Effect of Proton-Proton Scattering on an Initial Longitudinal Spin Polarization*

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The Wolfenstein A parameter for proton-proton scattering is measured for three angles at an incident laboratory energy of 316 Mev.

INTRODUCTION

THIS report is a description of the last in a series of measurements performed at Berkeley on the proton-proton system at 316 Mev. The object of this experiment is to obtain information concerning the scattered spin polarization when the initial polarization is along the direction of motion. Three scatterings are therefore required; the first and last act as polarizer and analyzer, by means of which the unknown parameters of the second scattering are to be determined. In this way we are able to measure another independent parameter of the proton-proton system, denoted by Wolfenstein[1] as $A(\theta)$.

To obtain a longitudinal component of polarization it is necessary to employ an auxiliary magnetic field, since the polarization produced at the first target is perpendicular to the plane of the scattering. This requirement may be compared to the conditions for the other triple-scattering experiments already performed at this laboratory; namely the measurements of the D and R parameters.[2] These are concerned with components of the final polarization when the initial polarization is perpendicular to the direction of motion.

THE POLARIZED BEAM

The polarized 316-Mev proton beam[3] was produced by a 13° left elastic scattering of the internal circulating beam of the 184-inch cyclotron from target No. 1, of beryllium. The polarization vector of the first scattered beam, $\langle\boldsymbol{\sigma}\rangle_1$, is directed upward, perpendicular to the plane of the scattering, and has the magnitude $P_1=0.69\pm0.05$. The beam is brought into the experimental area (cave) through a 2-inch-diameter collimator at a rate of 3×10^6 particles per second. Here it enters a horizontal magnetic field which deflects it upward by an angle $\Omega=28.4°$. Figure 1 outlines the experimental geometry in the cave. When the beam emerges from the magnet the polarization vector is no longer perpendicular to the direction of motion, but has been rotated backward through an angle χ, in the vertical plane. Consequently the beam has acquired a longitudinal component of polarization equal to $-P_1\sin\chi$, with respect to the direction of motion.

That the angle χ is different from zero after passage through the horizontal magnetic field is due to the anomalous part of the proton magnetic moment. The following formula, derived by Garren[4] and contained in the work of Mendlowitz and Case,[5] gives the relation between χ and Ω as defined in the previous paragraph:

$$\chi=\gamma(\mu_p-1)\Omega. \tag{1}$$

Here, γ represents the ratio of relativistic mass to rest mass and μ_p is the proton magnetic moment in units of the nuclear magneton. For 316±10-Mev protons and $\Omega=28.4°\pm0.25°$, χ has the value $66.5°\pm0.7°$.

SCATTERING GEOMETRY

The disposition of the second and third scattering planes is described below and depicted in Fig. 1. Proceeding upward from the magnet, the beam encounters the second target, composed of 1.13 g/cm² liquid hydrogen. We fix our attention on protons that are scattered at an angle Θ_2 in a plane π_2, which is perpendicular to the vertical plane passing through the

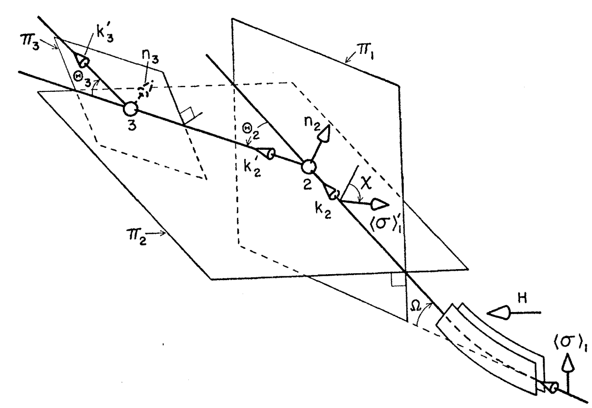

FIG. 1. Perspective drawing of the A experiment geometry. (Not to scale.) The circles labeled 2 and 3 represent the hydrogen target and the analyzing target respectively. The first scattering inside the cyclotron is not shown. The plane labeled π_1 is the vertical plane containing the deflected beam, while the planes π_2 and π_3 are, in order, the planes of second and third scattering. The planes π_1 and π_2 are perpendicular as are the planes π_2 and π_3. The vector $\mathbf{n}_2$ lies in the vertical plane. The vector $\mathbf{H}$ represents the horizontal magnetic field.

