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Elastic Scattering of Protons from Na^{23}[†]

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The elastic scattering of protons from Na^{23} has been obtained over the proton energy range from 0.58 to 1.50 Mev at the angles of 90° and 157.5°. Additional readings were made at angles of 125° and 140° at resonances. Thin metallic sodium targets on thin nylon backings were used. Out of 31 known resonances for Mg^{24} in this energy region, 12 unambiguous assignments of spin and parity were made. With the energy given in Mev, these are $E_p=0.594$ (3−); 0.725 (0−); 0.797 (1−); 0.815 (2+); 0.849 (4+); 0.877 (1+); 0.922 (2+); 1.022 (2−); 1.176 (1+); 1.258 (1+); 1.288 (1−); 1.398 (3+); and 1.460 (3−). Three tentative assignments were made for $E_p=1.206$ (2−); 1.213 (3+); and 1.365 (0−). The excitation curve for alpha-particle emission to the ground state of Ne^{20} was obtained simultaneously to give a differential cross section for this reaction. Resonance states were found which agreed in spin and parity as well as energy with those found before only in the elastic scattering of alpha particles from Ne^{20}.

INTRODUCTION

THE compound nucleus Mg^{24} has been the subject of many experiments. A discussion of much of the information known concerning it is given in a review article by Endt and Kluyver.[1] The present experiment covers resonances obtained by protons on sodium over the proton energy range from 580 to 1500 kev. Information regarding resonance parameters in this energy range has been found in various ways. A determination of resonance energies and total widths was made early by Burling[2] and more recently and accurately by Stelson and Preston.[3] Spin and parity assignments as well have been made for eleven states formed during elastic scattering of alpha particles from Ne^{20}.[4] Further assignments, mostly above this energy, have been made by Stelson,[5] Newton,[6] and Seed.[7]

The lack of agreement among the various assignments for given levels and the incompleteness of previous results prompted the present investigation. The first phase, a detailed investigation of the gamma rays arising from proton capture, has already been reported,[8] and is hereafter referred to as Part I.

The analysis of the elastically scattered protons from Na^{23} is a very general and powerful technique for obtaining many of the resonance parameters for the compound nucleus states of Mg^{24}. All combinations of spin and parity may be attained in contrast to the scattering of alpha particles from Ne^{20} which gives only the alternating series 0+, 1−, 2+, 3−, ⋯. Furthermore, all states formed by proton bombardment must decay to some extent by elastic scattering. Offsetting the advantages is the complexity of analysis necessarily accompanying the generality obtained from particles with nonzero spin.

EXPERIMENTAL PROCEDURE

This experiment was performed with the Kansas University Van de Graaff generator. Characteristics of its operation are given in Part I. The voltage was calibrated using a thick and a thin aluminum target on the well-known gamma resonance at a proton energy of 993.3 Mev.[9]

Target backings were made from nylon films formed by allowing a small drop of heated nylon to spread over a smooth surface of triply distilled water. Such films were suitably thin, approximately 500 A, were strong enough to withstand several hours of bombardment by the proton beam, took a uniform coating of the target material, and adhered readily to the nickel target frame. Metallic sodium was evaporated onto the nylon films inside the target chamber, Fig. 1. Uniformity of target thickness was maintained by timing the exposure of the film to the sodium vapor emanating from the furnace. An auxiliary oil-diffusion vacuum pump was used at the target chamber to improve deposition of target material as well as to minimize target deterioration after formation.

A zinc sulfide scintillation screen in conjunction with a DuMont 6291 photomultiplier tube was used as the particle detector. The screens were made by applying a thin uniform coat of clear Krylon acrylic spray to a disk of Lucite and blowing a cloud of zinc sulfide powder across the face while it was still wet. Screens made in this fashion were found to give a detection efficiency of 80 to 90% for protons elastically scattered at 1 Mev from a 2000-A nickel foil. Light arising from the fluorescence under proton bombardment of sodium hydroxide on the target made it necessary to put a 1000-A nickel

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[1] P. M. Endt and J. C. Kluyver, Revs. Modern Phys. **26**, 95 (1954).

[2] R. L. Burling, Phys. Rev. **60**, 340 (1941).

[3] P. H. Stelson and W. M. Preston, Phys. Rev. **95**, 974 (1954).

[4] Goldberg, Haeberli, Galonsky, and Douglas, Phys. Rev. **93**, 799 (1954).

[5] P. H. Stelson, Phys. Rev. **96**, 1584 (1954).

[6] J. O. Newton, Phys. Rev. **96**, 241 (1954).

[7] J. Seed, Phil. Mag. **44**, 921 (1953).

[8] Prosser, Baumann, Brice, Read, and Krone, Phys. Rev. **104**, 369 (1956), preceding paper.

[9] Herb, Snowden, and Sala, Phys. Rev. **75**, 246 (1949).

foil as a light shield in front of the screen. All measurements were made with this shield in place.

