Nuclear Levels in Dv¹⁶¹[†]

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A specimen of Gd¹⁶⁰ enriched from its normal 21.9% up to 95.4% was irradiated in the Argonne heavy water pile. The Gd¹⁶¹ formed by neutron capture decays by beta and gamma emission with a half-life of 3.7 min to radioactive Tb^{161} . Spectrometric studies of this activity show it to decay with a half-life of 7.15 days emitting both beta and gamma rays many of which previously have not been evaluated. There appear to be seven gamma rays in Dy, all internally converted, whose energies are: 25.6, 27.7, 48.9, 57.3, 74.8, 78.3, and 106.2 kev. These energies fit into a scheme with nuclear levels at 25.6, 74.8, 103.9, and 132.0 kev. Coincidence data with the scintillation spectrometer support the proposed plan. The beta spectrum, studied with the double focusing spectrometer, appears to be complex, having three components whose energies are 531, 447, and 405 kev, with relative intensities of 68%, 22%, and 10%, and log ft of 6.7, 6.9, and 7.2, respectively.

ADOLINIUM-161 formed by neutron capture in \mathbf{J} Gd¹⁶⁰ decays with a 3.7 minute half-life to terbium-161. This activity in turn emits beta and gamma radiation with a half-life found in the present investigation to be 7.15 days, terminating in dysprosium-161. A gamma transition of energy 49.0 kev had been reported¹ for this activity. An additional gamma ray of energy approximately 75 kev and a beta upper energy of 550 kev were observed² by the scintillation method. Coulomb excitation by alpha rays on unseparated dysprosium showed³ gamma rays with energies of 76 and 166 kev, the latter probably not being in Dy^{161} . A proportional counter study of low-energy photons indicated a 26-kev transition.⁴

With the stronger sources now available a reinvestigation of the activity seemed desirable. A specimen of separated Gd160, enriched up to 95.4 percent, was irradiated for a week in the maximum flux of the heavy water Argonne reactor. Beta energies were studied with the double focusing magnetic spectrometer, using a scotch tape backed source and 15 $\mu g/cm^2$ Zapon window. Gamma energies were evaluated from conversion electrons in magnetic spectrometers and coincidences observed with the scintillation spectrometer.

Some twenty-five electron conversion lines were observed and their energies measured as shown in column 1, Table I. The interpretation of these lines yields seven gamma energies, four of which had not been previously reported. The relative intensities of many of the lines were measured with the microphotometer and the relative values for each group shown in column 4. The clearly resolved 18.6-kev electron line was interpreted as an L_1 line for a gamma ray of 27.7 kev. Had this been a K line then an L_1 at 63.3 kev should have been expected, but it was not observed. In addition to the group of electron lines tabulated, all of which decayed

with a half-life of 7.15 days, certain other lines persisted with a longer half-life. These were recognized as due to gamma rays in europium, which must have been present as an impurity, even in the separated terbium.

The relative intensities of the three L lines of the 25.6 kev and the 48.9-kev gamma rays are shown in Table II. These are compared with the calculated Lshell coefficients of Rose *et al.*⁵ for a pure M1 transition at this energy and for a sufficient admixture of E2 to be in agreement with the observed data. The possibility that the 25.6-kev gamma is an electric dipole transition is not completely excluded. The L_2 line of the 57.3

TABLE I.	Conversion	electron	energies	$_{ m in}$	Dy^{161}	and	their	in-
terpretation.	. (Numbers	italicized	in colu	nn -	4 indi	cate	arbitr	ary
normalizatio	n for each g	roup of li	nes.)					

Electron energy, kev	Assignment	Energy sum, kev	Relative intensity	Gamma energy, kev
16.4 17.0 17.8 23.6	$egin{array}{c} L_1 \ L_2 \ L_3 \ M \end{array}$	25.5 25.6 25.6 25.6	10.0 7.5 6.3	
25.3	N	25.7		25.6
18.6	L_1	27.7		27.7
$\begin{array}{c} 39.9 \\ 40.3 \\ 41.0 \\ 46.9 \\ 48.6 \end{array}$	$egin{array}{ccc} L_1 \ L_2 \ L_3 \ M \ N \end{array}$.	49.0 48.9 48.8 48.9 49.0	10.0 1.6 0.7	48.9
48.1 48.6 49.4 55.6 57.1	$egin{array}{c} L_1 \ L_2 \ L_3 \ M \ N \end{array}$	57.2 57.2 57.2 57.6 57.5	10.0 2.5	57.3
20.8 65.8 66.9 73.0 74.5	$egin{array}{c} K \ L_1 \ L_3 \ M \ N \end{array}$	74.6 74.9 74.7 75.0 74.9	10.0 5.5 2.2	74.8
24.4 69.2	K L_1	78.2 78.3		78.0
52.4 97.3	$K L_1$	$106.2 \\ 106.4$		106.2

⁵ Rose, Goertzel, and Swift (unpublished).

