholomew, Can. J. Phys. **31**, 1051 (1954); and Adyasevish, Groshev, and Demidov, *Academy of Science, U.S.S.R., Atomic Energy Conference Report, 1955* (Academy of Science, Moscow, 1955), p. 270.

[27] The density of $g_-$ resonances would be determined by the number of available $J=I-\frac{1}{2}$ states in the compound nucleus. As is shown in reference 3, the density of states is a function of $J$. It should be noted also that if the density of states is less for the smaller $J$, then the average distance between levels, $D$, would be larger for $I-\frac{1}{2}$. Thus if $\bar{\Gamma}_n^0/D$ were the same for the two spin states, the level density difference would predict a *larger* $\bar{\Gamma}_n^0$ for the $I-\frac{1}{2}$ state.