Scattering of D(d,n)He^3 Polarized Neutrons by C^{12}†

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The right-left asymmetry of D(d,n)He^3 neutrons scattered by C^{12} was observed for various neutron emission angles, deuteron bombarding energies, and C$^{12}(n,n)$ scattering angles. Using a figure for carbon polarization based on a C$^{12}(n,n)$ phase shift analysis, the maximum neutron polarization was found to be (10.6 ± 1.0)% for neutrons produced at 53° (c.m.) by 600–700 kev deuterons on a thick target. Observation of the right-left asymmetry for different neutron scattering angles yielded a sin 2θ angular dependence of the carbon polarization in agreement with the prediction of the C$^{12}(n,n)$ phase shift analysis.

THE polarization of the neutrons from the D(d,n)He^3 reaction has been measured by several workers,[1] who have reported values of 10–40% for incident deuteron energies of 300–800 kev. The polarization is observable through the right-left asymmetry induced when the neutrons are elastically scattered by a nucleus involving strong spin-orbit coupling.[2] Two groups have reported statistically significant results. Meier, Scherrer, and Trumpy[1] observed the right-left asymmetry of the D(d,n) neutrons scattered by carbon. They inferred the carbon polarization P_c from a phase shift analysis of C$^{12}(n,n)$ zenith angular distribution data. Using this value of P_c, they found a maximum neutron polarization of (10.8 ± 1.2)% for 600-kev incident deuterons.

Levintov *et al.*[1] observed the right-left asymmetry in He$^4(n,n)$ scattering of the D(d,n) neutrons. Using the phase shifts determined by Seagrave[3] for He$^4(n,n)$ scattering, they deduced from their observed right-left asymmetry a neutron polarization of (17.5 ± 2.0)% for neutrons produced at 49° by 800-kev deuterons.

The reaction has been studied theoretically by Blin-Stoyle[4] and Fierz,[5] who find for the polarization of neutrons emitted at c.m. angle θ_1,

$$\mathbf{P}_n(\theta_1)=\mathbf{n}_1P_n(\theta_1)=\mathbf{n}_1\frac{\sum_m a_m \sin 2m\theta_1}{d\sigma/d\Omega}, \tag{1}$$

where

$$\mathbf{n}_1=\mathbf{k}_d\times\mathbf{k}_n/|\mathbf{k}_d\times\mathbf{k}_n|.$$

The coefficients a_m depend upon the "approach" cross sections[6] and the spin-orbit coupling. Owing to insufficient knowledge of nuclear forces, calculation of the coefficients is not possible.

We have performed further measurements of the D(d,n) neutron polarization at neutron emission angles near the polarization maximum for deuteron bombarding energies of 500, 600, and 700 kev. Using this source of polarized neutrons, we have also observed the dependence of the carbon polarization upon the scattering angle θ_2 (Fig. 1).

EXPERIMENTAL METHOD

The right-left asymmetry of neutrons scattered in carbon was employed in the polarization analysis as in the work of Meier *et al.* It can be shown[7] that for the scattering of neutrons of polarization $\mathbf{P}_n(\theta_1)$ by a scatterer of polarization $\mathbf{P}_c(\theta_2)$ (the degree of polarization induced by scattering unpolarized neutron at c.m. angle θ_2), the differential cross section can be written in terms of that for unpolarized neutrons as

$$\sigma(\theta_2,\phi_2)=\sigma_u(\theta_2)[1+\mathbf{P}_n(\theta_1)\mathbf{P}_c(\theta_2)]. \tag{2}$$

The phase analysis of Meier *et al.*[1] for C$^{12}(n,n)$ scattering shows that in the 2.4–3.6 Mev neutron energy range, the $P_{\frac{3}{2}}$ and $P_{\frac{1}{2}}$ phases are strongly split by spin-orbit coupling, yielding a maximum carbon polarization of 1 such that over most of the range,

$$\mathbf{P}_c(\theta_2)=\mathbf{n}_2P_c(\theta_2)\approx-\mathbf{n}_2\sin 2\theta_2, \tag{3}$$

where

$$\mathbf{n}_2=\mathbf{k}_n\times\mathbf{k}_n'/|\mathbf{k}_n\times\mathbf{k}_n'|.$$

If the C$^{12}(n,n)$ event lies in the D(d,n) reaction plane, the right-left asymmetry ratio before correction for finite geometry is then

$$R(\pm\theta_1,\theta_2)=\frac{1\mp P_n(\theta_1)P_c(\theta_2)}{1\pm P_n(\theta_1)P_c(\theta_2)}. \tag{4}$$

Here the choice of signs corresponds to neutron emission to the left or right (Fig. 1).

