completely rigorous). For $T > 0°\text{K}$, some fairly crude approximations, taken from Feynman's theory of the lambda-transition, have to be introduced. This part of the theory is therefore open to improvement—possibly in the form of a more rigorous proof that Feynman's implied criterion for B.E. condensation [the importance of long cycles in the sum (40) for the partition function] is equivalent to our criterion (4) at thermal equilibrium. Despite these imperfections, however, our analysis would appear to strengthen materially the case put forward previously by London[1,2] and Tisza[9] for the importance of B.E. condensation in the theory of liquid helium.

We have not considered here how B.E. condensation is related to superfluidity and to the excitation theory[16,17] of liquid helium. This will be done in another paper, where some of the results already obtained by Bogolyubov[12] for weakly repelling B.E. particles will be extended[40] to the case of interacting He^4 atoms.

9. ACKNOWLEDGMENTS

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[40] A brief account of this work was given at the National Science Foundation Conference on Low-Temperature Physics and Chemistry, Baton Rouge, Louisiana, December, 1955 (unpublished).

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Spectral Diffusion in Magnetic Resonance*

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Electron spin-spin interaction is discussed for the case of strong hyperfine broadening. The hyperfine interaction is represented by a resonance spectrum of width $\Delta\omega$ and the electron interactions, which are taken to be dipolar, are treated by time-dependent perturbation theory. A characteristic relaxation time for electron spins, $\tau_s = \Delta\omega/\Delta^2$, is found, where Δ is a measure of the strength of the dipolar interaction. The time-dependent theory suggests a modification of the Bloch equations to give a phenomenological description of systems of this kind. Spin-spin relaxation is represented by a term which gives diffusion of spin excitation through the resonance spectrum. Slow passage, rapid passage, and free relaxation are considered by using the modified equations.

I. INTRODUCTION

THE Bloch equations[1] have given a satisfactory description of saturation effects involving F centers in alkali halides.[2] One of the reasons for this success is that the dipolar interaction between F-center electrons is extremely weak compared with the lattice interaction.[3] The effects of spin-spin interaction have not been observed in room temperature studies. Although the interaction is quite weak, it should be detectable at liquid helium temperatures.

The purpose of this note is to consider the form expected for such an interaction. In the first part a treatment of dipolar interaction is developed in terms of time-dependent perturbation theory. The results of this treatment are applied to a macroscopic description of the interaction in terms of a modification of the Bloch equations. In the final section, these equations are applied in an examination of slow passage, rapid passage, and free relaxation.

II. MICROSCOPIC THEORY

We consider a system suggested by the interaction between F-center electrons in alkali halides. The interaction between electron spins is taken to be dipolar. There is a contact-type hyperfine interaction between electron and nuclear magnetic moments.[4] For simplicity electron spin-orbit coupling is neglected. The magnetic Hamiltonian in an external field $\mathbf{H}_0$ is given by

$$\mathcal{H} = \sum_i g\beta\mathbf{s}_i\cdot\mathbf{H}_0 - \frac{16\pi}{3}\sum_{il}\beta\mu_l|\psi_i(\mathbf{r}_l)|^2\mathbf{s}_i\cdot\mathbf{I}_l/I_l + \tfrac{1}{2}\sum_{ij} g\beta\mathbf{s}_i\cdot\left[\frac{g\beta\mathbf{s}_j}{r_{ij}{}^3} - \frac{3g\beta\mathbf{r}_{ij}(\mathbf{r}_{ij}\cdot\mathbf{s}_j)}{r_{ij}{}^3}\right]. \qquad (2.1)$$

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[1] F. Bloch, Phys. Rev. **70**, 460 (1946).

[2] A. M. Portis, Phys. Rev. **91**, 1071 (1953). The identification of T_2 in this reference with the mean dipolar field is in error. As established in the present treatment, T_2 should equal T_1 for F centers in KCl at room temperature. This yields a corrected value of 8×10^{-6} sec for T_1.

[3] A. G. Redfield, Phys. Rev. **98**, 1787 (1955).

[4] Kip, Kittel, Levy, and Portis, Phys. Rev. **91**, 1066 (1953).

