move out and decrease in intensity as the spinning frequency is increased and the two main components coalesce. An experiment approximating this uses two liquids with different bulk magnetic susceptibilities, a small cylinder of one inside a larger cylinder of the other. This assembly gives U-shaped resonances with splittings related to the differences in bulk susceptibility.[9] Upon spinning the assembly, the doublets are averaged out and the arrangement is useful in comparing the resonance positions of nuclei in the two samples.

The value of ν_s required to average out the two local fields depends on how weak one wishes the sidebands to be. The problem can be treated as a "double" square wave modulation of H with the result that if the first sideband is to be less than 1% of the intensity of the coalesced fundamental, the spinning frequency is $\nu_s \gtrsim 5\sqrt{2}\gamma H_l^0/\pi^2$. Random processes thus appear to be about twice as effective as coherent processes in reducing local field effects. This apparent difference in the effectiveness of the two types of motion arises mainly from the continuous distribution of frequencies for the random motions in combination with the nonlinear relation[3] between frequency and extent of averaging.

Another situation of interest concerns the effect upon a resonance line of a random modulation of H uniformly throughout the sample, arising say from fluctuations in the current of an electromagnet or in the field sweep system. Of course, if the fluctuations ΔH are of small amplitude and fast enough, they average out, the criterion being $\nu_f \gtrsim \gamma\Delta H/\sqrt{2}$. If the frequencies are too low or the amplitudes too great the resonance is broadened by the fluctuations, depending on how long a time is taken to scan the resonance. It is apparent that sample spinning will not average out these local fields because it produces no change in them.

TABLE I. Deviation ratios for a cylindrical sample rotating in a magnetic field with a linear gradient. k_c is the value calculated from an approximate calibration of the field gradient coil; k_i, that inferred from the relative intensities of the fundamental and sideband resonances.

Coil current	$\frac{\omega_s}{2\pi}$	k_c	k_i	k_i/k_c
0.25 amp	32.5 cps	0.48	0.60	1.25
0.50	32.5	0.94	1.19	1.27
0.50	64.7	0.48	0.60	1.25
0.75	32.2	1.43	1.9	1.33
0.75	66.8	0.69	0.92	1.33
1.00	31.6	1.95	2.6	1.33
1.00	32.0	1.93	2.6	1.35
1.00	36.2	1.70	2.3	1.36
1.00	39.8	1.54	2.1	1.36
1.00	59.0	1.00	1.4	1.40

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Magnetic Moment of the Neutron*

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An experiment has been performed to measure the magnetic moment of the neutron to a greater precision than has heretofore been obtained. The method is somewhat similar to that of atomic-beam magnetic resonance using separated oscillating fields. A beam of slow neutrons is polarized by reflection from a magnetized cobalt mirror and then reflected again from a similar mirror used as an analyzer. Between the two mirrors the neutrons pass through a region of uniform magnetic field where they may become depolarized by a resonant rf magnetic field. This results in a drop in intentisy of neutrons reflected from the second mirror. The resonance frequency for depolarization is compared with the proton moment resonance frequency in the same transition region. In order to achieve high resolution of the neutron resonance, the path length of the neutrons in the uniform magnetic field was 110 cm. The measured ratio of resonant frequencies $\nu_n/\nu_p = 0.685057 \pm 0.000017$. This corresponds to a value of $\mu_n = -1.913148 \pm 0.000066$ nm.

INTRODUCTION

OF the simpler nuclei and nucleons, the one whose magnetic moment has been measured with the least precision is the neutron. Although the previously determined values of the neutron magnetic moment are of sufficient accuracy to point out the deficiencies of present-day nucleon theory, it is to be hoped that more precise data will be of eventual value in view of the fundamental character of the nucleon.

The magnetic moment of the neutron has been measured in the past with various degrees of precision by Alvarez and Bloch,[1] Arnold and Roberts,[2] and Bloch,

* Work performed under the auspices of the U. S. Atomic Energy Commission.

[1] L. W. Alvarez and F. Bloch, Phys. Rev. **67**, 111 (1940).
[2] W. R. Arnold and A. Roberts, Phys. Rev. **71** 878 (1947); **70**, 766 (1940).

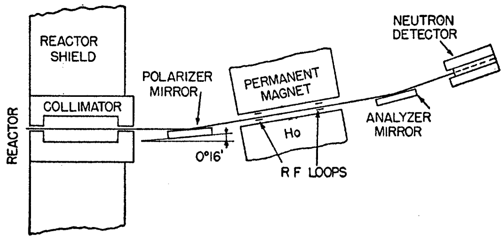

FIG. 1. Schematic arrangement of the components, not to scale.