From observed values of $R(\theta_1)$ with θ_2 fixed at $\pi/4$ (as described below), we have determined the neutron polarization $P_n(\theta_1)$, assuming Eq. (3) to be correct. The validity of these results, however, depends upon that of the phase shifts of Meier *et al.* which yield Eq. (3). If $P_c(\pi/4)$ were less than 1, the values of

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[7] L. Wolfenstein, Phys. Rev. 75, 1664 (1949); J. V. Lepore, Phys. Rev. 79, 137 (1950); W. Simon and T. A. Welton, Phys. Rev. 90, 1063 (1953); 92, 1050 (1953).

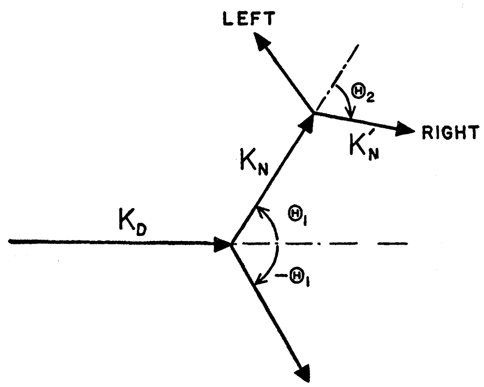

FIG. 1. Definition of the vector momenta and angles referred to in the text. K_D, K_N, and K_N' represent the momenta of the incident deuteron, produced neutron, and scattered neutron.

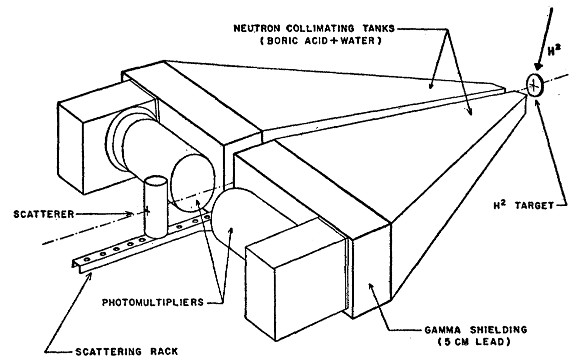

FIG. 2. Experimental arrangement.

$P_n(\theta_1)$ determined as described would be lower than the correct values.

In order to further investigate this point, we have observed $R(\theta_2)$ with θ_1 fixed at a value yielding maximum neutron polarization. From these data corrected for finite detector geometry, we determine $P_c(\theta_2)$. The absolute values are again dependent upon the phase analysis of Meier *et al.*; the angular dependence, however, is not and constitutes a test of that analysis.

EXPERIMENTAL DETAILS

The right-left asymmetry was measured with the arrangement shown in Fig. 2. Stilbene crystals ($\frac{3}{4}$-in. cubes) simultaneously detected neutrons scattered to the right and left, with wedges of $H_2O+H_3BO_3+Pb$ shielding the detectors from the deuterium target. The radius of the scatterer was one quarter of a mean free path. The difference between the "scatterer in" and "scatterer out" counting rates was taken as the scattered neutron rate. The "difference" pulse height spectrum with a carbon scatterer showed a well-defined plateau and sharp break characteristic of the proton energy spectrum in *s*-wave *n–p* scattering. For each deuteron energy and neutron emission angle, the counters were differentially biased to accept pulses just below the sharp break, corresponding to the maximum neutron energy. This made possible discrimination against neutrons inelastically scattered from the surroundings; the carbon scattering was wholly elastic. The "in" minus "out" difference rate varied from 12% to 40% of the "out" rate, depending upon the scattering angle θ_2.

The deuterium target was a thick "drive in" target, produced by prolonged bombardment of a water-cooled copper disk with 50 to 75 microamperes of deuterons. Among previous workers, only Meier *et al.*[1] attempt to employ a thin target. The close agreement between their results and those given below suggests that the backing of their thin heavy-ice target may have constituted a thick "drive in" target. Even with a thick target, the range of neutron energies at the various angles θ_1 (2.55−3.3 Mev) falls within the supposed range of validity of Eq. (2). The energy spectrum of the neutrons detected and contributing to the results reported herein is not well known. However, an estimate based on the variations of the reaction cross section and detector efficiencies with neutron energy indicates that a it is quite narrow. For neutrons produced at 53° (c.m.) by 600-kev deuterons, the estimated mean neutron energy is 3.10 Mev compared to the maximum energy of 3.22 Mev.