Here $\psi_i(\mathbf{r}_l)$ is the amplitude of the wave function of the ith electron at the lth nucleus. A time-dependent treatment is cast in the following way: Given that the ith spin is in the state $S_z=+\frac{1}{2}$ at $t=0$, we wish to know its mean life in this state. We separate the Hamiltonian into a zero-order part:

$$\mathcal{H}_0=\sum_i g\beta s_i^z H_0-(16\pi/3)\sum_{il}\beta\mu_l|\psi_i(\mathbf{r}_l)|^2 s_i^z I_l^z/I_l +\tfrac{1}{2}\sum_{ij}[g^2\beta^2(1-3\gamma_{ij}^2)/r_{ij}^3]s_i^z s_j^z, \quad (2.2)$$

where the z direction is the direction of $\mathbf{H}_0$ and γ_{ij} is the direction cosine of $\mathbf{r}_{ij}$ with the field. Of the remainder of the Hamiltonian, we need consider only the nearly secular part:

$$\mathcal{H}'=-\tfrac{1}{2}\sum_{ij}[g^2\beta^2(1-3\gamma_{ij}^2)/4r_{ij}^3] \times(S_i^-S_j^+ + S_i^+S_j^-). \quad (2.3)$$

The other terms in the magnetic Hamiltonian connect states differing by the order of the Zeeman energy. As long as the external magnetic field is large compared with the dipolar and hyperfine interactions, these terms may be neglected.

The zero-order Hamiltonian is diagonal in a representation characterized by the individual s_i^z. The local magnetic fields are distributed around the magnitude of the external field with a width dependent on the hyperfine and dipolar interaction. The wave function of the system may be written as a linear combination of the eigenfunctions of the zero-order Hamiltonian with coefficients which depend on the time:

$$\Psi=\sum_n a_n(t)\Psi_n e^{-iE_nt/\hbar}, \quad (2.4)$$

where

$$\Psi_n=\prod_j \psi_n(\mathbf{r}_j)\sigma_n(s_j), \quad (2.5)$$

and $\sigma_n(s_j)$ is the spin function of the jth electron in the nth state. The spin function may be α_j or β_j corresponding to $s_j^z=\pm\frac{1}{2}$. We take the system to be in the mth state at $t=0$. Since spin-orbit interaction is being neglected, we may write just the spin function:

$$\Psi_m=\alpha_i\prod_j \sigma_m(s_j).$$

In order to determine the mean life of the ith spin in the state α, we calculate the transition probability to all states

$$\Psi_n=\beta_i\prod_j \sigma_n(s_j)$$

for which, to first order,

$$a_n(t)=\frac{1}{i\hbar}\int_0^t \mathcal{H}_{nm}'(t')e^{i\omega_{nm}t'}dt'.$$

The perturbation connects states m and n which have apart from the ith spin only one other spin reversed in the opposite sense:

$$\Psi_m=\alpha_i\beta_j\prod_k \sigma_m(s_k),$$
$$\Psi_n=\beta_i\alpha_j\prod_k \sigma_n(s_k),$$

and $\sigma_m(s_k)=\sigma_n(s_k)$ for $k\neq i$ or j. The initial state is connected to approximately $\frac{1}{2}N$ states in this way, where N is the total number of spins and we have assumed equal numbers of up and down spins. The total initial transition probability for the ith spin:

$$w_i=\frac{1}{t}\sum_j B_{ij}^2\frac{\sin^2(\frac{1}{2}\omega_{ij}t)}{\omega_{ij}^2}, \quad (2.6)$$

where

$$B_{ij}=g^2\beta^2(1-3\gamma_{ij}^2)/2\hbar r_{ij}^3.$$

We now average the above expression over spins in approximately the same local field. If the electron spin concentration is sufficiently dilute, there is very little overlap of wave functions. Then $\omega_{ij}=\omega_j-\omega_i =(E_n-E_m)/\hbar$, the difference in Larmor frequencies of the ith and jth spins, is completely uncorrelated with r_{ij} and γ_{ij}. The average initial transition probability,

$$\langle w\rangle_{\text{Av}}=\frac{1}{Nt}(\sum_{ij} B_{ij}^2)\left\langle\frac{\sin^2(\frac{1}{2}\omega_{ij}t)}{\omega_{ij}^2}\right\rangle_{\text{Av}}, \quad (2.7)$$

where the average over ω_{ij} is a weighted average. The appropriate weighting function is the density of states given by the zero-order Hamiltonian. This is the same function of ω as $f(\omega)$, the normalized resonance absorption spectrum. Although the spectrum is, strictly speaking, discrete, the energy separation between states will usually be small compared with the mean life of a state and may be taken as continuous. At the other limit, the mean life must be long compared with the width of the spectrum. Under these conditions, we obtain

$$\left\langle\frac{\sin^2(\frac{1}{2}\omega_{ij}t)}{\omega_{ij}^2}\right\rangle_{\text{Av}}=\int_{-\infty}^{\infty}\frac{\sin^2[\frac{1}{2}(\omega_j-\omega_i)t]}{(\omega_j-\omega_i)^2}f(\omega_j)d\omega_j =\tfrac{1}{2}\pi t f(\omega_i). \quad (2.8)$$

