Physical Conditions for Ferromagnetic Resonance

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All but one of the multiple ferromagnetic resonances observed by White and Solt in a sphere are shown to correspond to modes in which the oscillating part of the magnetization is independent of r (the only case ordinarily considered) or varies linearly with r in the sample.

ORDINARILY the ferromagnetic resonance of a spherical sample is calculated under the assumption that the oscillating part of the magnetization is the same at all points of the sample. But multiple ferromagnetic resonances have been observed[1–3] under conditions which suggest that higher spacial modes were being excited, in which the magnetization in the plane (xy) perpendicular to the main external field varies with position in the sample. The frequencies of the simplest of these higher modes are calculated here, and it is shown that all but one line (line B) of White and Solt[1] can be explained in this way. To find these frequencies, note that the resonance equations involving the internal fields

$$\begin{aligned} i\omega M_x &= \gamma(M_y H_z - M_z H_y), \\ i\omega M_y &= \gamma(M_z H_x - M_x H_z), \end{aligned} \tag{1}$$

may be solved uniquely for ω whenever $\mathbf{H}(\mathbf{r})_{\text{induced}} = K\mathbf{M}(\mathbf{r})$. This is certainly true when both H and M are uniform, and in this case one gets the classical demagnetizing factors. However, there are other than the uniform distribution for which $\mathbf{H}(\mathbf{r})_{\text{induced}} = K\mathbf{M}(\mathbf{r})$.

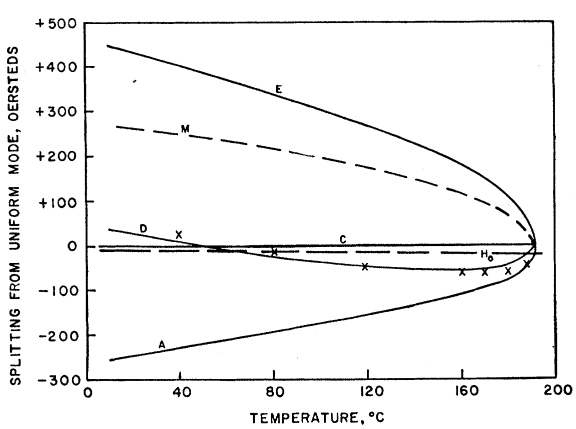

FIG. 1. The temperature dependence of the magnetic fields necessary for the various multiple resonances in a manganese ferrite sphere. Full lines are experimental data. Dashed lines and × are calculated values. The agreement of theory with experiment is shown by the correspondence of the line marked H_0 with line C, and the correspondence of the ×'s with line D.

We assume only slight deviations for M from the direction, z, of the applied field, H_0.

We consider, for example, the sphere. Here $H_z = H_0 - (4\pi/3)M_0$. However, in x, y we must solve the magnetostatic equation

$$\nabla^2\phi = -4\pi\nabla\cdot\mathbf{M},$$

where $\mathbf{H}_{\text{induced}} = \nabla\phi$, and we wish only those solutions where

$$H_x(r)_{\text{induced}} = KM_x(r), \quad H_y(r)_{\text{induced}} = KM_y(r).$$

There is an intrinsic time dependence since the M_x, M_y precess about the z axis. The system may precess from a configuration where $K = c_1$ to a configuration where $K = c_2$ one-quarter cycle later. Under these circumstances, Eq. (1) yields

$$\omega = \gamma\left\{\left[H_0 - \left(\frac{4\pi}{3} - c_1\right)M_0\right]\left[H_0 - \left(\frac{4\pi}{3} - c_2\right)M_0\right]\right\}^{\frac{1}{2}}.$$

Some of the lower order solutions to the eigenvalue problem have been found by essentially guessing the $M_{x,y}$ distribution, guided by the symmetry of the appropriate driving field. For example, a radial distribution $M_x = ax$, $M_y = ay$ contributes both a volume and a surface divergence. The internal potential is found to be $\phi_{\text{int}} = (8a\pi/5)(-x^2 - y^2 - \frac{1}{2}z^2)$ and $H_{x,y} = (-16\pi/5)M_{x,y}$. After $M_{x,y}$ precesses by $\pi/2$, the distribution becomes circular, i.e., $M_x = -ay$, $M_y = ax$. This gives no divergence anywhere and thus $H_{x,y} = 0$. Combining these two results, $H_r = (-16\pi/5)M_r$, $H_\theta = 0$ and the resonant frequency becomes

$$\omega_1 = \gamma\{[H_0 - (4\pi/3)M_0][H_0 + (28\pi/15)M_0]\}^{\frac{1}{2}}.$$

Similarly $M_x = 0$, $M_y = az$ gives $H_{x,y} = (-4\pi/5)M_{x,y}$ and $\omega_2 = \gamma[H_0 - (8\pi/15)M_0]$. For $M_x = -ax$, $M_y = ay$ one finds $H_{x,y} = (-8\pi/5)M_{x,y}$, yielding

$$\omega_3 = \gamma[H_0 + (4\pi/15)M_0].$$

Of course the usual resonance $\omega_0 = \gamma H_0$ occurs when $M_x = a$, $M_y = 0$.

The results are shown to be consistent with experimental data[1,2] in Fig. 1. Frequencies were converted into fields and ω_1 corresponds to line D, ω_2 to line E, and ω_3 to line A. These correspondences also correctly predict the position in the cavity where the field configuration is optimal for inducing the mode. Since $M_0(T)$ was not available, M_0 and H_0 were calculated from ω_2 and ω_3 and used in ω_1 to calculate the resonant field for this distribution.

[1] R. L. White and I. H. Solt, Jr., Phys. Rev. **104**, 56 (1956), preceding paper.
[2] White, Solt, and Mercereau, Bull. Am. Phys. Soc. Ser. II, **1**, 12 (1956).
[3] L. R. Walker, Bull. Am. Phys. Soc. Ser. II, **1**, 125 (1956).