Acoustodynamic Effects in Semiconductors

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(Received July 5, 1956)

The presence in a semiconductor of both signs of current carriers makes it possible to disturb the spatial distribution of these carriers appreciably without giving rise to electrical space charge. As a result, the interaction of particles with acoustic waves leads to certain effects which, at least at low frequencies, are peculiar to semiconductors; these include a (complex) addition to the elastic modulus and the "acoustoelectric effect."

I. INTRODUCTION

THE concept of a "deformation potential" has proved very fruitful in discussing the motion of electrons and holes in a crystal in the presence of acoustic deformations of the crystal. Stated in its simplest form, which is in fact applicable to low-energy electrons and holes whose bands have a "simple" structure, the assumption is that such a particle has a potential energy V_I proportional to the dilatation of the crystal:

$$V_I = -\mathcal{E}_1\Delta, \tag{1}$$

where $\mathcal{E}_1$ is a constant and Δ the dilatation. This interaction leads not only to forces exerted on particles by acoustic waves, but also in certain cases to radiation of acoustic waves by particles. The word "acoustodynamics" has been coined for the description of the motion of this field-particle system.

The theory of the effect of conduction electrons on acoustic propagation has been studied by a number of people,[1–5] all of whom, however, directed their attention to metals rather than semiconductors. The peculiar feature of semiconductors is that, with both signs of carriers present, it is possible to produce appreciable spatial "bunching" of carriers without concomitant space charge. Since the effects to be described here depend on such bunching, they would be very much smaller for metals at frequencies well below the dielectric relaxation frequency (as presently attainable ultrasonic frequencies are).

The "acoustoelectric effect" is a related phenomenon which was named and first discussed by Parmenter[6]; the term refers to the appearance of a dc electric current when an acoustic wave is passing through a conducting medium. Parmenter's treatment seems deficient in two respects. First, he applies a Boltzmann factor to find the distribution of electrons among states which are not eigenstates of energy. In fact, the situation under discussion is not an equilibrium situation but a "steady state," which has to be treated by a transport equation or some equivalent formalism which contains transition rates. Second, he uses a one-electron approximation, and thus does not consider the space-charge forces which strongly resist the bunching of electrons (see previous paragraph). This leads him to predict an acoustoelectric effect in metals, where it may in fact be expected to vanish.*

In the present paper, we shall treat all phenomena classically; moreover, since attainable acoustic wavelengths are much larger than carrier mean free paths (and the periods much longer than mean free times), we can describe the net particle currents j in the usual macroscopic way as being composed of a drift term and a diffusion term:

$$j = D(F/KT - \nabla)n, \tag{2}$$

where n is the particle density, F the force applied to the particles, KT the termal energy, and D the diffusion constant. The single constant D thus incorporates the result of solving the appropriate transport equation for the rate at which transitions occur. Equation (2), together with the equation of continuity, will allow us to calculate the redistribution of carriers which takes place when an external force field, in the form of an acoustic wave, is applied.

To avoid space-charge difficulties, we shall have to consider not only the deformation potential force $-\nabla V_I$, but also the force exerted by electric fields resulting from the redistribution of charges. Here we shall avail ourselves of a good (and common) approximation which is valid when the period of the waves is much longer than the dielectric relaxation time and their wavelength much larger than a Debye length, and which states that charge neutrality is maintained exactly, the electric fields necessary for this purpose being set up automatically and instantaneously. For small sinusoidal disturbances we can further state that this induced electrostatic potential is proportional to the deformation potential of the applied acoustic wave (the proportionality constant being in general complex).

The change in the acoustic propagation properties of the medium can be thought of as the continual addition

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* *Note added in proof.*—A calculation of the acoustoelectric effect similar to the one given here has also been made by T. Holstein (unpublished). In addition, Van Den Beukel [Appl. Sci. Research, B5, 459 (1956)] has calculated the effect, but without taking into account the spatial redistribution of carriers.

to the original wave of a wave which is radiated by the redistributed carriers. To solve this problem, we shall first formulate the field equations of acoustodynamics, i.e., the acoustic field equations in the presence of sources which are coupled to it by the deformation potential, Eq. (1). This derivation will be followed by a discussion of the carrier bunching which takes place in the presence of an acoustic wave. In fact, the problem will be generalized somewhat by the inclusion of an applied constant longitudinal electric field, since the dependence of the results on this applied field are rather interesting. We shall next derive the acoustodynamic effect on the acoustic propagation properties, and conclude with a brief discussion of the acoustoelectric effect.

