Modes of Acceleration of Ions in a Three-Dee Cyclotron*

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Formulas are presented for the maximum energy gain per turn of ions in a fixed-frequency cyclotron with three identical dees symmetrically spaced. Calculations have been made for dees of general angle θ and specific angles 60°, 90°, and 120°. Dee excitations in which the dee voltages are all in phase or phased at 120° with respect to each other have been considered. The type of ions that can be accelerated is dependent upon the mode of excitation. In a 60° three-dee cyclotron with variable rf phasing, protons, deuterons, and tritons can be accelerated in the nonrelativistic energy region at substantially the same magnetic field and oscillator frequency. A 20-inch cyclotron of this type has been operated successfully.

INTRODUCTION

IT is possible for a symmetrical three-dee cyclotron to accelerate ions of differing q/m ratios below relativistic velocities without a change in the frequency of the electrical power supplied to the dees or the value of the magnetic field. This is demonstrated in the following article by calculating the energy gain per turn for various ions in a cyclotron with three identical dees of arbitrary widths. For these calculations it has been assumed that a step-function voltage change occurs at each edge of the dees.

THEORY[1]

Consider a three-dee cyclotron as schematically indicated in Fig. 1. The three dees of equal angles are labeled A, B, and C. The angular extent of each dee is θ. For the initial calculations the dee voltages are of equal amplitudes. The term "mode of acceleration" or, more simply, "mode" is used to describe the differing types of dee excitation, depending upon the relative phases of the dee voltages. This is in keeping with a previous publication.[2]

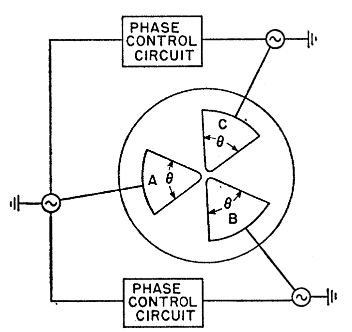

FIG. 1. A schematic of a three-dee cyclotron.

Calculations have been restricted to the cases where (a) the three dee voltages are all in phase, called the in-phase mode; (b) the voltages are phased 120° with respect to each other, with the phase of B dee following that of A dee, and the phase of C dee following that of B dee, designated the ABC mode; and (c) the voltages are phased 120° from each other, but in the reverse sequence, namely ACB. A phasing of type (b) will be considered first in analyzing the possibilities for ion acceleration.

The dee voltages for this ABC mode can be expressed:

$$V_A = Ve^{i\omega t},$$
$$V_B = Ve^{i(\omega t - 2\pi/3)},$$
$$V_C = Ve^{i(\omega t - 4\pi/3)}. \tag{1}$$

Here V is the amplitude of the dee voltage with respect to ground, ω is the angular frequency of the electrical power supplied to the dees, and t is time.

As an ion of charge q and mass m moves along its quasi-circular path in the cyclotron, it completes a revolution in a time $T = 2\pi m/qB = 2\pi/\Omega$, where Ω is the ion's angular frequency.

An ion that passes through the center of A dee at a time t_0 will cross the edge of A dee at a time

$$t_1 = t_0 + \frac{\theta}{2\Omega}. \tag{2}$$

It will enter B dee at a time

$$t_2 = t_0 + \frac{2\pi}{3\Omega} - \frac{\theta}{2\Omega}. \tag{3}$$

The times of crossing of the other dee edges follow similarly. The energy received by the ion in crossing one edge is the product of the charge on the ion and the voltage on the dee at that instant. The energy received by the ion in going around the cyclotron once is the sum of the energy contributions at the six edges. Carrying out this sum, we have the energy gain per turn, ΔW, given by

$$\Delta W = 2qV \sin\left(\frac{\theta\omega}{2\Omega}\right) \mathrm{Re}\left[e^{i\phi_0} \sum_{r=0}^{2} e^{irf}\right], \tag{4}$$

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[1] It has been brought to the authors' attention that Professor Edwin M. McMillan has obtained similar results as a special case of a general multi-dee cyclotron calculation. This general solution will appear soon [E. M. McMillan, *Experimental Nuclear Physics*, edited by E. Segrè (John Wiley and Sons, Inc., New York, to be published), Vol. 3].
[2] M. Jakobson and F. H. Schmidt, Phys. Rev. 93, 303 (1954).

where $f=\frac{2}{3}\pi(\omega/\Omega-1)$; ϕ_0 is the initial phase angle ωt_0 of the ion with respect to the dee voltage.