* This work was done under the auspices of the U. S. Atomic Energy Commission.
[1] L. Wolfenstein, Phys. Rev. **96**, 1654 (1954).
[2] Ypsilantis, Wiegand, Tripp, Segrè, and Chamberlain, Phys. Rev. **98**, 840 (1955).
[3] Chamberlain, Segrè, Tripp, Wiegand, and Ypsilantis, Phys. Rev. **102**, 1659 (1956).
[4] A. Garren, Atomic Energy Commission Report NYO-7102 (unpublished).
[5] H. Mendlowitz and K. M. Case, Phys. Rev. **97**, 33 (1955).

deflected beam. These scattered protons are allowed to scatter again at the third target, in a plane π_3 chosen perpendicular to π_2 and containing targets 2 and 3. The third or analyzing target is composed of beryllium and located 4 ft from the hydrogen target. By measurement of an asymmetry after the last scattering, the component of the scattered beam polarization at right angles to the plane π_3 is determined, and thereby the quantity A.

The relation between the measured asymmetry at the third target and the parameter A is obtained from Eq. (1.4b) of Wolfenstein[1]:

$$I_2\langle\boldsymbol{\sigma}\rangle_2\cdot\mathbf{s}_2=I_{20}[A\langle\boldsymbol{\sigma}\rangle_1'\cdot\mathbf{k}_2+R\langle\boldsymbol{\sigma}\rangle_1'\cdot(\mathbf{n}_2\times\mathbf{k}_2)]. \quad (2)$$

The subscript refers to the particular scattering, so that I_j and I_{j0} are the intensities out of the jth target for polarized and unpolarized incident beams respectively, and $\mathbf{k}_j$ and $\mathbf{k}_j'$ are the unit vectors in the incident and outgoing directions, respectively. A system of coordinates is defined for the scattered particles at any target by the unit vectors $\mathbf{n}_j$, $\mathbf{k}_j'$, $\mathbf{s}_j$, where $\mathbf{n}_j=\mathbf{k}_j\times\mathbf{k}_j'/|\mathbf{k}_j\times\mathbf{k}_j'|$, and $\mathbf{s}_j=\mathbf{n}_j\times\mathbf{k}_j'$. The symbol $\langle\boldsymbol{\sigma}\rangle_1'$ represents the polarization produced at target No. 1 but rotated in the horizontal magnetic field, whereas $\langle\boldsymbol{\sigma}\rangle_2$ is the polarization vector, of unknown direction and magnitude, after scattering at target No. 2. For the geometry chosen, $\langle\boldsymbol{\sigma}\rangle_1'$ is perpendicular to $\mathbf{n}_2\times\mathbf{k}_2$, which means that the effect of the parameter R does not appear. Since we wish to measure the component of $\langle\boldsymbol{\sigma}\rangle_2$ along the direction $\mathbf{s}_2$, the analyzing plane is chosen perpendicular to $\mathbf{s}_2$ and therefore perpendicular to the plane of second scattering. The scattered intensity out of the third target is[6]

$$I_3=I_{30}(1+\langle\boldsymbol{\sigma}\rangle_2\cdot\mathbf{n}_3P_3). \quad (3)$$

Let $I_3(\pm)$ denote the intensity when $\mathbf{n}_3$ is parallel to $\pm\mathbf{s}_2$; then the asymmetry at target No. 3 is defined to be

$$e_{3s}\equiv[I_3(+)-I_3(-)]/[I_3(+)+I_3(-)]=\langle\boldsymbol{\sigma}\rangle_2\cdot\mathbf{s}_2P_3. \quad (4)$$

If we insert $I_2=I_{20}(1+\langle\boldsymbol{\sigma}\rangle_1'\cdot\mathbf{n}_2P_2)$ in Eq. (2) and use the geometrical facts, $\langle\boldsymbol{\sigma}\rangle_1'\cdot\mathbf{k}_2=-P_1\sin\chi$ and $\langle\boldsymbol{\sigma}\rangle_1'\cdot\mathbf{n}_2=\pm P_1\cos\chi$, we obtain the following expression for e_{3s}:

$$e_{3s}=-P_1P_3A\sin\chi/(1\pm P_1P_2\cos\chi). \quad (5)$$

The $\pm$ refers to left or right[7] scatter from target No. 2.