The summation in Eq. (2.7) is just one-half the second moment of the transverse dipolar interaction, $\langle\Delta\omega^2\rangle_{\text{Av}}$. The factor of two arises because the sum is only over down spins. Finally we may write for the initial transition probability of spins with resonant frequency ω,

$$\langle w\rangle_{\text{Av}}=\tfrac{1}{4}\pi\langle\Delta\omega^2\rangle_{\text{Av}}f(\omega). \quad (2.9)$$

Now that we have an expression for the average initial transition probability, we wish to find the mean life of a spin state. In order to relate these two quantities, we must investigate the distribution in local dipolar fields. For a regular array of dipoles, the distribution in transverse dipolar fields is Gaussian and the mean life $\tau_s\sim 1/\langle w\rangle_{\text{Av}}$. However, if the dipoles are randomly distributed, τ_s may be quite different from $1/\langle w\rangle_{\text{Av}}$. The case of dipoles distributed randomly over a regular lattice has been investigated by Kittel and Abrahams[5] by the method of moments. We briefly

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[5] C. Kittel and E. Abrahams, Phys. Rev. **90**, 238 (1953).

review their method, which we apply to the present problem. The fourth moment of the dipolar interaction is

$$\langle\Delta\omega^4\rangle_{\text{Av}}=\frac{3}{N}\sum_i\left(\sum_j B_{ij}^2\right)^2-\frac{2}{N}\sum_{ij}B_{ij}^4, \qquad (2.10)$$

where the sums are over those lattice sites actually occupied by magnetic dipoles. If f is the fraction of sites occupied,

$$\langle\Delta\omega^2\rangle_{\text{Av}}=f\textstyle\sum_j B_{ij}^2, \qquad (2.11)$$

$$\langle\Delta\omega^4\rangle_{\text{Av}}=3f^2(\textstyle\sum_j B_{ij}^2)^2+f(1-3f)\textstyle\sum_j B_{ij}^4, \qquad (2.12)$$

where the sums are now over all lattice sites. Performing the sums in Eqs. (2.11) and (2.12) for a simple cubic lattice with the magnetic field along a cube edge, we obtain:

$$\langle\Delta\omega^4\rangle_{\text{Av}}/3(\langle\Delta\omega^2\rangle_{\text{Av}})^2=0.796+0.068/f.$$

For a Gaussian distribution in local fields, the right side should be unity. For small fractional concentrations, the ratio deviates considerably from what is expected for a Gaussian. Anderson[6] has argued from the statistical theory of line broadening that the distribution in local fields should in this case be Lorentz near the center with a Gaussian falloff in the wings. We fit the calculated second and fourth moments by a distribution

$$I(\omega)=1/(\omega^2+\Delta^2), \quad \omega^2<\alpha^2;$$
$$I(\omega)=0, \quad \omega^2>\alpha^2.$$

We obtain $\Delta=7.2fA$ and $\alpha=2.8A$, where $A=g^2\beta^2/2\hbar a^3$.

We are now able to see the nature of the error introduced by taking $\tau_s=1/\langle w\rangle_{\text{Av}}$ for randomly distributed spins. The distribution in local dipolar fields has a half-width corresponding to the mean separation between dipoles. The distribution drops off slowly above this value with contributions extending up to the order of the nearest neighbor interaction. The transition probability $\langle w\rangle_{\text{Av}}$ is the average over all local dipolar fields. The main contributions to this integral arise from spins with local fields well out in the wings of the distribution. These spins, few in number, undergo transitions very rapidly. The main body of spins, however, undergo transitions at a rate associated with the half-width in the energy distribution. Then

$$\frac{1}{\tau_s}\sim\langle w\rangle_{\text{Av}}\frac{\Delta^2}{\langle\Delta\omega^2\rangle_{\text{Av}}}=\Delta^2 f(\omega). \qquad (2.13)$$

In obtaining this expression for the mean life of a spin with resonance frequency ω, we have made a number of assumptions which it may be well to summarize. In order to treat the problem by time-dependent perturbation theory, the lifetime must be long compared with the width of the spectrum $\Delta\omega$,

$$\frac{1}{\tau_s}\sim\frac{\Delta^2}{\Delta\omega}<\Delta\omega.$$

This then requires that $\Delta<\Delta\omega$; the concentration must be low enough that the mean dipolar field is small compared with the width of the spectrum. We have further assumed that the electron spins are randomly distributed and that the difference in Larmor frequencies of any two spins is completely uncorrelated with their relative positions in the lattice.