Multiple scattering effects were considered insignificant by virtue of the following argument. In addition to (1) neutrons singly scattered into the detectors, the total counting rates included (2) once-scattered neutrons which subsequently scattered into the solid angles of the detectors and excluded (3) those which scattered out of these solid angles. Both of these latter scattered intensities should be roughly proportional to the rate of singly scattered neutrons. Thus the right-left ratio of total rates should closely approximate the ratio of singly scattered rates, which is the quantity of interest. The influence of multiple scattering is discussed more fully by Meier *et al.*[1]

The right-left asymmetry was measured in the following way. The ratio of the right to left "difference" counting rates yielded an observed ratio r which was related to the "true" ratio R by a constant involving the ratio of the over-all detector efficiencies ($r=kR$). This constant k was eliminated by observing r at both θ_1 and $-\theta_1$ [corresponding to the two choices of signs in Eq. (4)]. Thus,

$$R(-\theta_1)=[R(\theta_1)]^{-1};\quad R(\theta_1)=\left[\frac{r(\theta_1)}{r(-\theta_1)}\right]^{\frac{1}{2}}.$$

The only geometrical asymmetry not eliminated by this procedure was that due to the finite angle subtended at the target by the scatterer and the anisotropy of the D(d,n) neutrons. The scatterer subtended an angle of only 1° over which the neutron intensity changes by only 1.5% for 600-kev incident deuterons on a thin target. Taking into account the fact that the target was thick and the scatterer was cylindrical, the maxi-

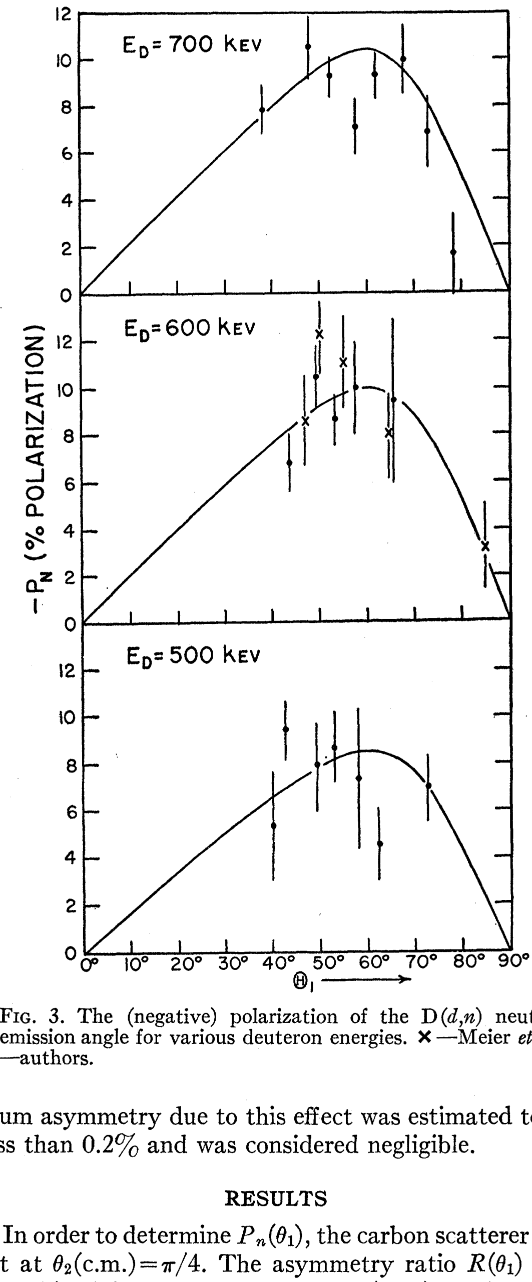

FIG. 3. The (negative) polarization of the D(d,n) neutrons *vs* emission angle for various deuteron energies. × —Meier *et al.*[1]; ●—authors.

mum asymmetry due to this effect was estimated to be less than 0.2% and was considered negligible.