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¹ Cork, LeBlanc, Nester, and Stumpf, Phys. Rev. 88, 685 (1952).
² R. Barloutaud and R. Bellini, Compt. rend. 241, 389 (1955).
³ N. Heydenberg and G. Temmer, Phys. Rev. 100, 150 (1955).
⁴ Scharff-Goldhaber, der Mateosian, McKeown, and Sunyar, Phys. Rev. 78, 325(4) (1950).

Phys. Rev. 78, 325(A) (1950).

TABLE II. Theoretical and observed relative L-shell intensities for the 25.6- and 48.9-kev gamma rays.

25.6-kev gamma		48.9-kev gamma		
	$\alpha L_1: \alpha L_2: \alpha L_3$		α_{L1} : α_{L2} : α_{L3}	
Pure $M1$ M1+E2(2%) Observed	10:0.9:0.16 10:6.0:8.5 10:7.5:6.3	Pure $M1$ $M1+E2(\frac{1}{2}\%)$ Observed	10:0.8:0.2 10:1.1:0.6 10:1.6:0.7	

gamma falls together with the N line of the 48.9-kev gamma ray so that its intensity and hence the multipolarity of the transition cannot be expressed with certainty, although the possibility of electric quadrupole is eliminated. The K/L ratio for the 74.8-kev gamma is compatible with an E2 transition. The multipolarities of the other weaker gamma rays were not determined.

A nuclear level scheme as shown in Fig. 1 satisfies both the observed energies and the relative intensities of the gamma rays. In this plan the 25.6-kev gamma ray should be the strongest. Actually its L lines photographically appear weaker than the L lines for the 48.9kev gamma, but when the proper correction for the variation of emulsion sensitivity with energy is made, the expected relationship appears reasonable. To check the proposed level scheme coincidence measurements were made with a scintillation spectrometer. The fine resolution obtainable with the magnetic spectrometers could of course not be obtained with the scintillation device. The singles gamma-ray spectrum is found to consist of three broad peaks at 25, 50, and 76 kev, each of which is believed to be composite. Coincidences were



FIG. 1. Proposed nuclear energy level scheme for Dy¹⁶¹. The heavy lines indicate the strongest transitions.

observed between (76, 50); (50, 25); and (50, 50) kev gammas. When lead shielding is used so as to minimize the effect of backscattered iodine x-rays (28 kev), there still appeared to be a (25, 25) kev coincidence. These data are in accord with the arrangement shown.

The Kurie plot of the beta spectrum was found to be complex. Since all gamma rays observed were of low energy, it was expected that the maximum energies of all beta components would lie rather close together. In order to avoid the large uncertainties resulting from



FIG. 2. Analysis of the Kurie plot of the beta spectrum of Tb¹⁶¹. The set of points A represents the composite spectrum. The lines B, C, and D indicate the separate Kurie plots of the three components whose energies are 531, 447, and 405 kev. The points adjacent to C and D indicate the result of previous subtractions.

determining each component by only a few points near its upper limit, the set of experimental points was fitted as the sum of a number of components, assumed to be of allowed form, an assumption subsequently justified by the log ft values. It was found that two components were not sufficient but the data above 200 kev could be fitted well by three components, as shown in Table III and Fig. 2. Deviations below 200 kev are probably due, at least in part, to backscattering in the source.

The spin of the ground level in Dy^{161} has been observed and reported to be probably 7/2 in one report⁶ and 5/2 in the other.⁷ Odd parity is predicted by the shell model. Similarly the ground level in Tb^{161} is predicted to have even parity but the spin may have values 3/2, 5/2, or 7/2. This isotope lies in a region where rotational bands are to be expected. However, the levels at 25.6 and 74.8 kev cannot be the first two excited levels of a rotational band with a ground state spin

⁶ K. Murakawa and T. Kamei, Phys. Rev. 92, 325 (1953).

