Higher Order Corrections to the Field Emission Current Formula

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In this paper, higher order terms in expansions made to obtain the field emission current formula are retained. Analytic expressions for first-order correction terms to the current formula are derived.

I. INTRODUCTION

IN a recent extension of the Fowler-Nordheim theory[1,2] to finite temperatures,[3] based on the free electron and image force barrier model, an approximate expression for the current emitted from a metal as a function of temperature and applied electric field was obtained. The values of field and temperature at which the formula for the current applies were established, but a quantitative discussion of the approximations involved was not made.

In the present paper, higher order terms in expansions made to obtain the current formula are retained. Analytic expressions for first-order corrections to the current formula are derived. The calculations are valid at all fields and at somewhat elevated temperatures. The results provide an estimate of the size of the higher order corrections to the current formula. In the zero-temperature limit (Fowler-Nordheim region), the correction to the current formula amounts to five or ten percent of the total current.

II. THE CORRECTION TERMS

The starting point for the calculations is the following expression for the current density:

$$j(F,T,\zeta)=\frac{kT}{2\pi^2}\int_{-\infty}^{\infty}\frac{\ln\{1+\exp[-(W-\zeta)/kT]\}dW}{1+\exp[(4/3)\sqrt{2}F^{-\frac{1}{4}}y^{-\frac{3}{2}}v(y)]}. \quad (1)$$

The notation used here is the same as used by Murphy and Good[3]; their paper will be referred to as I. Equation (1) above is Eq. (I-20) with the integration range and integrand modified in a way appropriate to the field emission process. The usual approximation is to discard the 1 compared to the exponential in the denominator and to expand the denominator exponent about the Fermi energy ζ. If these two processes are extended to the next order, one obtains the following first-order approximation for the current:

$$j_1=\frac{kT}{2\pi^2}\int_{-\infty}^{\infty}[1-f(W-\zeta)^2-e^{-b+c(W-\zeta)}]\times e^{-b+c(W-\zeta)}\ln(1+e^{-(W-\zeta)/kT})dW. \quad (2)$$

This is the same expansion as in I, based on Eqs. (I-36) and (I-37), but with terms of one higher order retained. The zero-order approximation j_0 is found by replacing the square brackets in Eq. (2) by unity and is given in Eqs. (I-54) and (I-55).

The integrals in Eq. (2) can be evaluated in terms of elementary functions. The result is

$$j_1/j_0=1-2c^{-2}f\{1+\tfrac{1}{2}(\pi ckT)^2+\pi ckT\cot(\pi ckT)+[\pi ckT\cot(\pi ckT)]^2\}-\tfrac{1}{4}e^{-b}\sec(\pi ckT). \quad (3)$$

These correction terms are not valid within the entire field emission region defined in I; they apply at low temperatures but not above the $ckT=\frac{1}{2}$ line. An exact determination of the limits of the region of applicability has not been made. At zero temperature, in which case j_0 reduces to the Fowler-Nordheim expression, the correction becomes

$$(j_1/j_0)_{T=0}=1-6c^{-2}f-\tfrac{1}{4}e^{-b}\cong 1-\frac{3\sqrt{2}F}{8\phi^{\frac{3}{2}}t^4}-\tfrac{1}{4}\exp\left(-\frac{4\sqrt{2}\phi^{\frac{3}{2}}v}{3F}\right), \quad (4)$$

where the approximation of Eq. (I-65) is used to simplify the result. The arguments of v, t are $F^{\frac{1}{2}}/\phi$.

The correction ranges from 3 to 16% at fields of 2×10^7 to 10×10^7 volts/cm, 4.5 ev work function, and zero temperature. With these fields and work function, the corrections are the same at 300°K.

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