III. MACROSCOPIC THEORY

The form of a macroscopic theory is suggested by the arguments leading to Eq. (2.8). The main contribution to the integral arises from ω_j within $1/\tau_s$ of ω_i; that is to say, when the ith spin flips, and this happens in a mean time τ_s, another spin within $1/\tau_s$ of the same place in the spectrum flips in the opposite direction. The resonance frequency of this second spin may be higher or lower than ω_i with equal probability. We may now start over with this second spin and watch it flip. It will now flip within a time τ_s against some third spin which will be with $1/\tau_s$ of its position in the spectrum. What has been described is evidently just a one-dimensional random walk along the resonance spectrum of one unit of spin angular momentum. The mean free path $\Lambda=1/\tau_s$ and the velocity $u=1/\tau_s^2$. We define a spectral spin density at frequency ω:

$$S^z(\omega)=Nf(\omega)\langle s^z\rangle_\omega, \qquad (3.1)$$

where $\langle s^z\rangle_\omega$ is the expectation value of the z component of an individual spin at ω. The transfer of spin excitation through the resonance spectrum may be represented by the diffusion equation:

$$\frac{\partial S^z(\omega)}{\partial t}=\frac{\partial}{\partial\omega}\left\{\frac{1}{\tau_s^3}\left[\frac{\partial S^z(\omega)}{\partial\omega}-\frac{S^z(\omega)}{S_s^z(\omega)}\frac{\partial S_s^z(\omega)}{\partial\omega}\right]\right\}, \qquad (3.2)$$

where

$$S_s^z(\omega)=\tfrac{1}{2}Nf(\omega)\tanh[\hbar(\omega+\omega_s^z)/2kT_s].$$

This equation automatically conserves angular momentum. Also, when $S^z(\omega)$ becomes proportional to $S_s^z(\omega)$ there is no further diffusion of spin excitation. The form of $S_s^z(\omega)$ is indicated by the general requirement that the equilibrium distribution be the most probable distribution under the existing constraints and that spin-spin interaction cannot displace the equilibrium distribution of spins in contact with a heat reservoir, no matter how weak the contact. T_s is called the effective spin temperature. The spin frequency ω_s^z is introduced as a second parameter to fix the final spin distribution. Conservation of both energy and angular momentum requires two adjustable parameters. That Eq. (3.2) is not strictly correct may be seen from the fact that it does not automatically conserve energy in

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[6] P. W. Anderson, Phys. Rev. **82**, 342 (1951).

the same way that it does angular momentum. The best that we can do in trying to describe the relaxation by a diffusion-type equation is to choose T_s and ω_s^z in such a way that energy is conserved between the initial and final states. However, if the initial state differs from the equilibrium state over only a small region of the spectrum, the diffusion equation approximately conserves energy for the initial spread of the excitation.

We next consider the behavior of the spin system in the presence of a small transverse radio-frequency magnetic field. The macroscopic spectral spin density $\mathbf{S}(\omega)$ will now depart from the instantaneous field direction. We write the rate of change of the spin as the sum of three terms:

$$\frac{d\mathbf{S}(\omega)}{dt}=\left[\frac{d\mathbf{S}(\omega)}{dt}\right]_{\text{fields}}+\left[\frac{d\mathbf{S}(\omega)}{dt}\right]_{\text{lattice}}+\left[\frac{d\mathbf{S}(\omega)}{dt}\right]_{\text{spins}}. \tag{3.3}$$