The incident protons were collimated to give a beam $\frac{3}{16}$ inch in diameter. The beam current was limited to 40 millimicroamperes, which gave a target life of from four to five hours. The current was measured by a separate collector cup and a current integrator of a conventional type.[10] The circuit was estimated to be accurate within two percent over the range used. The aperture to the counter was $\frac{1}{4}$ inch in diameter and four inches from the center of the target. Two identical counters were mounted on the lid of the target chamber at right angles to each other, thus allowing measurements to be taken at two angles simultaneously. The zinc sulfide screen clearly separated the alpha particles from the protons, despite the comparatively poor energy resolution of this type of scintillator. A typical differential bias curve showing the resolution of the counter is shown in Fig. 2 for resonance 18 (1.012 Mev) which decays by both ground-state and first excited-state alpha particles. The resolution was not, however, good enough to separate the elastically scattered protons from the inelastically scattered protons. The contribution of this group was estimated from known yields of the associated gamma rays reported in Part I to be negligible with respect to the elastic scattering. For each detector one integral discriminator was set to accept all pulses just greater than those from elastically scattered protons and another set to accept those just above the noise level. The first count was a measure of alpha particles, the difference a measure of the elastically scattered protons.

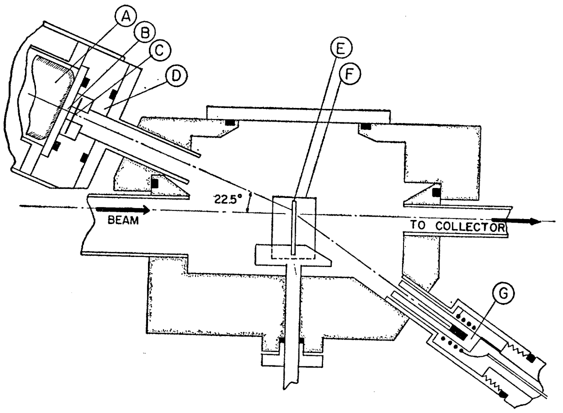

FIG. 1. Full scale sectional view of the target chamber and counter assembly. (A) Photomultiplier tube; (B) Lucite light pipe on the front of which a zinc-sulfide screen has been deposited; (C) Light shield for photomultiplier tube; (D) Lucite insulator, allowing electrical isolation of the target chamber from ground; (E) Target; (F) Shield that could be placed to intercept the sodium from the sodium furnace (G) during the initial phase of target preparation. The target chamber was actually equipped with two counter assemblies, at right angles to each other, to allow simultaneous measurements at two angular positions. The entire counter assembly which is fastened to the movable lid could be rotated under vacuum.

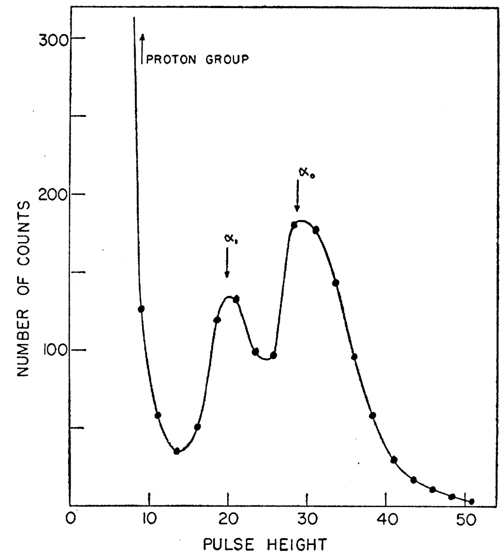

FIG. 2. Pulse-height distribution of all the protons and alpha particles emitted at proton bombarding energy $E_p = 1.012$ Mev (resonance 18).

Although a proportional counter is capable of better resolution of the different particle groups, the scintillator was selected for its much better resolving time. The energy resolution was sufficiently good to separate the two groups of interest, the elastically scattered protons and the ground-state alpha particles. Zinc sulfide was used in preference to other scintillators because of its extremely low sensitivity to the many resonance and background gamma rays.

The count from protons elastically scattered from elements other than sodium was subtracted from the observed readings. The contribution of each individual target backing to the observed scattering was determined by scattering protons from the bare target before the sodium was deposited. The amount of this scattering from the backing was between 10 and 40% of the scattering from sodium for the targets used. The effect of target aging, notably from carbon buildup and oxidation, was determined by returning at frequent intervals to the initial proton energy and noting the increase in count rate. The elastic scattering cross-section curve[11] for protons on C^{12} was invaluable for making this correction for carbon. Proton scattering from O^{16} is nonresonant in the energy range used here, and hydrogen does not scatter at laboratory angles greater than 90°. The subtraction process was thus considered quite reliable.

The differential scattering cross section was obtained by normalizing the observed scattering against the known Coulomb scattering for sodium at energies well away from any resonance. The procedure should be

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[10] H. T. Gittings, Rev. Sci. Instr. **20**, 325 (1949).

[11] Jackson, Galonsky, Eppling, Hill, Goldberg, and Cameron, Phys. Rev. **89**, 365 (1953).