II. FIELD EQUATIONS

We assume that the deformation potential is correctly given by Eq. (1), and that the crystal is isotropic. Also, in writing the Hamiltonian density of the field we shall omit the shear energy. This omission will not lead to any difficulty as long as there are no boundaries which can couple compressional waves to shear waves, and in the problems which we shall solve the boundaries will all be perpendicular to the propagation vector of the waves.

In terms of the displacement $\mathbf{R}$, the density of the crystal μ, and the sound velocity c, it is convenient to define

$$\mathbf{\Psi}\equiv\mu^{\frac{1}{2}}d\mathbf{R}/dt, \tag{3}$$

$$\mathbf{Q}\equiv\mu^{\frac{1}{2}}\mathbf{R}, \tag{4}$$

$$\Phi\equiv c\nabla\cdot\mathbf{Q}. \tag{5}$$

$\mathbf{\Psi}$ and $\mathbf{Q}$ can be regarded as canonically conjugate field quantities. The set $\mathbf{\Psi}$, Φ is what we shall regard as the "field strengths"; note that $\mathbf{\Psi}^2$ and Φ^2 both have dimensions of energy density.

The interaction energy between field and particles can now be written

$$U_I=-\textstyle\sum_i q_i\Phi_i, \tag{6}$$

where Φ_i is the value of Φ at the position of the ith particle, and q_i is the deformation potential of that particle divided by $\mu^{\frac{1}{2}}c$. We shall refer to q_i as the *acoustic charge* of the particle. If we choose to think of the particles as a continuous distribution, U_I becomes an integral containing the *acoustic charge density* ρ.

The Hamiltonian density for the field which includes the interaction is

$$H=\tfrac{1}{2}\mathbf{\Psi}^2+\tfrac{1}{2}c^2(\nabla\cdot\mathbf{Q})^2-c\rho\nabla\cdot\mathbf{Q}, \tag{7}$$

from which the following field equations for $\mathbf{\Psi}$ and Φ can be derived:

$$\partial\mathbf{\Psi}/\partial t=c\nabla\Phi-c\nabla\rho, \tag{8a}$$

$$\partial\Phi/\partial t=c\nabla\cdot\mathbf{\Psi}. \tag{8b}$$

Equivalently, if we define a new field quantity

$$\Phi'\equiv\Phi-\rho, \tag{9}$$

we can introduce a potential Π such that

$$\mathbf{\Psi}\equiv\nabla\Pi, \tag{10a}$$

$$\Phi'=(1/c)\partial\Pi/\partial t, \tag{10b}$$

$$\nabla^2\Pi-(1/c^2)\partial^2\Pi/\partial t^2=(1/c)\partial\rho/\partial t. \tag{11}$$

With these equations we are in a position to calculate the field radiated by an arbitrary acoustic charge distribution.

III. BUNCHING OF CARRIERS IN A COMPRESSIONAL WAVE

We consider a semiconducting body which is traversed by a plane compressional wave,

$$\Phi=\Phi_0e^{ik(x-ct)}, \tag{12}$$

moving in the positive x direction. In addition, a longitudinal constant electric field is applied such as to give electrons a drift velocity βc in the positive x direction. Let n, p be the concentrations of electrons and holes, respectively, and b the ratio of their mobilities; we can then write for the particle current densities

$$j_n=n\beta c+\left(\frac{nD_n}{KT}\right)\frac{\partial}{\partial x}(q_n\Phi+e\gamma_k\Phi)-D_n\frac{\partial n}{\partial x}, \tag{13a}$$

$$j_p=-\frac{p\beta c}{b}+\left(\frac{pD_n}{bKT}\right)\frac{\partial}{\partial x}(q_p\Phi-e\gamma_k\Phi)-\frac{D_n}{b}\frac{\partial p}{\partial x}, \tag{13b}$$

where q_n, q_p are the acoustic charges of an electron and a hole, $\mp e$ the electric charges, and $\gamma_k\Phi$ is the induced electrostatic potential discussed in the introduction.

The continuity equations take the form

$$\frac{\partial n}{\partial t}+\frac{\partial j_n}{\partial x}+\frac{1}{\tau}\left[n-n_0\left(1+\frac{q}{1+s}\frac{\Phi}{KT}\right)\right]=0, \tag{14a}$$

$$\frac{\partial p}{\partial t}+\frac{\partial j_p}{\partial x}+\frac{1}{\tau}\left[p-p_0\left(1+\frac{sq}{1+s}\frac{\Phi}{KT}\right)\right]=0, \tag{14b}$$

where $s\equiv n_0/p_0$ is the ratio of equilibrium concentrations of electrons and holes, τ is the lifetime, and $q\equiv q_n+q_p$. The last term expresses the first-order change in equilibrium concentrations due to the fact that in a field Φ the energy gap is decreased by the amount $q\Phi$.