RESULTS

In order to determine $P_n(\theta_1)$, the carbon scatterer was set at $\theta_2(\text{c.m.})=\pi/4$. The asymmetry ratio $R(\theta_1)$ was determined for several values of θ_1(c.m.) at deuteron bombarding energies of 500, 600, and 700 kev. After correction for the finite size of the scatterer and detectors (see Appendix), one obtains from Eq. (4)

$$P_n(\theta_1)=1.09\left[\frac{1-R(\theta_1)}{1+R(\theta_1)}\right].$$

The values of $P_n(\theta_1)$ obtained from the data through this relation are shown in Fig. 3. The solid curves are $P_n(\theta_1)$ as computed from Eq. (1), using the results of Chagnon and Owen[8] for $d\sigma/d\Omega$, taking only the $m=1$ term in the numerator, and adjusting a_1 to fit the experimental results. The agreement is seen to be only fair. The fact that the spread of the points is greater than the indicated statistical deviations suggests the presence of some instrumental fluctuation.

[8] P. R. Chagnon and G. E. Owen, Phys. Rev. **101**, 1798 (1956).

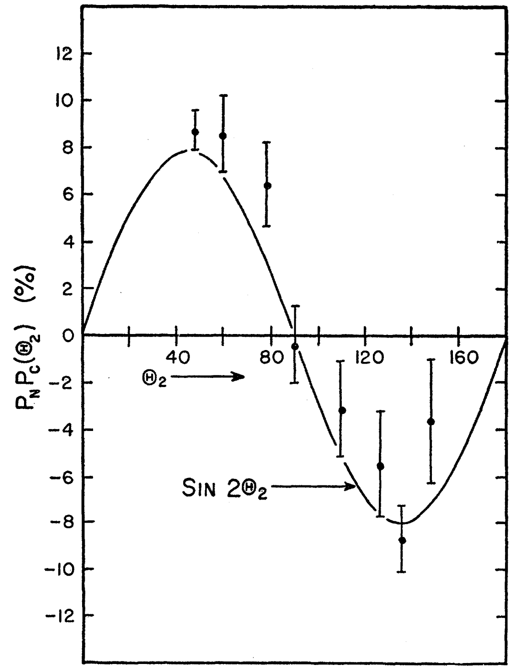

FIG. 4. The product of the neutron polarization and carbon polarization *vs* the carbon scattering angle. Neutrons produced at 53° (c.m.) by 600-kev deuterons were employed.

In order to observe $P_c(\theta_2)$, neutrons produced by 600-kev deuterons and emitted at $\theta_1=53°$ were allowed to strike the carbon scatterer, and again by the procedure outlined above, $R(\theta_2)$ was observed for values of θ_2 from 45° to 145°, by moving the scatterer along the rack shown in Fig. 2. After correction for the finite geometry which in this case will change with θ_2 (see Appendix), one obtains from Eq. (4),

$$P_nP_c(\theta_2)=\gamma'(\theta_2)\left[\frac{1-R(\theta_2)}{1+R(\theta_2)}\right],$$

where $\gamma(\theta_2)$ is the finite-geometry correction. Through this relation, the data yield the values of $P_nP_c(\theta_2)$ shown in Fig. 4. The agreement with the solid curve, which is simply $\sin 2\theta_2$ from Eq. (3), is fairly good and supports the view that the origin of the carbon polarization is the splitting of the $P_{\frac{1}{2}}$ and $P_{\frac{3}{2}}$-phase shifts.

The method of correcting for finite geometry outlined in the Appendix is due to Dr. John S. Plaskett. We are much indebted to him for his help in this regard.

APPENDIX I. CORRECTION FOR THE FINITE SIZE OF THE SCATTERER AND DETECTOR

The angles subtended by the scatterer at the target were only a few degrees and involve completely negligible corrections for the finite size of the scatterer. Therefore, in considering the scattering geometry we can assume the neutrons to be a parallel beam in the $+x_3$ direction, scattering from a right circular cylinder (symmetry axis along x_1) into a cubic detector with edges parallel to the coordinate axes and with the center lying in the (x_2,x_3) plane. The differential cross section may be considered a function of the direction cosines l_i of the scattered neutron direction, and in view of Eqs. (1), (2), and (3) may be written,

$$\sigma(l_1l_2l_3)=\sigma_u(l_3)(1+2P_nl_2l_3). \tag{5}$$

If the detector efficiency is assumed to be proportional to the volume traversed by the scattered neutrons, the number per second scattered at $x_1x_2x_3$ and detected at $x_1'x_2'x_3'$ in the detector is $I\sigma(l_i)dx_idx_i'$, where l_i, x_i, and x_i' each represent all three direction cosines and coordinates, and where I is a constant involving the incident neutron intensity and the detector efficiency. The integral of this expression over the scatterer and detector volumes yields the counting rate,

$$C=I\int_{V_s}\int_{V_d}\sigma\left(\frac{x_i'-x_i}{|\mathbf{r}'-\mathbf{r}|}\right)dx_idx_i'.$$