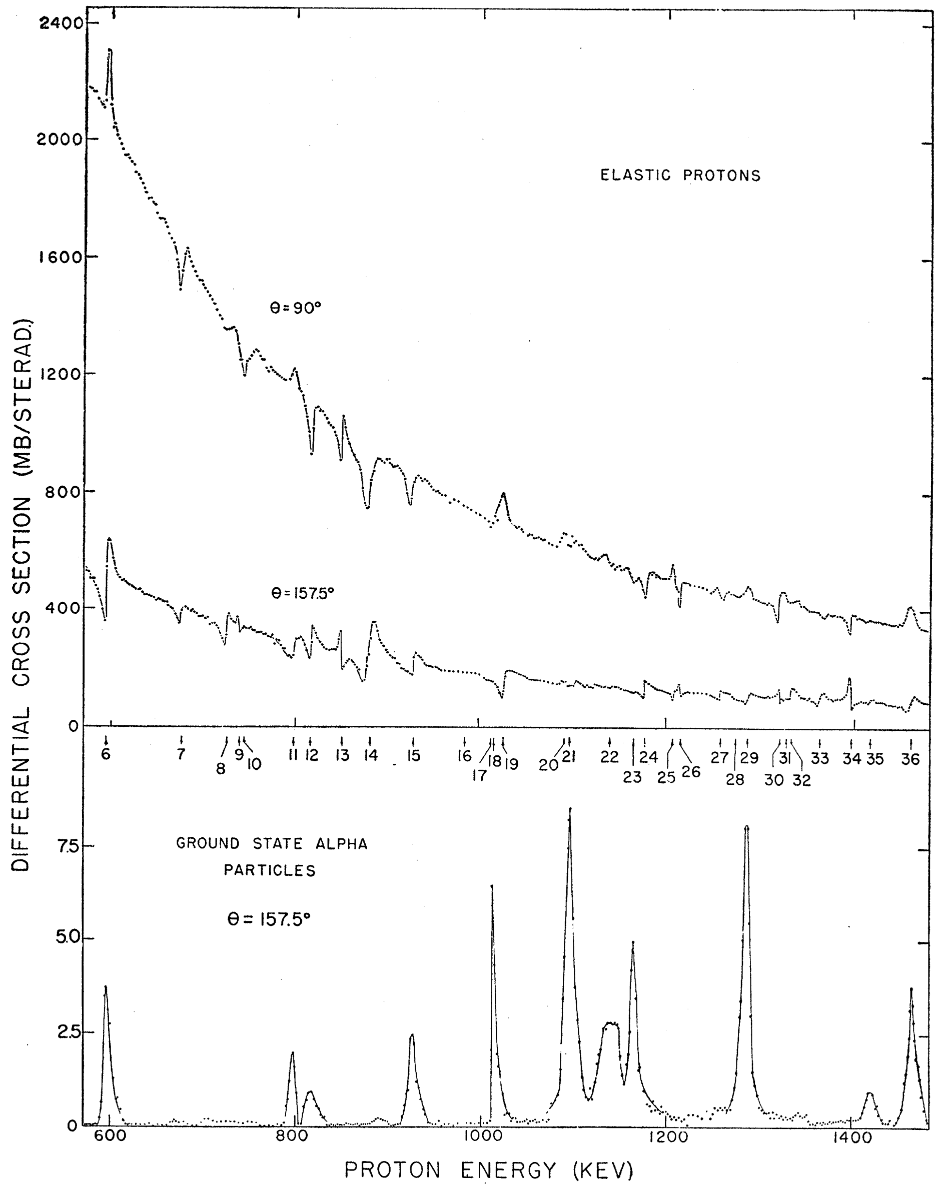

FIG. 3. Differential cross section for $Na^{23}(p,p)Na^{23}$ as a function of proton bombarding energy at angles of 90° and 157.5° in the center-of-mass system and the excitation curve for the ground-state alpha particles at the 157.5° setting.

valid because the "hard sphere" contribution to scattering is negligible for sodium at these energies.

EXPERIMENTAL RESULTS

The excitation curve for elastic scattering at the center of mass angles of 90° and 157.5° and for formation of ground-state alpha particles at 157.5° is shown in Fig. 3. The resonance numbers indicated include all known resonances in the same energy region. A summary of some of the salient features of these resonances and those at lower energies is given in Table I. It is of interest to note that resonances 11, 12, 13, and 15, for which there are no detectable gamma rays, match in energy and resonance parameters those resonances[4] found in the elastic scattering of alpha particles from Ne^{20}. This explains the difficulty in trying to match the $Ne^{20}+\alpha_0$ resonances with $Na^{23}(p,\gamma)$ resonances.

The yield curve of ground-state alpha particles is included for the sake of completeness. The $Na^{23}(p,\alpha_0)Mg^{24}$ differential cross section was obtained by comparing the yield of alpha particles to that of the known Coulomb scattering of protons and making the necessary center-of-mass solid angle corrections. In the region of overlap above 1 Mev the curve agrees with that previously reported by ionization chamber measurements.[3] The alpha-particle resonances below 1 Mev have also been previously reported[12] except that resonances 11 and 12 are not resolved. For those

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[12] J. M. Freeman and A. S. Baxter, Nature **162**, 696 (1948).

resonances which show strong decay by elastic scattering, the scattering was observed at the additional laboratory angles of 125° and 140°. The results for typical resonances together with the theoretical fit at 90° and 157.5° are shown in Figs. 4 through 7. Theoretical curves were also plotted for 125° and 140°, though they are not included here. A good fit at all four angles was required for any assignment to be considered certain.