If we assume $n-n_0\ll n_0$, $p-p_0\ll p_0$, and let

$$n=n_0+n_1e^{ik(x-ct)}, \tag{15a}$$

$$p=p_0+p_1e^{ik(x-ct)}, \tag{15b}$$

we obtain expressions for n_1 and p_1; at this point we

can invoke charge neutrality, which implies that $n_1=p_1$, and eliminate the unknown γ_k. The result is

$$n_1=p_1=\frac{n_0}{1+s}\frac{q\Phi_0}{KT}M, \tag{16}$$

where

$$M\equiv\frac{1+\omega_0(1+sb)/\omega^2\tau(1+s)}{1+\omega_0(1+sb)/\omega^2\tau(1+s)-i(\beta_0-\beta)(1-s)\omega_0/(1+s)\omega},$$

$$\omega\equiv kc,$$

$$\omega_0\equiv c^2/D_n,$$

$$\beta_0\equiv(1+sb)/(1-s).$$

The factor M incorporates the dependence of the bunching on frequency, applied field (through β), and properties of the material. We shall discuss this dependence for a few cases in connection with the resulting acoustic propagation properties.

IV. ACOUSTIC PROPAGATION PROPERTIES

From Eq. (16), we can immediately obtain the acoustic charge density:

$$\rho=(q_nn_1+q_pp_1)e^{ikx-i\omega t}$$

$$=\frac{qn_0}{1+s}\frac{q\Phi_0}{KT}Me^{ikx-i\omega t}. \tag{17}$$

This can be inserted into Eq. (11), the solution of which, if we assume it harmonic in space, turns out to be linearly increasing in time; this radiated wave is then to be added to the original wave. Since, however, ρ is proportional to the acoustic amplitude, it will itself change as the wave changes, giving rise to a change with time which is exponential rather than linear. By carrying out the appropriate calculations, we find that the variation of the wave can be represented by the exponential

$$\exp[ikx-i\omega(1+\epsilon+i\eta)t],$$

where

$$\epsilon=-(q^2n_0/2KT)\,\mathrm{Re}[M]/(1+s), \tag{18a}$$

$$\eta=-(q^2n_0/2KT)\,\mathrm{Im}[M]/(1+s). \tag{18b}$$

Quantitatively, ϵ is the fractional increase in phase velocity of the wave, and η the fractional increase in amplitude per radian of oscillation, or half the inverse negative "Q." The meaning of Eqs. (18) will now be illustrated by some simple examples.

A. Limiting Frequencies

As $\omega\to0$ and as $\omega\to\infty$, $M\to1$. We conclude that in both cases (a) the phase velocity of waves is decreased by a fractional amount $q^2n_0/2KT(1+s)$; (b) the attenuation, as measured by the equivalent "Q", vanishes. However, (c) for $\omega\to\infty$, the attenuation *per unit length* does not vanish, but approaches the constant value

$$\frac{q^2n_0}{KT}\frac{\omega_0(\beta_0-\beta)}{c}\frac{1-s}{(1+s)^2}.$$

B. Extrinsic Material with Long Lifetime

If we assume $s\ll1$, $sb\ll1$ (strongly p-type material); also $k(D_n\tau)^{\frac{1}{2}}\gg1$ (diffusion length long compared to wavelength), we find

$$M\cong[1-i(1-\beta)\omega_0/\omega]^{-1}.$$

We conclude that, as β is varied, (a) the phase velocity of waves is always decreased; the maximum effect occurs at $\beta=1$ and is given by

$$\delta c/c=q^2n_0/2KT;$$

(b) the wave-carrier interaction endows the system with a finite Q; the maximum effect occurs at $\beta=1\pm\omega/\omega_0$ and is given by

$$Q=\mp2KT/q^2n_0.$$

The fact that the attenuation is negative when $\beta>1$ is somewhat analogous to the amplifying action of an electromagnetic traveling-wave tube.

C. Intrinsic Material

By setting $s=1$, we obtain

$$M=\frac{1+\omega_i/\omega^2\tau}{1+\omega_i/\omega^2\tau-i\omega_i/\omega},$$

where

$$\omega_i\equiv\tfrac{1}{2}c^2(1/D_n+1/D_p).$$

We conclude that the acoustic behavior of intrinsic material is unaffected by an applied electric field.