Nicodemus, and Staub.[3] The general method involved in these experiments is the reorientation of the moments in a beam of polarized neutrons by an alternating magnetic field in resonance with the Larmor frequency of precession of the neutrons in a superimposed uniform magnetic field.

Developments of the past few years in the field of neutron physics have permitted a measurement of the neutron magnetic moment to a higher order of accuracy. The previous limitations in accuracy have been due to (1) the degree of polarization of the neutrons attainable by transmission through magnetized iron; (2) the low neutron flux attainable; (3) the short path of the neutrons in the magnetic field. The particular developments contributing to greater accuracy in this experiment were (1) the discovery that neutrons reflected from saturated magnetized cobalt mirrors are polarized to well over 90%[4]; (2) the fact that the external neutron beam obtainable from the Brookhaven reactor is considerably stronger than the beams used by previous experimenters, hence permitting adequate counting statistics to be obtained in reasonable time intervals; (3) the discovery that the separated oscillatory field[5] method of observing a resonance in a beam of magnetic particles produces a much narrower resonance line than those previously available, permitting a more precise measurement of its center.

The nature of the present experiment is illustrated in Fig. 1. A beam of neutrons is defined by a collimator, then polarized by reflection from a magnetic mirror. The mirrors M_1 and M_2 are the analogs of the polarizer and analyzer of optics. In the intermediate region, there is a uniform steady magnetic field upon which is superimposed a perpendicular alternating magnetic field of variable frequency. When the frequency of this field is equal to the Larmor frequency of the neutron spin-precession in the uniform field, the beam becomes depolarized and the intensity as detected will decrease. This frequency ν is related to the magnetic moment by the relation

$$2\mu H_0 = h\nu,$$

where μ is the magnetic moment, H_0 the steady magnetic field, h is Planck's constant, and ν the frequency of the Larmor precession. Since the absolute value of a magnetic field is extremely difficult to measure, we do a resonance type of experiment on the proton in the same magnetic field and thereby obtain a value of the ratio,

$$\mu_n/\mu_p = \nu_n/\nu_p.$$

PRODUCTION AND DETECTION OF THE NEUTRON BEAM

The source of neutrons was the Brookhaven National Laboratory Reactor. This reactor is graphite moderated and air cooled. In the shield was placed a collimator which defined a beam of neutrons by two slits in a block of graphite separated by about 60 inches. The width of the beam at the shield was $\frac{1}{8}$ inch, the height one inch, and the angular divergence was about eight minutes of arc. Vertical stops were placed at each end of the magnet in order to confine the height of the beam to the small region over which the magnetic field had adequate uniformity. Additional vertical strips of cadmium were placed approximately one mm away from each of the mirrors, midway along its length. These served to eliminate from the detector those neutrons which may not have been reflected from both mirrors.

The velocity distribution of the neutrons in the beam is quite similar to that expected for the effusion from a reservoir with a Maxwellian distribution. The characteristic temperature is that of the graphite moderator. The velocity most probable in the beam is approximately 2200 meters/sec. It will appear, from the equation relating critical angle with scattering amplitudes, that the neutron mirror acts as a velocity selector as well as a polarizer. In our experimental arrangement, only neutrons with speeds less than 1300 meters/second are reflected. See Fig. 2. As will be discussed further, the neutron mirror acts as a velocity selector as well as a polarizer and in our experimental

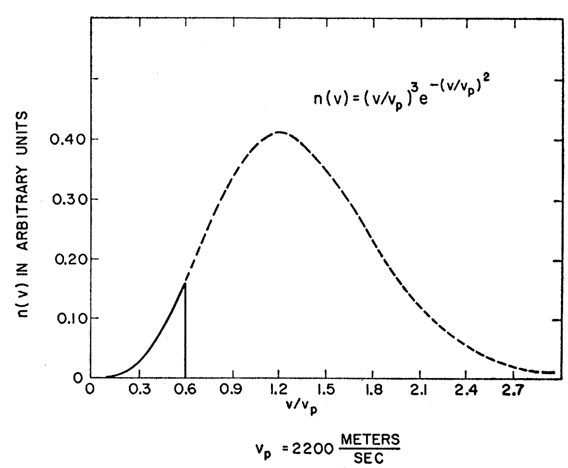

FIG. 2. The solid curve shows roughly the velocity spectrum of the neutron beam reflected from the mirror at $\theta=0.005$ radian (17′). The broken line shows the approximate spectrum of neutrons in the original beam.