After S turns, the phase angle becomes

$$\phi=\phi_0+2\pi S(\omega/\Omega-1).$$

The term $\sin(\theta\omega/2\Omega)$ indicates the dependence of the energy gain per turn on the angular extent of the dees. The summation term expresses the dependence of the energy gain per turn on the 120° phasing of the similar dees. The term $e^{i\phi_0}$ indicates the dependence on the initial phase of the ion. Maximum energy gain occurs when the ion is at the center of any dee when the instantaneous voltage on that dee is zero. The gain drops to zero as the rf phase varies to $\pm\pi/2$, corresponding to $\pm\Omega\pi/2\omega$ angular displacement of the ion from the center of the dee when the instantaneous voltage is zero. The angular extent of each ion bunch in the cyclotron is then $\Omega\pi/\omega$. Other factors, such as field falloff with radius, would limit the angle to values less than this. The number of ion bunches present per ion cycle is ω/Ω. The total energy gain for the S turns is

$$W=2qV\sin\left(\frac{\theta\omega}{2\Omega}\right)\mathrm{Re}\left[\sum_{r=0}^{3S-1}e^{i\phi}e^{irf}\right]. \qquad (5)$$

If, after the S turns, the ion reaches the center of A dee at an rf phase identical with that at which the energy summation was begun, the series can be terminated to give a unique value for the energy gain per revolution. If no such repeat interval exists, the series does not terminate and the average energy gain per turn approaches zero. If the ion does reach the center of the first dee at an rf phase identical to that at which the summation was begun, then $\phi-\phi_0=2\pi m$. However, not only must this repeat interval exist, but by inspection, the summation of Eq. (5) will be zero unless the series terminates at $S=1$, which has the physical meaning that the ion makes one revolution in an integral number of rf cycles. In this case the summation is

$$e^{i\phi_0}\sum_{r=0}^{2}e^{irf}=3e^{i\phi_0}, \qquad (6)$$

and the quantity f is restricted to the value $f=2\pi n$. The energy gain per revolution becomes

$$\Delta W=6qV\sin\left(\frac{\omega}{\Omega}\frac{\theta}{2}\right)\cos\phi_0 \qquad (7)$$

for those ions satisfying the equation $f=\frac{2}{3}\pi(\omega/\Omega-1)=2\pi n$, n an integer. The acceptable ratios ω/Ω are thus found to be $\omega/\Omega=3n+1$ for this mode with the ions traveling in the ABC dee sequence.

For a given magnetic field, positive ions will circulate in one direction in the cyclotron and negative ions in the other, so that a given phase sequence ABC will be in the direction of rotation of the positive ions but opposite that of the negative ions. Consequently, the foregoing analysis is valid for positive ions when the rf phase sequence is ABC, and for negative ions when the phase sequence is ACB. The quantity n is positive for positive ions and negative for negative ions.

Table I. Energy gain per turn for various ions in a three-dee cyclotron. ω=rf angular frequency; Ω=ion frequency=qB/m; n=integer.

Dee angle	Mode	Ions accelerated ω/Ω	Maximum energy gain per turn n even	n odd
60°	ABC	$3n+1$	$3qV$	$3\sqrt{3}qV$
	ACB	$3n+2$	$3\sqrt{3}qV$	$3qV$
	In-phase	$3n$ except $6n$	$6qV$	$6qV$
90°	ABC	$3n+1$ except $4n$	$3\sqrt{2}qV$	$6qV$
	ACB	$3n+2$ except $4n$	$6qV$	$3\sqrt{2}qV$
	In-phase	$3n$ except $4n$	$6qV$	$3\sqrt{2}qV$
120°	ABC	$3n+1$	$3\sqrt{3}qV$	$3\sqrt{3}qV$
	ACB	$3n+2$	$3\sqrt{3}qV$	$3\sqrt{3}qV$
	In-phase	none	none	none
θ	ABC	$3n+1$	$6qV\lvert\sin[(3n+1)\theta/2]\rvert$	
	ACB	$3n+2$	$6qV\lvert\sin[(3n+2)\theta/2]\rvert$	
	In-phase	$3n$	$6qV\lvert\sin(3n\theta/2)\rvert$	

The geometric factor $\sin(\omega\theta/2\Omega)$ will, in addition to the requirement $\omega/\Omega=3n+1$, further limit the types of ions that may be accelerated. It is a modulating factor for energy gain, which may be zero for a given dee angular width for some type of ion that would otherwise be accelerated.

For the case in which the ion travels in a direction opposite to the previous phase sequence, the foregoing analysis still holds, but with the redefinition of the quantity f:

$$f=\left(\frac{\omega}{\Omega}+1\right)\frac{2\pi}{3}.$$

Using the requirement that $f=2\pi n$, we find that the permissible values of ω/Ω are

$$\omega/\Omega=3n+2,$$

where n may take integral values including zero. This is valid for positive ions when the phase sequence is ACB or for negative ions when the sequence is ABC, using the convention established earlier that a positive ion passes through the A, B, and C dees in consecutive order. The energy gain per revolution is the same as for the ABC mode and is given by Eq. (7).