Strictly speaking, absorption in the scatterer and detector introduces a dependence of I upon x_i and also shifts the "effective centers" of the scatterer and detector (to be used below). However, evaluation of this effect for the setup employed reveals that it is negligible. It changes the mean scattering angle, for example, by one degree at the most.

It is convenient to express the cross section in the integrand by a function of $x_i'-x_i$, *viz.*, $f(x_i'-x_i)$; and to introduce a vector $\boldsymbol{\xi}$ from scatterer center (X_i) to detector center (X_i'). To compute the integral we expand $f(x_i'-x_i)$ in a Taylor's Series about ξ_i, retaining terms through the second derivatives. The integral can then be evaluated in terms of the moments of the scatterer and detector and the mixed second derivatives of the cross section evaluated at ξ_i, *viz.*, $f_{jk}(\xi_i)$. Using the summation convention, the result may be written,

$$C=I[f(\xi_i)+\tfrac{1}{2}(M_{jk}+M_{jk}')f_{jk}(\xi_i)]V_sV_d,$$

where

$$M_{jk}=\frac{1}{V_s}\int_{V_s}(x_j-X_j)(x_k-X_k)dx_i,$$

$$M_{jk}'=\frac{1}{V_d}\int_{V_d}(x_j'-X_j')(x_k'-X_k')dx'.$$

The first moments conveniently do not appear; also for the orientation of the scatterer and detector employed, the third moments would vanish had the Taylor's Series been extended to third-order derivatives.

The evaluation of the counting rate is somewhat laborious but is simplified by the fact that for the case at hand only f_{11}, f_{22}, and f_{33} are required. The final result for the counting rate can be written in terms of the direction cosines of $\boldsymbol{\xi}$, which we denote λ_i,

$$C(\lambda_i)=\epsilon_1(\lambda_i)+\epsilon_2(\lambda_i)P_n,$$

where

$$\epsilon_1(\lambda_i)=\sigma_u-\sigma_u'(A+B)\lambda_3+\sigma_u''B\lambda_2^2,$$

$$\epsilon_2(\lambda_i)=2\sigma_u\lambda_2\lambda_3(1-2A-4B) -2\sigma_u'\lambda_2[(A+5B)\lambda_3^2-2B]+2\sigma_u''\lambda_2^3\lambda_3B.$$

In these expressions the primes denote derivatives of σ_u with respect to l_3 evaluated at λ_3, and A and B involve the moments and the magnitude ξ as follows:

$$A=(1/2\xi^2)(M_{11}+M_{11}'),$$

$$B=\frac{1}{2\xi^2}(M_{22}+M_{22}')\approx\frac{1}{2\xi^2}(M_{33}+M_{33}').$$

The approximation in this last equation is quite good for the geometry employed. Since $\boldsymbol{\xi}$ always lies in the (x_2,x_3) plane, all terms containing λ_1 are zero and do not appear. Also, scattering to the right $(\phi=\pi/2)$ and left $(\phi=-\pi/2)$ correspond respectively to λ_2 (and, hence, ϵ_2) being positive or negative. The observed right-left asymmetry ratio is then

$$R=(\epsilon_1+\epsilon_2P_n)/(\epsilon_1-\epsilon_2P_n)$$

or, letting $\epsilon_1/\epsilon_2=\gamma$,

$$P_n=\gamma(1-R)/(1+R).$$

The factor γ was evaluated for the measurements described, obtaining σ_u and its derivatives from the data of Meier *et al.*[1] Fortunately γ is not strongly dependent upon these quantities and $\sigma_u(l_3)$ could be closely approximated by a parabola.

In the case of the second group of experiments, in which P_nP_c was measured for various $\theta_2=\cos^{-1}l_3$, we used the correction factor $\gamma'=\gamma\sin 2\theta_2$. The uncorrected results for $P_nP_c(\theta_2)$ fit $\sin 2\theta_2$ quite well, which justifies somewhat this procedure. In any case the corrections were only about 10% or smaller.