The first two terms are just those given by Bloch[1] except for the use of spin densities here:

$$\left[\frac{d\mathbf{S}(\omega)}{dt}\right]_{\text{fields}}=\gamma\mathbf{S}(\omega)\times\mathbf{H}, \tag{3.4}$$

where $H_x=2H_1\cos\omega_1 t$, $H_y=0$, and $H_z=\omega/\gamma$; and

$$\left[\frac{d\mathbf{S}(\omega)}{dt}\right]_{\text{lattice}}=-\frac{\mathbf{S}(\omega)-\mathbf{S}_l(\omega)}{\tau_l}, \tag{3.5}$$

where for equilibrium with the lattice the spins point along the z direction with a spectral density

$$S_l(\omega)=\tfrac{1}{2}Nf(\omega)\tanh(\hbar\omega/2kT_l).$$

In order to discuss the form of the spin-spin interaction in the presence of a radio-frequency field, it is useful to requantize the Hamiltonian along the effective field direction in the rotating frame. As long as the radio-frequency field is large compared with the magnitude of the dipolar fields, the secular part of the transformed Hamiltonian continues to have the form of Eq. (2.3).[3] The spin-spin interaction then continues to conserve angular momentum and we may retain Eq. (3.2) for the z component of the angular momentum. The only modification is that $S_s^z(\omega)$ is now a function of the time and must be evaluated from the instantaneous energy and z component of the total angular momentum. The equations for the transverse components of spin take the form:

$$\frac{\partial S^{x,y}(\omega)}{\partial t}=\frac{\partial}{\partial\omega}\left\{\frac{1}{\tau_s^3}\left[\frac{\partial S^{x,y}(\omega)}{\partial\omega}-\frac{S^{x,y}(\omega)}{S_s^{x,y}(\omega)}\frac{\partial S_s^{x,y}(\omega)}{\partial\omega}\right]\right\}, \tag{3.6}$$

where

$$S_s^{x,y}(\omega)=\tfrac{1}{2}Nf(\omega)\tanh(\hbar\omega_s^{x,y}/2kT_s).$$

Since all spins see the same transverse magnetic fields there is automatic conservation of energy for diffusion of the transverse spin components.

IV. APPLICATIONS

As applications of the modified Bloch equations, we consider three experimental situations, slow passage, rapid passage, and free relaxation. We first consider what happens when a portion of the spectrum is saturated by an intense radio-frequency field. If there were no spin-spin interaction, we would obtain as a steady state solution of Eq. (3.4):

$$S^z(\omega)=\frac{1+(\omega-\omega_1)^2\tau_l^2}{1+(\gamma H_1\tau_l)^2+(\omega-\omega_1)^2\tau_l^2}S_l(\omega); \tag{4.1}$$

and in a frame rotating at frequency $-\omega_1$:

$$S^{x'}(\omega)=\gamma H_1\tau_l\frac{(\omega-\omega_1)\tau_l}{1+(\omega-\omega_1)^2\tau_l^2}S^z(\omega), \tag{4.2}$$

$$S^{y'}(\omega)=\gamma H_1\tau_l\frac{1}{1+(\omega-\omega_1)^2\tau_l^2}S^z(\omega). \tag{4.3}$$

At frequencies far from ω_1, the transverse components of $\mathbf{S}(\omega)$ will be small. In looking at the variation in $S^z(\omega)$ well away from the radio-frequency, we can neglect the field term. Assuming that $S_l(\omega)$ and τ_s are slowly varying compared with $S_z(\omega)$, we have

$$\frac{dS^z(\omega)}{dt}\cong-\frac{S^z(\omega)-S_l(\omega)}{\tau_l}+\frac{1}{\tau_s^3}\frac{\partial^2S^z(\omega)}{\partial\omega^2}. \tag{4.4}$$

Under steady state conditions, we have

$$S_l(\omega)-S_z(\omega)\sim\exp[\pm(\tau_d/\tau_l)^{\frac{1}{2}}\omega/\gamma H_1], \tag{4.5}$$

where $\tau_d=(\gamma H_1\tau_s)^2\tau_s$ is the time required for the spin excitation to diffuse a distance γH_1. The approximations made in obtaining an exponential approach of $S^z(\omega)$ to $S_l(\omega)$ require that the distance along the spectrum to which energy can diffuse in a time τ_l be large compared with γH_1 but small compared with the width of the spectrum. Under steady state conditions, energy is transferred to the lattice at a rate

$$P_l=\int\hbar\omega\frac{S_l(\omega)-S^z(\omega)}{\tau_l}d\omega \sim 2g\beta\omega_1H_1S_l(\omega_1)/(\tau_l\tau_d)^{\frac{1}{2}}. \tag{4.6}$$