ANALYSIS

The formulation of Blatt and Biedenharn[13] was followed in analyzing the individual resonances. The general formula for elastic scattering is given by Eq. (7.12) of their article. This expression can be greatly simplified for the present application. From tables of Coulomb functions of light elements[14] the phase shift ϕ_l from the finite size of the nucleus was found to be less than 1° for proton energies below 1.5 Mev; hence the correction terms for the finite size of the nucleus could be neglected. The terms which contribute significantly to the elastic scattering are pure Rutherford scattering, pure resonance scattering, and an interference term between the two. In general the latter two terms involve sums over the various channel spins and orbital angular momenta allowed for formation of the state. The parameters for describing a particular resonance are the energy E_0, the spin J, and the parity of the compound nucleus, the total resonance width Γ, and the elastic proton partial widths Γ_{sl} for each allowed channel spin and orbital angular momentum. The Γ_{sl} decrease rapidly with l; hence usually only the lowest value of l need be considered since the next value of l allowed is two units higher. Frequently only one value of channel spin is commensurate with the lowest value of l and the summations are greatly simplified.

Where available, the values of the total width Γ for fitting the elastic scattering data were taken from the more accurate determinations from gamma-ray excitation data.[3] For the remaining resonances the total width was chosen to best fit the scattering data. The partial widths Γ_{sl} were made consistent with other experiments. Where such information was unknown, partial widths for the minimum value of orbital angular momentum only were considered and the ratio Γ_1/Γ_2, where the subscripts denote channel spins, was taken as an arbitrary parameter.

A simplification of the expression for differential scattering cross section is made by dividing out the Coulomb scattering and thus obtaining the dimensionless variables $(E-E_0)/\Gamma$ and the ratios Γ_{sl}/Γ. This procedure greatly facilitates fitting the experimental points to the theoretically predicted curves.

Two of the angles selected for the counters were those allowing the fullest use of the scattering data. The angle of 157.5° was selected because it was the largest back angle attainable with the target chamber and because none of the low-order Legendre polynomials up to and including the fifth order is near a zero. The angle 90° was chosen since all odd Legendre polynomials vanish at 90°. An inspection of the expression for elastic scattering shows that for protons on Na^{23} this imposes a severe restriction on the shape of the resonance at this angle. This arises because the only term which is not symmetrical about E_0 is the interference term. For this term the orders of the Legendre polynomials are specified by the values of the orbital angular momentum, which in the case of protons on sodium, are either all even (if the parity of the compound state is positive) or all odd (if the parity is negative). Hence the differential scattering cross section for all odd-parity states must be symmetrical about E_0 at 90°. Even-parity states will show this symmetry only for cases of accidental symmetry in the interference term. The resonance parity may thus be fixed with a fair degree of certainty for those resonances having a well-defined shape at 90°.

DISCUSSION

Resonance 6

This resonance agrees in energy with a resonance reported in the elastic scattering of alpha particles from neon for which an assignment of 3− was made. The proton scattering at 90° indicates negative parity. The presence of ground-state alpha particles makes a further restriction to the states 1−, 3−, 5−, $\cdots$. The only fit possible for $l<4$ was with a 3− assignment. This fit is very good at all four angles with $\Gamma = 2$ kev and $\Gamma_p/\Gamma = 0.9$. The reaction goes almost entirely by $l=1$ and $s=2$.

Resonance 8

The choice of 0− for this state agrees well with the observed decays and gives an excellent fit of the scattering data, especially at 90° where there is almost no deviation from Coulomb scattering.

Resonance 11

This resonance, which decays in part by ground-state alpha particles, agrees in energy with a 1− state observed in the scattering of alpha particles from neon. The 1− assignment gives an excellent agreement with the proton scattering under the assumption that the resonance decays primarily by elastically scattered protons, and that decay goes equally by both channel spins.

Resonance 12

Decay by ground-state alpha particles is observed. The state agrees in energy with a 2+ state for the

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[13] J. M. Blatt and L. C. Biedenharn, Revs. Modern Phys. **24**, 258 (1952).

[14] Bloch, Hull, Broyles, Bouricius, Freeman, and Breit, Revs. Modern Phys. **23**, 147 (1951).

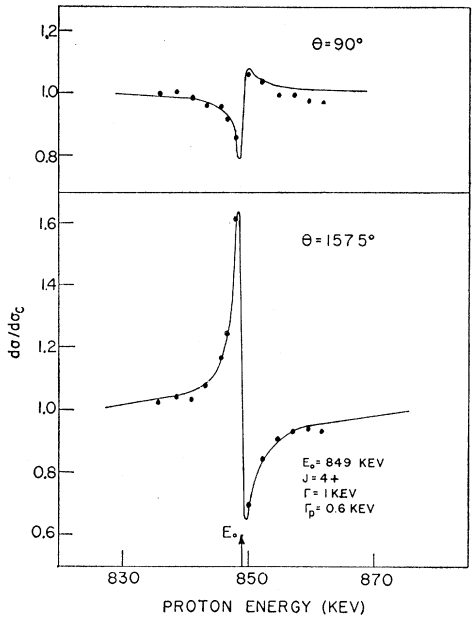

FIG. 4. Ratio of the observed cross section to Rutherford cross section as a function of proton energy for resonance 13. The curves represent theoretical fits for the parameters indicated.

scattering of alpha particles from neon. This same assignment is in good agreement with the proton scattering data for a ratio $\Gamma_p/\Gamma=0.6$.