For the case in which the three dees are excited in phase, the quantity f becomes $f=2\pi\omega/3\Omega$, and the types of ions accelerated are determined by the relation $\omega/\Omega=3n$. In this case, except for direction of rotation, it is immaterial whether the ions are positive or negative. The energy gain per revolution is again that of Eq. (7).

TABLE II. Hydrogen, deuterium, tritium, and helium ions that can be accelerated in a three-dee cyclotron with 60° dees with $\omega=\Omega_{proton}$.

Mode	Ion	Energy gain per turn	Energy at radius R	Relative threshold voltage
ABC	H^+	$3qV$	W	V_0
ABC	D_2^+	$3\sqrt{3}qV$	$W/4$	$V_0/\sqrt{3}$
ABC	He^+	$3\sqrt{3}qV$	$W/4$	$V_0/\sqrt{3}$
ABC	D^-	$3\sqrt{3}qV$	$W/2$	$V_0/\sqrt{3}$
ACB	H_2^+	$3\sqrt{3}qV$	$W/2$	$V_0/\sqrt{3}$
ACB	D^+	$3\sqrt{3}qV$	$W/2$	$V_0/\sqrt{3}$
ACB	He^{++}	$3\sqrt{3}qV$	W	$V_0/\sqrt{3}$
ACB	H^-	$3qV$	W	V_0
In-phase	$T^{\pm}$	$6qV$	$W/3$	$V_0/2$

RESULTS AND DISCUSSION

Table I gives the results of this analysis in tabular form, with numerical values computed for the maximum energy gain per revolution for 60°, 90°, and 120° dees. For dees of other angular widths or dees in which the fringing fields are included, the same ω/Ω ratios will give the ions eligible for acceleration, but with other values of energy gain per revolution, including zero in particular cases, as indicated by the θ^0 column. Negative values of ω/Ω indicate negative ions and positive values of ω/Ω indicate positive ions.

Table II indicates the ions of hydrogen, deuterium, tritium, and helium that can be accelerated in the three modes of a three-dee cyclotron with 60° dees and rf frequency,

$$\omega=\Omega_{proton}.$$

These calculations have been made for conditions of exact balance of amplitudes of the voltages and exact phase conditions. In practice these conditions are not ordinarily met; slight unbalances in amplitudes or deviations in phase may occur. However, any arbitrary voltage distribution among the three dees (either as to amplitude or as to phase) may be expressed as the sum of the three symmetrical components as analyzed earlier—the ABC mode, the ACB mode, and the in-phase mode (termed in reference 3 the forward mode, reverse mode, and neutral mode).

If the dee voltages are nearly balanced in one of the phase sequences, then the amplitudes of the other two components will be small. Since, to a first approximation, the effects of these voltages on accelerating ions are linear, each component can be considered separately, and the effect of the unbalanced dee voltages can be found by adding the separate effects of each component. It is observed from the formulas for different modes of acceleration that no ion can be accelerated by more than one of the three components, for a given magnetic field and electrical frequency.

These latter results are significant in that under identical operating conditions several different types of ions may be accelerated, either intentionally or as contamination products. For example, if the cyclotron is driven in phase sequence ABC to accelerate H^+ in its lowest mode ($\omega=\Omega$), then it will also accelerate He^{4+} and N^{14++} at the same time. In addition, if the voltages on the dees are unbalanced so that the ACB and in-phase components are also present, then many other ions can be accelerated simultaneously. An ion with a nonintegral ratio of ω/Ω, however, such as He^{3++}, cannot be accelerated under these conditions of operation, but can be accelerated only if the magnetic field or the electrical frequency of the cyclotron is changed. Since submultiples of ω/Ω are excluded, if the cyclotron is adjusted to accelerate H_2^+ or D^+ in the lowest mode, then H^+ cannot be accelerated.

In practice the voltage will not change in a step-function manner at the dee edges, but will change gradually over a distance comparable to the vertical aperture of the dees. When this fringing distance is comparable to the distance that an ion travels in an rf cycle—which will be the case for modes of operation where ω/Ω is large—the ion will gain very little energy in crossing any of the dee edges, and there will be no significant acceleration of these ions relative to those of lower ω/Ω ratios.