[3] Bloch, Nicodemus, and Staub, Phys. Rev. **74**, 1025 (1948).
[4] D. J. Hughes and M. T. Burgy, Phys. Rev. **81**, 498 (1951).
[5] N. F. Ramsey, Phys. Rev. **78**, 695 (1950).

arrangement cuts off those neutrons having speeds of greater than 1300 meters/second.

The neutrons are detected in a standard manner with a $B^{10}F_3$ ionization chamber. The chamber was 18 in. long, 1 in. in diameter and filled to a pressure of 45 cm Hg. The output pulses were fed into a linear amplifier and a scale-of-128 counter.

NEUTRON MIRRORS

It has been shown by Hughes and others[4] that it is possible to obtain beams of highly polarized neutrons by reflection from magnetically saturated mirrors of cobalt. This effect arises as a result of the relationships between the nuclear and magnetic coherent scattering amplitudes, a_n and a_m. The critical angle, θ_c, for specular reflection is related to these amplitudes in the manner

$$\theta_c^2 \sim \lambda^2(a_n \pm a_m),$$

where the $\pm$ signs refer to the two possible orientations of the neutron moment in the magnetic field of the mirror. For fully magnetized cobalt a_m is greater than a_n and a reflected beam should be completely polarized. A mirror was constructed of hot-rolled cobalt but, at an internal H of 4500 oersteds, it was far from saturated and produced only about 50% polarization. Alloying cobalt with 6% Fe, however, gives an alloy with a cubic structure that saturates at much lower fields and gave a beam of 90% polarization.

The extent of polarization was measured by determining the decrease in intensity due to depolarizing the beam by interposing a thin sheet of unmagnetized iron in the beam between the two mirrors.

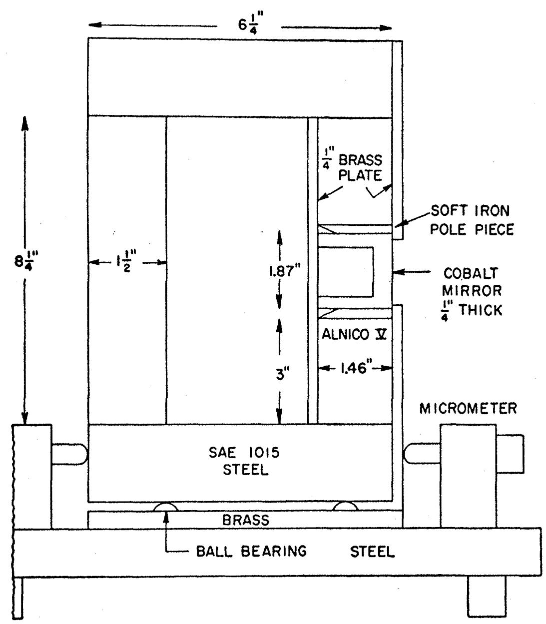

FIG. 3. Cross section of polarizing magnetic mirror, showing mounting arrangement.

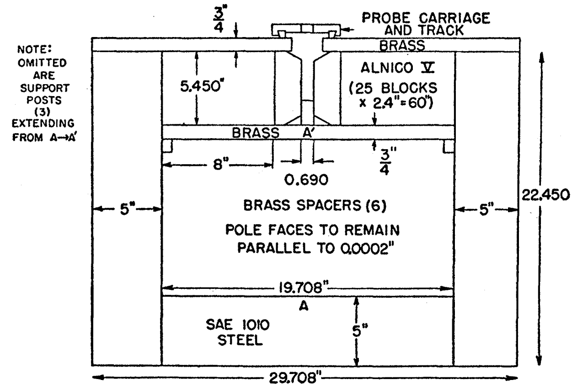

FIG. 4. Vertical section through the magnet which furnishes the steady field. The field strength is approximately 8500 gauss.

The mirrors were 14 in.×1.87 in.×¼ in. thick. They were polished using techniques characteristic of the preparation of high-grade optical instruments. The magnetic circuit consisted of two "Alnico V" sections in series with a yoke of mild steel. See Fig. 3. The system was magnetized by pulsing 1300 amperes through a coil composed of 25 turns of 1 in.×0.022 in. copper ribbon.

The mirrors were mounted on horizontal planar ball bearings and each one was positioned by three micrometer screws.