Resonance 13

This resonance coincides energetically with a 4+ state reported from alpha-particle scattering.[4] The nonappearance of long-range alpha particles in the present experiment arises from the high angular momentum barrier compared to those of the observed reactions. The asymmetric curve at 90° requires even parity. The best fit, as shown in Fig. 4, is obtained with a 4+ assignment. A 3+ assignment can also be made to give a fair fit with proper choice of Γ_1/Γ_2, but there is little doubt that 4+ is the correct assignment in view of the alpha-particle scattering results.

Resonance 14

Proton capture radiation is the only reaction observed other than elastic scattering. The absence of ground-state alpha particles limits the choices to 0−, 1+, 2−, 3+, ···. Of these the only one giving a good fit is 1+. A 1+ state is reached with *s*-wave protons, which accounts for the strength and width of the scattering resonance. The theoretical fit of the data as shown in Fig. 5 is very good for this resonance. The assignment also agrees with the angular distribution of the capture radiation as reported in Part I.

Resonance 15

An assignment of 2+ was made for this state by alpha-particle scattering from neon.[4] As a preliminary check of the coincidence of the resonances as seen by the two different experiments, the angular distribution of the ground-state alpha particles was run. A distribution isotropic to within 2% together with the formation of ground-state alpha particles implied a 0+ or 2+ state. A reasonable fit of the proton scattering data was obtained only with a 2+ assignment.

Resonance 19

The angular distribution of the capture radiation at this resonance allows a 1+, 2− or 3+ state, as reported in Part I. Of these, only the 2− assignment fits the scattering data. The fit is quite good if the assumption is made that the state decays primarily by elastic scattering. The two very narrow unresolved resonances 17 and 18 cause a scattering of the data points on the low-energy side of resonance 19.

Resonances 23 and 24

The angular distribution of the ground-state alpha particles has been found to be isotropic[5] for resonance 23. As in the case of resonance 15, a 0+ or 2+ state is implied. The proton scattering data, though not sufficiently good to make assignment certain, are consistent with a 2+ assignment. Resonance 24, which has no ground-state alpha particles, is well fitted as a 1+ state.

Resonances 25 and 26

Because these two resonances are barely resolved, assignment is difficult. For resonance 25, the lack of

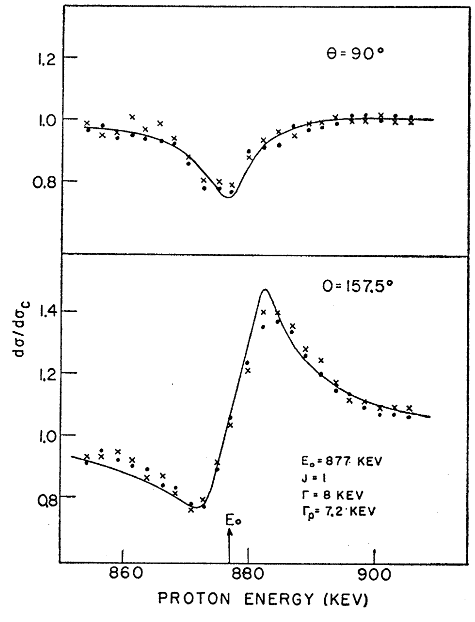

FIG. 5. Ratio of observed to Rutherford cross section as a function of proton energy for resonance 15. The dots and crosses represent different data runs. The theoretical fits for the parameters indicated are given by the curves.

ground-state alpha particles and the general shape of the elastic scattering curve make an assignment of 2− most likely. Resonance 26 is fitted by a 3+ assignment by the same argument.

Resonance 27

The resonance decays strongly by short-range alpha particles with no long-range ones. It was selected as the initial state in the determination of the spin and parity of the first excited state of Ne^{20} by means of the angular correlation between the short-range alpha particles and the resultant gamma rays.[7] On the basis of this work an assignment of 2+ was made for the excited state in Ne^{20} and 1+ for the compound state in Mg^{24}. As seen in Fig. 6, the fits at 157.5° for the two most likely assignments, 1+ and 2−, are about equally good, or bad. However, the behavior at 90° shows a marked preference for the 1+ assignment in agreement with the results reported by Seed.[7]

Resonance 29

This resonance decays measurably by all five possible modes, and very strongly by the p_1 channel. A 1− assignment is consistent with these decays and gives a good fit for the proton scattering data. This fit is shown in Fig. 7 with the assumption of a mixing of channel spins $\Gamma_1/\Gamma_2=2$. These values were used for determining the spin and parity of the first excited state of Na^{23} from the angular distribution of the gamma rays accompanying the inelastic protons.[15]

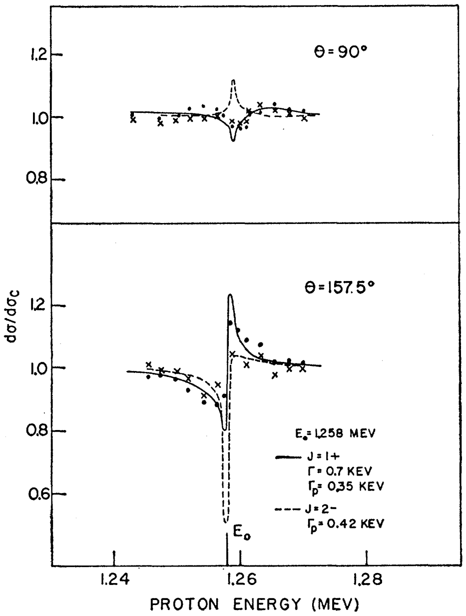

FIG. 6. Ratio of observed to Rutherford cross section as a function of bombarding proton energy for resonance 27. Theoretical fits for $J=1+$ and $J=2-$ are given by the solid and dashed curves, respectively. The dots and crosses indicate different data runs.