The power absorbed from the radio-frequency field may be written as

$$P_a=\tfrac{1}{2}\omega_1\chi''(\omega_1)(2H_1)^2.$$

Since these two powers must be equal,

$$\chi''(\omega_1) \sim g\beta S_l(\omega_1)/H_1(\tau_l\tau_d)^{\frac{1}{2}}. \tag{4.7}$$

The imaginary part of the susceptibility may be obtained alternatively from the expression

$$\chi''(\omega_1) = \frac{g\beta}{2H_1}\int S^{y'}(\omega)d\omega.$$

We obtain from Eqs. (4.2) and (4.3) the corresponding result in the absence of spin diffusion

$$\chi''(\omega_1) \sim \tfrac{1}{2}\pi g\beta S_l(\omega_1)/H_1\tau_l. \tag{4.8}$$

By comparison we see that when the diffusion time is short compared with τ_l, $\chi''(\omega_1)$ is increased by a factor $(\tau_l/\tau_d)^{\frac{1}{2}}$. We expect from this that saturation effects set in for $\gamma H_1 \sim 1/(\tau_l\tau_d)^{\frac{1}{2}}$ rather than for $\gamma H_1 \sim 1/\tau_l$ as in the absence of spin diffusion. These results are very similar to what one obtains from the Bloch equations with a transverse relaxation time τ_d. The present theory gives a dependence of τ_d on H_1 which is not present in the Bloch theory, however.

We now reduce the magnitude of the radio-frequency field out of the saturation range rapidly compared with the spin-lattice relaxation but slowly enough that the decrease is adiabatic. The relaxation will now proceed with essentially the time constant τ_l even though the diffusion time τ_d may be much shorter than τ_l. This may be seen from the fact that during the steady state period the spectrum is saturated over an interval equal to the distance spin excitation can diffuse in a time τ_l. When the radio-frequency field is reduced, the saturated region relaxes by two mechanisms, transfer of energy out to the unsaturated regions and transfer of energy to the lattice. But the time for spin excitation to diffuse out of the saturated region is just τ_l.

In order to observe the effect of spin diffusion on the time dependence of the response of the spin system, we consider rapid-passage conditions. If the frequency is swept through the resonance spectrum of a system of noninteracting spins so rapidly that it traverses γH_1 in a time short compared with τ_l, the effect of the passage is to reverse the sign of $S_z(\omega)$ through the spectrum. If we now introduce spin diffusion effects and assume that τ_d is short compared with τ_l, we see that the situation is modified. It is now possible for spins in advance of the radio-frequency which have not been inverted to exchange with spins behind which have already been inverted. This exchange acts like the partial saturation obtained at passage rates of the order of τ_l. In order to avoid saturation, passage must be rapid compared with both τ_l and τ_d. This result is somewhat different from that of the Bloch theory. The reason for this difference is that there is no place in the Bloch theory for a spin-spin interaction which affects the z spin components. Here all three spin components are affected symmetrically by the interaction.

We finally make an estimate of the strength of diffusion effects based on resonance studies of F centers in KCl.[2] Taking the resonance spectrum to be Gaussian, we obtain $f(\omega) = [(2\pi)^{\frac{1}{2}}\Delta\omega]^{-1} \sim 10^{-9}$ sec at the line center. For an F-center concentration of $10^{18}/\text{cm}^3$, $\Delta \sim 10^6$ sec^{-1}. Then $\tau_s = [\Delta^2 f(\omega)]^{-1} \sim 10^{-3}$ sec. In a microwave field $2H_1 \sim 0.02$ oe, $\tau_d = (\gamma H_1\tau_s)^2\tau_s \sim 30$ sec. Compared with a spin lattice time of 8×10^{-6} sec, it is clear that spin diffusion effects are unobservable at room temperature as long as the interactions are purely dipolar. The condition that diffusion effects be observable under saturation conditions is simply that $\tau_l > \tau_s$, as may be seen from the following argument. We want $\tau_d < \tau_l$ with $\gamma H_1 \sim (\tau_l\tau_d)^{-\frac{1}{2}}$. Then $1/\tau_l < \gamma H_1 < 1/\tau_d$. But from the expression for τ_d we obtain $\gamma H_1 < 1/\tau_s$ and this establishes the condition. At liquid helium temperatures the spin lattice time should be longer than at room temperature by at least two orders of magnitude. This brings the spin lattice time into the millisecond range. Diffusion effects arising from pure dipolar interaction should then be observable for F centers of moderate concentration.

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