The parameter A may be related to the coefficients of the proton-proton scattering matrix, as given in reference 1. The result[8] is

$$\begin{aligned}I_0A&=\mathrm{Im}C^*(B+G-N)\cos(\theta/2)\\&\quad-\tfrac{1}{2}\,\mathrm{Re}[(N-H)B^*+(G-N)^*(N+H)]\sin(\theta/)2, \quad (6)\end{aligned}$$

where θ is the c.m. scattering angle, and I_0 is the unpolarized differential cross section. In this expression, relativistic corrections are neglected.

It may be noted that another geometry is available to measure A besides the one employed here. It involves magnetic deflection in the manner described above, but the second scattering takes place in the vertical plane, while the analyzing plane is again perpendicular to the plane of second scattering. To eliminate the parameter R from the picture a magnet sufficiently powerful to rotate the spin through 90° must be used.

TABLE I. Relative values of background and effect.[a]

Lab angle Θ_2	R_B/R_H	R_A/R_H
11.8°	0.17	0.07
24°	0.06	0.025
36°	0.04	0.026

[a] R_B = blank counting rate, R_A = accidental counting rate, R_H = counting rate with the hydrogen target in place. The third scattering angle, Θ_3, has different values for each Θ_2. The corresponding values of θ_2 are listed in Table II.

EXPERIMENTAL DETAILS

An ionization chamber was placed before the entrance to the deflecting magnet for the purpose of monitoring the polarized beam. Four plastic scintillation counters connected in coincidence, together with their associated electronic circuits, were used to record the desired events. Two of these counters, called A and B, each having the dimensions 3 by 3 by $\frac{1}{4}$ inches, were placed between targets No. 2 and 3 to define the second scattered beam. Counter A was at the midway point, while B formed part of the third target. The last two counters, labeled 1 and 2, were placed after target No. 3. The distances of Counters 1 and 2 from target 3 were 25.5 and 33 inches; their dimensions were 2.5 by 8 by $\frac{3}{8}$ inches and 3 by 9 by $\frac{3}{8}$ inches, respectively. Between the last two counters was located a variable amount of copper absorber for the purpose of reducing inelastic scattering from the third target. When Counters A, B, 1, and 2 all fire within approximately 3×10^{-8} sec of each other, a triple-scattering event is said to have occurred. The four counters and target No. 3 were mounted on a supporting frame located behind the hydrogen target.

The following quantities must be known before A may be calculated from Eq. (5): e_{3s}, P_1P_3, P_1P_2, and χ. Determination of e_{3s}, which is the primary experimental quantity, involves measuring the coincidence counting rate between counters A, B, 1, and 2 with the hydrogen target in place, and then replacing the target by a blank and measuring the background counts arising from scattering at the vacuum-jacket windows. In addition, background arising from accidental coincidence events was measured. Table I gives the relative magnitude of the background effects.

The factor P_1P_3, which may be called the calibration asymmetry, was determined separately at the end of

[6] L. Wolfenstein and J. Ashkin, Phys. Rev. **85**, 947 (1952).

[7] The directions denoted by left or right are those that would be discerned by an observer moving with the beam incident on target 2 and oriented in the direction of the component of the spin normal to the trajectory.

[8] L. Wolfenstein, Seminar lectures, University of California, Spring semester 1955 (unpublished).