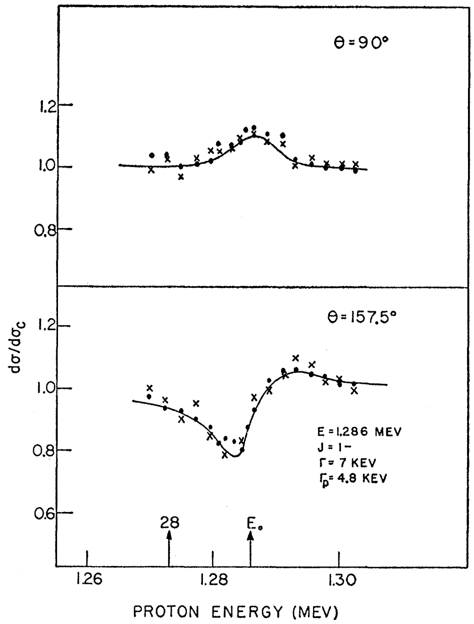

FIG. 7. Ratio of observed to Rutherford cross sections for resonance 29. Different data runs are indicated by the dots and crosses. The curves represent theoretical fits for the parameters indicated.

Resonance 33

Resonance 33 does not decay by alpha-particle emission, implying one of the series 0−, 1+, 2−, ⋯. The absence of short-range alpha particles makes the state 0− the most likely. The data at 90° are not good enough to determine parity in this case. A fit may be obtained by either 0− or 1+ of which the former is favored.

Resonance 34

No ground-state alpha particles are seen at this resonance, which favors the series 0−, 1+, 2−, 3+, ⋯. The asymmetry in the scattering at 90° indicates positive parity. The only assignment which meets these requirements and gives a possible fit of the data is 3+. An excellent fit is obtained with the assumption of equal contributions from the two channel spins.

Resonance 36

Resonance 36 decays partly by ground-state alpha particles requiring the series 0+, 1−, 2+, ⋯, and very strongly by inelastically scattered protons. A 3− assignment fits the data very well, if the assumption is made that the reaction goes almost entirely by elastic scattering. This should be valid since elastic

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[15] R. W. Krone and W. G. Read, Bull. Am. Phys. Soc. Ser. II, **1**, 212 (1956).

TABLE I. The 36 low-energy resonances in Mg^{24} formed by the $Na^{23}+p$ reaction are listed. Proton energies are in the laboratory system. Energies for the compound nucleus $E_{\mathrm{C.N.}}$, are based on a Q value of 11.703 Mev. For resonances 11, 12, 13, and 15 the total width Γ is that given by Goldberg *et al.*; for resonances 18 through 36 the values are those reported by Stelson. The ratio of the elastic proton partial width to the total width Γ is that which best fits the scattering data and includes both channel spins. For the observed modes of decay p_0 refers to elastic scattering, p_1 to inelastic protons, α_0 to ground-state alpha particles, α_1 to short-range alpha particles, and γ to proton capture. Boldface type indicates the most prominent modes of decay at a particular resonance. The present spin and parity assignments are either from the elastic scattering (p_0) or from angular distribution of capture gamma rays (γ) reported in Part I.