For the mirrors set at an angle of 17.5 minutes from the incident beam, the maximum neutron velocity was 1300 meters per second. For the various conditions of the beam, the counting rates were as follows:

Direct unreflected beam	700 000 counts/min
Beam reflected from M_1	24 000
Beam reflected from M_1 and M_2	7500
Beam stopped down in height	900

THE MAGNET

In order to attain high resolution in the neutron resonance line, it is necessary to make the time of measurement large. This means that the time spent by the neutron during its path in the effective magnetic field should be as large as possible. We chose to make the magnet 150 cm long and to use a permanent magnet, thereby eliminating current control problems. In order to maintain a high degree of field uniformity, the height of the pole faces was chosen to be 4½ in. A gap thickness of 0.690 in. was chosen to give adequate space for the insertion of a field-measuring probe. A sketch of the magnet is shown in Fig. 4. A magnetizing winding of 25 turns of copper ribbon 5 in.×0.025 in. was wound over each section of the Alnico. A pulse of 2000 amperes served to magnetize the magnet.

An attempt was made to render the field along the beam trajectory fairly uniform and also to make it a maximum with respect to the vertical coordinate.

Shims of stainless steel, rendered partially ferromagnetic by heat treatment or cold work, were placed along the inner surface of the pole faces. While this procedure was in the main successful, a defect arose from the fact that as the shims were cut to size, the metal was cold-worked to an unusual degree along the cut edge. This, we believe, gave rise to correspondingly large permeability along the edge, which caused the sharp irregularities in the field, as shown in Fig. 5.

MAGNETIC FIELD MEASUREMENTS

Since the purpose of the experiment was to measure the ratio of the neutron to proton moments, it was necessary to measure the field in terms of the proton resonance frequency over the neutron path between the rf loops. For this purpose, we used a proton probe which was suspended from a slide which was free to move along a track mounted on the magnet. In order to align the probe so that it measured the field over the cross section of the neutron beam, a small piece of cadmium sheet was placed over the proton sample. The location of the probe was checked by making sure that the cadimum could cut off more than 90% of the neutron beam. When the neutron resonance was to be observed, the probe could be moved to one side of the gap.

The proton resonance device was a copy of that described by Thomas and Huntoon.[6] While this type is not the most sensitive, it was quite adequate for our purposes. It had the advantage over the oscillator type in that only a single setting was made both to produce the resonant frequency and to measure it. The probe was fed from an exceedingly stable, calibrated signal generator, described below.

The proton sample was contained in a glass cylinder approximately two mm in diameter and 4 mm long. It consisted of distilled water with Fe^{++} ion added (approximately $0.25M$) to cut down the relaxation time

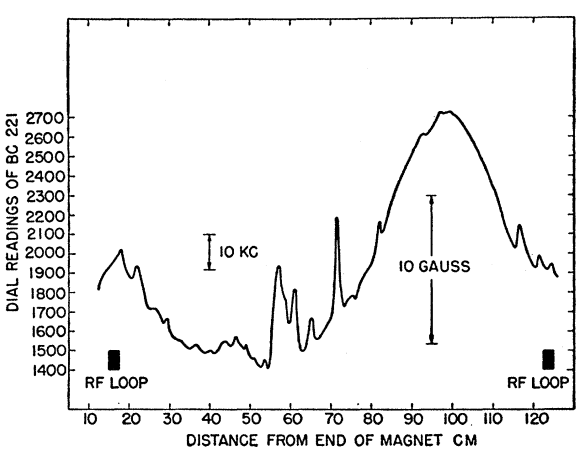

FIG. 5. Plot of the magnetic field along the beam trajectory.

[6] H. A. Thomas and R. Huntoon, Rev. Sci. Instr. **20**, 516 (1949).

so that the natural half-width of the line was comparable to the width contribution due to the field inhomogeneities. The rf tank coil was wound around the sample tube and consisted of 22 turns of No. 34 Formvar-coated copper wire. The magnetic field at the probe was modulated at a frequency of 350 cycles. The amplitude of modulation was made small compared to the line width. The audiofrequency output of the probe was amplified by a narrow-band 350-cycle amplifier and then displayed on a cathode-ray oscilloscope. The over-all arrangement was such that when one swept through resonance a large sine wave would appear on the scope, first in one phase and then reversed. The null point between the two was taken to be the resonance center.