In the foregoing analysis it has been assumed that the magnetic field is constant as a function of radius, resulting in a constant value of the ion frequency Ω. In order to focus the beam, however, the magnetic field ordinarily is decreased with increasing radius, resulting in a lowering of the ion frequency as the orbit of the ion expands. For an ion that is in resonance at the center of the cyclotron the phase angle ϕ is no longer constant but varies as the ion moves outward. The permissible limits of ϕ are $\pm 90°$; outside these limits the ion will lose energy to the electric field. Since the total phase shift ϕ with respect to the electrical frequency is the sum of the increments per ion revolution, a certain threshold dee voltage is required to limit the number of revolutions necessary to obtain the desired energy with the phase angle ϕ within the limits given. The last column of Table II indicates the relative threshold voltages for various ions in a cyclotron with three 60° dees.

CONCLUSION

A three-dee three-phase 20-inch cyclotron with 60° dees has been constructed and operated successfully.[3,4] Stable operation was maintained with 6 ma of 1-Mev protons accelerated in the ABC mode. From programmed orbits the gain in energy per turn was measured to be $2.5qV$. The discrepancy between this value and the calculated value of $3qV$ is probably due to the fringing electric fields at the dee edges. Grounded dummy dees were not used. When the phase sequence

[3] Ruby, Heusinkveld, Jakobson, Smith, and Wright, Rev. Sci. Instr. **27**, 490 (1956).
[4] B. H. Smith and K. R. Mackenzie, Rev. Sci. Instr. **27**, 485 (1956).

was changed from ABC to ACB, 6.5 ma of 0.5-Mev deuterons were obtained. With helium ions, in the ACB mode, 1.5 ma of 1-Mev alpha particles was obtained. The measured energy gain per turn for He^{4++} and D_2^+ ions in this mode was approximately $5qV$.

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Hyperfine Structure in the l-Type Doubling Spectrum of HCN†

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Direct l-type transitions for HCN have been observed for $J=1$, 2, 3, 4, and 5. Values obtained of the asymmetry parameter $\eta=-0.082\pm0.005$ and the magnetic coupling constant $c_l=13\pm5$ kc/sec are in good agreement with previous work. For N^{14} the value $eq_mQ=-4.81\pm0.02$ Mc/sec is 5% higher than that obtained in the ground vibrational state. This change in eq_mQ can be explained by a decrease in the hybridization of the σ bond.

INTRODUCTION

DIRECT l-type doubling transitions of the linear molecule HCN have been observed by several investigators in the microwave K band.[1-3] For smaller values of J the spectrum lies in the L and S bands where some work has been reported.[4,5] The work reported herein is confined to the L and S bands.[6]

In the first excited bending mode the degenerate vibrational levels are split and it is the transition between these levels that gives rise to the l-type doubling spectrum. N^{14}, with its spin of one, produces a quadrupole splitting. This splitting normally results in a single $\Delta F=0$ line. White[3] has shown that this main line is further split due to an asymmetry of the electric field gradient at the nitrogen nucleus produced by the bending of the molecule.

At lower J values the relative intensities of the $\Delta F=\pm1$ lines increase and as a result these transitions for $J=3$ and 4 were observed. With this added information it was possible to evaluate the quadrupole coupling constant, the asymmetry parameter and the magnetic $\mathbf{I}\cdot\mathbf{J}$ interaction constant.

EXPERIMENTAL TECHNIQUE

A Stark-modulated spectrometer whose cell is a 20-foot S-band wave guide was used throughout. The guide operated in the usual TE_{01} mode for the $J=3$, 4, and 5 lines. Since the guide cutoff is 2000 Mc/sec, for the $J=1$ and 2 lines it was run as a transmission line in a TEM mode. A description of the apparatus is found elsewhere.[7]

EXPERIMENTAL RESULTS

Table I lists the lines observed, their frequencies and the parameters evaluated from this data. All the $\Delta F=\pm1$ transitions were observed for $J=3$. The $\Delta F=0$ splitting is theoretically threefold. The line found, however, was split twofold with the two higher frequency components unresolved. This is due to the fact that the lower of the two unresolved components has twice the intensity of the upper, plus the fact that the lines are separated by only 60 kc/sec. This observation is approximately true for all the reported lines save $J=1$. Since the $F=0\rightarrow F=0$ transition is forbidden, the $J=1$ $\Delta F=0$ splitting is theoretically twofold. The intensity ratio of these two lines is 5:1 and as a result only one line was observed for $J=1$. The $J=4$ line $\Delta F=\pm1$ transitions were seen only at the lower end of the spectrum because of interference from Stark components at the upper frequency end.

The second harmonic of a 707A klystron was employed as the signal source for the $J=4$ line. The result was gratifying in that not only was the $J=4$ line seen but the $J=5$ line as well. (See Fig. 1.) This came about as follows: The frequency of the l-type doubling transition is given as $\nu=qJ(J+1)$ where (see following section) q is approximately constant. Since the fundamental frequency used was $10q$, the frequency of the $J=4$ line $20q$, and the frequency of the $J=5$ line $30q$, the second and third harmonics of the signal source

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