No.	E_p (Mev)	$E_{\mathrm{C.N.}}$ (Mev)	Γ (kev)	Observed decays	Γ_p/Γ	Present assignments	Previous assignments
1	0.255	11.947		$\boldsymbol{\gamma}$			
2	0.305	11.995	~0.5	$\boldsymbol{\gamma}$			
3	0.375	12.062		$\boldsymbol{\gamma}$			
4	0.445	12.129		$\boldsymbol{\gamma}$			
5	0.515	12.197		$\boldsymbol{\gamma}$			
6	0.594	12.273	2 ± 1	$\mathbf{p}_0$ $\boldsymbol{\alpha}_0$ $\boldsymbol{\gamma}$	0.9 ± 0.1	$3-$ (p_0)	$3-$[a]
7	0.675	12.349	$\leqslant1$	p_0 $\boldsymbol{\gamma}$			
8	0.725	12.398	7 ± 2	$\mathbf{p}_0$	>0.9	$0-$ (p_0)	
9	0.740	12.412	<3	p_0 $\boldsymbol{\gamma}$			
10	0.744	12.416	<3	$\boldsymbol{\gamma}$			
11	0.797	12.467	5 ± 2	$\mathbf{p}_0$ $\boldsymbol{\alpha}_0$	0.8 ± 0.1	$1-$ (p_0)	$1-$[a]
12	0.815	12.484	4 ± 1	$\mathbf{p}_0$ $\boldsymbol{\alpha}_0$	0.6 ± 0.2	$2+$ (p_0)	$2+$[a]
13	0.849	12.517	$\leqslant1$	$\mathbf{p}_0$	>0.6	$4+$ (p_0)	$4+$[a]
14	0.877	12.543	8 ± 2	$\mathbf{p}_0$ $\boldsymbol{\gamma}$	$\geqslant0.9$	$1+$ (p_0) (γ)	
15	0.922	12.587	4 ± 1	$\mathbf{p}_0$ $\boldsymbol{\alpha}_0$	0.6 ± 0.2	$2+$ (p_0) (α_0)	$2+$[a]
16	0.989	12.651	<1	$\boldsymbol{\gamma}$		$4\pm$ (γ)	
17	1.011	12.672	$\leqslant0.5$	p_1 γ			
18	1.012	12.673	0.8 ± 0.1	$\mathbf{p}_1$ $\boldsymbol{\alpha}_0$ $\boldsymbol{\alpha}_1$			$3-$,[b] $2+$[c]
19	1.022	12.682	6.6 ± 0.5	$\mathbf{p}_0$ p_1 α_1 $\boldsymbol{\gamma}$	0.9	$2-$ (p_0) (γ)	
20	1.087	12.745	1.1 ± 0.5	p_1 $\boldsymbol{\gamma}$			
21	1.094	12.751	7.9 ± 0.5	$\mathbf{p}_1$ $\boldsymbol{\alpha}_0$ α_1			$2+$[c]
22	1.137	12.793	30 ± 5	$\boldsymbol{\alpha}_0$			$0+$[b]
23	1.166	12.820	1.2 ± 0.1	p_1 $\boldsymbol{\alpha}_0$ $\boldsymbol{\alpha}_1$ γ		$2+$? (p_0)	$2+$,[b] $1-$[c]
24	1.176	12.830	2.5 ± 0.5	$\mathbf{p}_0$ p_1 α_1 $\boldsymbol{\gamma}$	0.7 ± 0.2	$1+$ (p_0)	
25	1.206	12.859	0.3 ± 0.1	$\mathbf{p}_0$ p_1 α_1		$2-$? (p_0)	
26	1.213	12.865	0.4 ± 0.1	$\mathbf{p}_0$ $\mathbf{p}_1$ $\boldsymbol{\alpha}_1$ γ	0.7 ± 0.2	$3+$? (p_0)	
27	1.258	12.909	0.6 ± 0.1	$\mathbf{p}_0$ $\mathbf{p}_1$ $\boldsymbol{\alpha}_1$	0.5 ± 0.2	$1+$ (p_0)	$1+$[d]
28	1.273	12.923		γ			
29	1.288	12.937	7.1 ± 0.2	$\mathbf{p}_0$ $\mathbf{p}_1$ $\boldsymbol{\alpha}_0$ α_1	>0.9	$1-$ (p_0)	$1-$[b]
30	1.321	12.969	2.1 ± 0.5	p_0 p_1 α_1 $\boldsymbol{\gamma}$		$3+$ (γ)	
31	1.329	12.978	3.5 ± 0.2	p_0 $\mathbf{p}_1$ $\boldsymbol{\alpha}_1$			
32	1.334	12.982	6.7 ± 0.2	p_0 $\mathbf{p}_1$			
33	1.365	13.012	1.2 ± 0.2	$\mathbf{p}_0$ $\mathbf{p}_1$		$0-$? (p_0)	
34	1.398	13.044	0.5 ± 0.1	$\mathbf{p}_0$ $\mathbf{p}_1$ α_1 $\boldsymbol{\gamma}$	0.7 ± 0.1	$3+$ (p_0)	$3+$[c]
35	1.419	13.064	$\leqslant0.3$	p_1 α_0 α_1 $\boldsymbol{\gamma}$		$4+$ (γ)	
36	1.460	13.103	9.8 ± 0.2	$\mathbf{p}_0$ $\mathbf{p}_1$ $\boldsymbol{\alpha}_0$ α_1	0.9 ± 0.1	$3-$ (p_0) (α_0)	$2-$[c]

[a] See reference 4.
[b] See reference 5.
[c] See reference 6.
[d] See reference 7.

scattering to the $3/2+$ state of Na^{23} and inelastic scattering to the $5/2+$ state of Na^{23} would then both require the same minimum orbital angular momentum, and elastic scattering would be strongly favored energetically. The assignment of $2-$ for this state by Newton[6] brings up the possibility that there may be two close-lying resonances at this energy. To check this possibility a fit for $2-$ was attempted, but no adjustment of parameters for a $2-$ state could be made to fit the scattering data at 157.5°. There seems little doubt of the $3-$ value.

The summary of the present work is shown in Table I, which includes the results of Part I. Included also are the results of some of the other experiments dealing with the resonances in the same energy interval. From the present work no assignments could be given for the very narrow resonances, for very close-lying resonances such as 30, 31, and 32, or for those which decay predominantly by some other channel, such as resonance 22 which decays almost entirely by ground-state alpha particles. The analysis of resonance 7 was not considered feasible because of the proximity (6 kev) of a strong and broad fluorine resonance.[16] Even a very small contamination of the target (known to be always present) would have altered the yield at this proton energy appreciably, thereby invalidating the results obtained. However, for those cases for which elastic scattering analysis could be made, the assignments made seem certain.