⁷ A. H. Cooke and J. G. Park, Proc. Phys. Soc. (London) A435, 282 (1956).

greater than $\frac{1}{2}$. If the ground state spin is assumed to be $\frac{1}{2}$, then these two excited levels can be used to calculate a decoupling parameter a = -0.071, and the next rotational level is predicted at 134.5 kev. The vibration-rotation interaction correction, which is proportional to $I^2(I+1)^2$, for this 7/2 level amounts to about -2 kev, bringing the energy of the expected level into good agreement with the one observed at 132 kev. This apparent sequence of rotational levels seems to be fortuitous, however, since it is not supported by the

TABLE III. The resolution of the beta spectrum of Tb¹⁶¹.

Maximum energy, kev	Percent abundance	Log fl	ΔI , parity	
531 ± 10	68%	6.7	0 or 1, yes	
447 ± 10	22%	6.9	0 or 1, yes	
405 ± 10	10%	7.2	0 or 1, yes	

observed ground state spin, Coulomb excitation studies,³ or character of the beta transitions.

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Fluctuations of Nuclear Reaction Widths*

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The fluctuations of the neutron reduced widths from the resonance region of intermediate and heavy nuclei have been analyzed by a statistical procedure which is based on the method of maximum likelihood. It is found that a chi-squared distribution with one degree of freedom is quite consistent with the data while a chi-squared distribution with two degrees of freedom (an exponential distribution) is not. The former distribution corresponds to a Gaussian distribution of the reduced-width amplitude, and a plausibility argument is given for it which is based on the consideration of the matrix elements for neutron emission from the compound nucleus and of the central limit theorem of statistics. This argument also suggests that within the framework of the compound-nucleus theory all reduced-width amplitudes have Gaussian distributions, and that many of the distributions for the various channels may be independent. One consequence of the latter suggestion is that the total radiation width for a given spin state which is formed in neutron capture will be essentially constant, in agreement with some observations, because it is the sum of many partial radiation widths. The fluctuations of the provisional fission widths of U²³⁵ are best described by a chi-squared distribution with about $2\frac{1}{2}$ degrees of freedom, indicating that there are effectively only a few independently contributing fission channels.

I. GENERAL REMARKS

S EVERAL hundred resonances have been observed in the Brookhaven fast chopper work on total neutron cross sections of intermediate and heavy nuclei in the neutron energy range up to several hundred electron volts.¹ For many of these resonances it has been possible to deduce the neutron width Γ_n and the velocity-independent reduced width $\Gamma_n^0 = \Gamma_n/E_0^{\frac{1}{2}}$, where E_0 is the resonance energy.^{2,3} In a typical sample of from ten to fifteen resonances the reduced widths are observed to fluctuate violently, the ratio of the largest to the smallest being as high as several hundred. Indeed, Hughes and Harvey⁴ have recently shown that the aggregate of the reduced-width data for fourteen nuclides is reasonably consistent with exponential-like distributions, one of the form $x^{-\frac{1}{2}} \exp(-\frac{1}{2}x)$ and another of the form $\exp(-x)$, where $x = \Gamma_n^{0} / \langle \Gamma_n^{0} \rangle_{Av}$. In view of the importance to nuclear reaction theory and to nuclear engineering of knowing which of the two distributions is more likely to be the correct one, we have made a more quantitative statistical analysis of the data. This analysis shows that the former distribution is quite consistent with the data, whereas to the latter one it assigns a very small probability of being correct. The most significant consideration of our analysis which enables this distinction to be made is the accounting for the possibility that levels with small widths, of which there are predicted to be a relatively large number in the former distribution, will not be observed. A second consideration which also enhances this distinction is the accounting for the errors introduced when finitesample averages are used as estimates for the infinitesample (population) averages $\langle \Gamma_n^0 \rangle_{AV}$.⁵

^{*} Work performed under the auspices of the U. S. Atomic Energy Commission.

¹ Seidl, Hughes, Palevsky, Levin, Kato, and Sjöstrand, Phys. Rev. 95, 476 (1954).

² Harvey, Hughes, Carter, and Pilcher, Phys. Rev. 99, 10 (1955). ³ D. J. Hughes and J. A. Harvey, *Neutron Cross Sections*, Brookhaven National Laboratory Report BNL-325 (Superintendent of Documents, U. S. Government Printing Office, Washington, D. C., 1955).

⁴ D. J. Hughes and J. A. Harvey, Phys. Rev. 99, 1032 (1955).

⁵ The existence of a reasonably well-defined average neutron reduced width is assumed here. The existence of such an average is suggested by the work of Feshbach, Porter, and Weisskopf, Phys. Rev. 96, 448 (1954) and of Lane, Thomas, and Wigner, Phys. Rev. 98, 693 (1955).