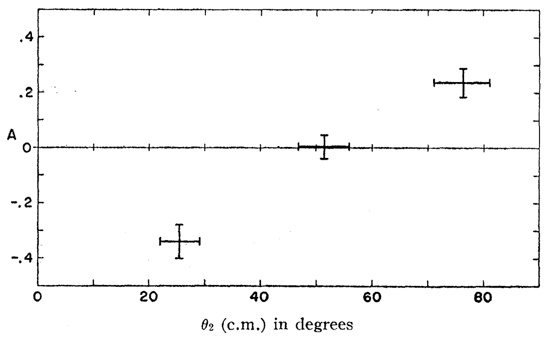

FIG. 2. Experimental values of the parameter A plotted against the center-of-mass scattering angle θ_2.

the run. The procedure used was as follows: The hydrogen target and deflecting magnet were removed from the cave. The supporting frame was then lowered so that counters A and B were in line with the undeflected polarized beam. The frame had also been rotated in such a manner that the analyzing plane was parallel to the plane of the cyclotron. The beam energy was degraded to a value corresponding to the energy of a proton after scattering from hydrogen at one of the angles used previously. Under these circumstances an asymmetry was measured at the beryllium target which is equal to P_1P_3 for the required experimental conditions.

P_1P_2 was calculated from previously known information on the beam polarization and from data for P_2,[9] the hydrogen polarization function.

Knowledge of the beam energy and the angle of deflection, Ω, enables one to compute χ. The angle Ω was determined by locating the deflected and undeflected beams by use of photographic films and measurement of the angle between them by use of transits. This method checked quite well an estimate for Ω obtained by constructing a current-carrying wire orbit through the magnet for the given value of the field.

RESULTS

The parameter A was determined for three values of the second scattering angle Θ_2(lab): 11.8°, 24°, and 36°. The last two points are averages for a left and a right scatter from hydrogen. This procedure provides a check on the reliability of the data; mechanical limitations, however, confined measurement of the 12° point to a left scatter only. Figure 2 shows the experimental values of the parameter A plotted against the c.m. angle θ_2, while Table II gives values for the pertinent experimental information.

[9] Chamberlain, Segrè, Tripp, Wiegand, and Ypsilantis, Phys. Rev. (to be published).

TABLE II. Experimental quantities for the A experiment.[a]

θ_2(c.m.)	25.4°±3.6°	51.36°±4.5°	76.26°±4.7°
Θ_3(lab)	13.85°	12.25°	19.9°
Target 3	2 in. Be	2 in. Be	1 in. Be
e_{3s} left	−0.155±0.028	−0.018±0.028	0.129±0.034
e_{3s} right	···	0.025±0.032	0.103±0.034
P_1P_3	0.543±0.021	0.515±0.022	0.537±0.027
P_2	0.335±0.025	0.317±0.025	0.142±0.025
A (average)	−0.339±0.064	0.007±0.045	0.236±0.050

[a] The quoted errors in e_{3s}, P_1P_3, and P_2 are expressed in terms of standard deviations. The spread in θ_2 is due mainly to geometrical resolution, with some contribution from multiple scatter in target No. 2. Values of P_2 are extracted from reference 9.

The uncertainty in A was calculated by combining the errors in the various measured quantities. The errors listed for e_{3s} and P_1P_3, although predominantly due to counting statistics, include a contribution from estimated false asymmetries induced by misalignment in the zero-setting of Θ_3. Misalignment contributes approximately 15% to the total error in A at the two smaller angles, and about 5% at the larger angle.

At the time of this experiment the first stage of a phase-shift analysis on the proton-proton system at 310 Mev had been completed.[10] This and subsequent work[11] gave six sets of phase shifts which fit the experimental results (not including the measurements reported here). Only one of these six solutions however was in reasonable agreement with the results of the present experiment. Continuation of the phase-shift analysis, with the A data included, gave the following results. Of the six solutions found in the earlier analysis only the one mentioned above remained essentially unaltered while four others changed materially so as to remain acceptable solutions. The sixth solution could be definitely excluded.

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[10] Henry Stapp, University of California Radiation Laboratory UCRL-3098, August 1955 (unpublished).

[11] For a complete discussion of the phase-shift analysis and definition of the terms used above, see Stapp, Ypsilantis, and Metropolis, Phys. Rev. (to be published).