It is of interest to compare the results of the elastic scattering of alpha particles from Ne^{20} with the present work. Li's[17] adjusted Q value of 11.703 Mev for the $Na^{23}(p,\gamma)Mg^{24}$ reaction was used and is the basis for the energies in Mg^{24} given in Table I. If one also takes

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[16] Webb, Hagedozn, Fowler, and Lauritsen, Phys. Rev. **99**, 138 (1955).

[17] C. W. Li, Phys. Rev. **88**, 1038 (1952); Li, Whaling, Fowler, and Lauritsen, Phys. Rev. **83**, 512 (1951).

Li's value of 9.332 Mev for the energy difference of $Ne^{20}+\alpha-Mg^{24}$ a systematic error is observed between the two sets of resonances. The energies as obtained by the $Ne^{20}+\alpha$ reaction are consistently 14 kev higher. If this 14-kev correction is made, resonances 6, 11, 12, 13, and 15 coincide perfectly between the two experiments, both in energy and in spin and parity. Only one resonance in the region of overlap has not yet been obtained by the $Na^{23}+p$ reaction. This is a 2+ resonance which should fall at a proton energy of 280 kev. It seems likely that when elastic scattering of protons is extended to this region that it too will be found.

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Approximation for Deuteron Stripping Reactions on Heavy Target Nuclei*

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The (d,p) stripping reaction for $l_n=0$ is discussed for the case where Coulomb effects predominate, employing a zero-range neutron-proton interaction and neglecting the finite nuclear size. The angular distribution of the emergent proton is shown to change drastically from forward to backward peaking as the Sommerfeld number η increases. The case $l_n\neq 0$ is discussed.

INTRODUCTION AND SUMMARY

THE characteristic feature of the deuteron stripping process, involving discrete levels for the residual nuclei, is an angular distribution that shows pronounced forward-to-backward asymmetry, with associated maxima and minima. The Butler discussion of the stripping process,[1] and the Born approximation treatment[2] as well, give an adequate explanation of this phenomenon, subject to certain approximations. Among these approximations is the neglect of the effect of Coulomb forces on the incident deuteron and the emergent proton for (d,p) reactions. There have been subsequent treatments which have taken the Coulomb forces into account. The most comprehensive has been that of Tobocman and Kalos.[3] These authors took into account not only Coulomb effects but also nuclear effects on the incident and emergent particles. Such a treatment necessitates a partial wave expansion and rather extensive calculations tailored to each particular reaction under consideration, but, compensating for this difficulty, the results agree much better with the experiments considered. Somewhat earlier, Butler and Austern[4] had discussed the Coulomb effects by means of a numerical example of $l=2(d,p)$ stripping on $Z=15$. Qualitatively, it is clear that when the Coulomb forces can be considered small, the effect should be primarily a smearing out of the otherwise well-defined incident and emergent momenta. Principally, then, one would expect a smoothing out of the distribution and a filling in of the minima, just as observed.

It is the purpose of this note to consider the opposite limiting case,[5] namely the situation where the Coulomb effects dominate, that is, for large values of the parameters η_d and η_p. This case shows a great many simplifications over the usual situation. Because of the large Coulomb repulsion, nuclear effects on the incident and emergent particles are minimized. As a result, the partial wave expansion, which the nuclear effects would require, can be avoided. Moreover, the nuclear radius which enters as a parameter in the usual theory, is seen by the same argument to be of slight concern.

Even this case is, of course, intractable without further assumptions. We shall assume that the n-p interaction for the deuteron has zero range. For the usual stripping development, this assumption is of minor effect. Furthermore, we shall employ only the first Born approximation, neglecting the interaction interior to the nucleus. This, as discussed in many papers, is equivalent to the (perhaps more convincing) Butler approach. Finally we shall, for reasons that will

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[1] S. T. Butler, Phys. Rev. **80**, 1095 (1950); Proc. Roy. Soc. (London) **A208** (1951); Phys. Rev. **88**, 685 (1952). A more complete list of references will be found in the reviews by R. Huby, Progr. Nuclear Phys. **3**, 177 (1953) and by W. Tobocman, Naval Research Laboratory Report (unpublished).

[2] Bhatia, Huang, Huby, and Newns, Phil. Mag. **43**, 485 (1952); R. Huby, Proc. Roy. Soc. (London) **A215**, 385 (1952); Fujimoto, Hayakawa, and Nishijima, Progr. Theoret. Phys. (Japan) **10**, 113 (1953); F. L. Friedman and W. Tobocman, Phys. Rev. **92**, 93 (1953); P. B. Daitch and J. B. French, Phys. Rev. **87**, 900 (1952).

[3] W. Tobocman and M. H. Kalos, Phys. Rev. **97**, 132 (1955). Recently I. P. Grant, Proc. Phys. Soc. (London) **A68**, 244 (1955) has discussed Coulomb effects in a detailed manner similar to Tobocman and Kalos, employing, however, an approximate form for the deuteron Coulomb wave function that is not well adapted to heavy target nuclei.

[4] S. T. Butler and N. Austern, Phys. Rev. **93**, 355 (1954).

[5] Before the write-up of our results was completed, it came to our attention that K. A. Ter-Martirosyan, Zhur. Eksptl. i Teort. Fiz. **29**, 713, ff. (1955) has also discussed stripping in this approximation, and arrives at similar conclusions. We have accordingly abbreviated our work, in the overlapping discussion of the approximation for very large η.