V. ACOUSTOELECTRIC EFFECT

The material of Sec. III enables us to calculate the acoustoelectric effect, i.e., the electric current which is caused by an acoustic wave. Because of the requirement of charge neutrality, no net ac electric current can be carried by the wave. However, there is a possibility of a nonvanishing dc current; from Eqs. (13) we see that it will appear in our perturbation calculations as a second-order effect in $q\Phi/KT$. The actual calculation is performed by substituting (15) into (13), using the solution (16), and computing the time-average value of the middle term of (13). This yields the dc particle currents $\bar{j}_n$ and $\bar{j}_p$; the net acoustoelectric current is then given by

$$I_{ae}=-e(\bar{j}_n-\bar{j}_p).$$

Before this calculation can be done one has to reconsider Eqs. (14), since the recombination term there is only correct to first order and thus inadequate for a second-order calculation. However, rather than encumber our arithmetic, we shall here assume that τ is very large, so that the whole recombination term is

negligible. The result is

$$I_{ae}=-\tfrac{1}{2}\frac{en_0c}{(1+s)^2}\left(\frac{q\Phi_0}{KT}\right)^2 \times\frac{(1-s)-\beta(1+s/b)}{1+(\beta_0-\beta)^2[(1-s)/(1+s)]^2(\omega_0/\omega)^2}. \quad (19)$$

The following conclusions are then easily verified.

A. *Extrinsic material, no applied field.*—If $\beta=0$, the direction of the current is that of minority carriers carried with the wave; that is, $I_{ae}<0$ if $s>1$ and $I_{ae}>0$ if $s<1$. If $s\ll1$ (strongly p-type material), I_{ae} becomes simply proportional to n_0 and independent of s, showing this to be a pure minority carrier effect. This may be understood from the following argument. The ac concentrations of holes and electrons are equal; but if there are many more holes than electrons, the ac force which brings about this concentration is much smaller for the former than for the latter. Thus the second-order effect, which is a product of ac force and ac concentration, is much greater for the minority carriers. If there are no minority carriers, the acoustoelectric effect vanishes.

B. *Intrinsic material, no applied field.*—It is equally easy to see that for intrinsic material ($s=1$) the effect vanishes. This is not surprising, since we know I_{ae} to reverse sign when going from n-type to p-type material.

C. *Effect of applied field.*—The acoustoelectric effect can be thought of as a drag by the wave on the particles, and will depend on the relative velocity between the wave and those particles. The application of a field will pull one type of carrier against the wave, and the other type with it; it will thus increase one acoustoelectric particle current and decrease the other. Since, however, the associated electric currents have opposite signs, the two effects on the net electric current will add. Thus, for example, Eq. (19) shows that even in intrinsic material an acoustoelectric current is to be expected if β does not vanish.

VI. ACKNOWLEDGMENTS

The author is grateful to D. Kleinman, C. Herring, and C. A. Lee for helpful and stimulating discussions.

PHYSICAL REVIEW VOLUME 104, NUMBER 2 OCTOBER 15, 1956

Proposal for a New Type Solid State Maser*

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(Received July 6, 1956)

The Overhauser effect may be used in the spin multiplet of certain paramagnetic ions to obtain a negative absorption or stimulated emission at microwave frequencies. The use of nickel fluosilicate or gadolinium ethyl sulfate at liquid helium temperature is suggested to obtain a low noise microwave amplifier or frequency converter. The operation of a solid state maser based on this principle is discussed.

TOWNES and co-workers[1,2] have shown that microwave amplification can be obtained by stimulated emission of radiation from systems in which a higher energy level is more densely populated than a lower one. In paramagnetic systems an inversion of the population of the spin levels may be obtained in a variety of ways. The "180° pulse" and the "adiabatic rapid passage" have been extensively applied in nuclear magnetic resonance. Combrisson and Honig[2] applied the fast passage technique to the two electron spin levels of a P donor in silicon, and obtained a noticeable power amplification.

Attention is called to the usefulness of power saturation of one transition in a multiple energy level system to obtain a change of sign of the population difference between another pair of levels. A variation in level populations obtained in this manner has been demonstrated by Pound.[3] Such effects have since acquired wide recognition through the work of Overhauser.[4]

Consider for example a system with three unequally spaced energy levels, $E_3>E_2>E_1$. Introduce the notation,

$$h\nu_{31}=E_3-E_1 \quad h\nu_{32}=E_3-E_2 \quad h\nu_{21}=E_2-E_1.$$

Denote the transition probabilities between these spin levels under the influence of the thermal motion of the heat reservoir (lattice) by

$$w_{12}=w_{21}\exp(-h\nu_{21}/kT), \quad w_{13}=w_{31}\exp(-h\nu_{31}/kT), \quad w_{23}=w_{32}\exp(-h\nu_{32}/kT).$$

The w's correspond to the inverse of spin lattice relaxation times. Denote the transition probability caused by a large saturating field $H(\nu_{31})$ of frequency

* Supported by the Joint Services.
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