Since Alnico V has an appreciable temperature coefficient of magnetization, it was necessary to build a heat-insulating jacket of Celotex and aluminum foil around the magnet. This served to reduce the rate of change of temperature of the magnet. However, on days when the outdoor temperature varied widely, it was impossible to keep down the field fluctuations. On favorable days, the temperature drifts as measured with a Beckmann thermometer appeared to be less than 0.005°C per hour. The effect of this temperature drift upon experimental procedure is discussed below.

rf SYSTEM

The rf signal generator was planned with the view that the errors in frequency measurement should be kept small compared to both the proton and neutron resonance line widths. It was designed and built largely by the Electronic Instrumentation Division of Brookhaven under the direction of Dr. J. B. H. Kuper and Mr. Lloyd Nevala. A block diagram is shown in Fig. 6. The heart of the system is a 100-kc crystal standard oscillator made by General Radio Company. This was compared to transmissions from Station WWV and showed a drift of less than one part in 10^7 in its fiftieth harmonic in several days. The crystal standard fed a system of multipliers, dividers, and mixers so chosen as to give one fixed frequency in the neighborhood of each of the resonances namely, 25 and 36 Mc. Each of these in turn was mixed with a variable frequency obtained from a Signal Corps frequency meter type B.C. 221. This variable component was stable to within a one cycle per second, per hour, and, consequently, so was the total frequency. The B.C. 221 variable frequency could be measured by comparison with harmonics of a 10-kc multivibrator which was controlled by the crystal standard. Interpolation between 10-kc harmonics was done with the aid of a 0–5000 cycle interpolation oscillator which was accurate to one or two cycles. The final output frequency could therefore be controlled and measured to an accuracy of two or three cycles in 25 or 36 million. In actual practice, at each setting of the signal generator during the neutron run,

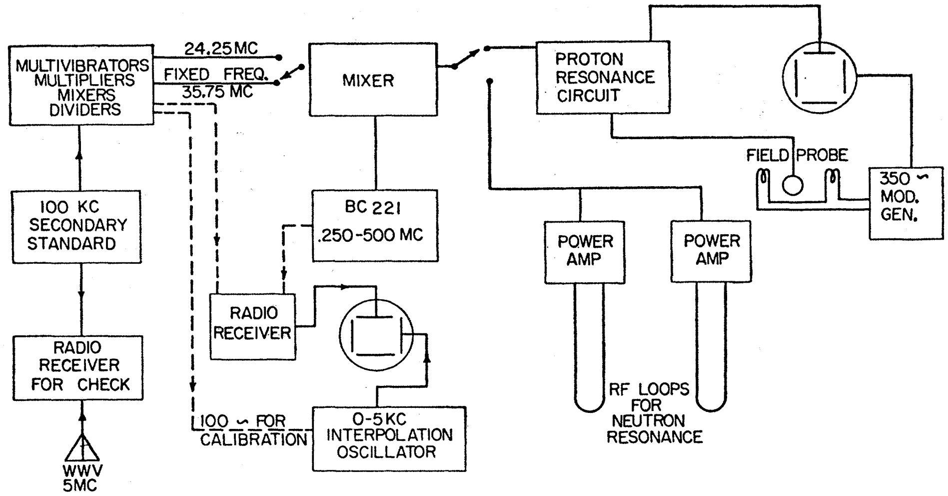

FIG. 6. Block diagram of the signal generator showing the alternate channels to the neutron loops and the magnetic field probe.

the frequency was measured precisely. In making a proton run, however, the time drift of the magnetic field made it necessary to make the observations as quickly as possible. As each resonance was observed, a reading was made in terms of the scale of the frequency meter. The readings were later translated into frequencies. In this case the precision of the calibration was of the order of 10 cycles. The signal generator could readily be switched from the neutron to the proton frequency range.

rf TRANSITION LOOPS

The principle and advantages of the separated rf transition loops have been described by Ramsey.[5] This system affords a feasible method of obtaining a much narrower resonance line than could be obtained by the single-loop technique. In the present experiment, each loop (see Fig. 7) was constructed of hollow copper tubing flattened to some extent; the loops could be water cooled. Each loop was made a part of a resonant circuit by the placing of a condenser at the upper open end of the "U." The design of the loops was also due to Dr. Kuper and Mr. Nevala. The loops were so mounted that they could be set in the region of the field which would be most advantageous for the neutron run and then be readily removed when the proton run was made. In order to monitor both amplitude and phase of the currents, each loop was fitted with an identical pickup loop. These were connected by accurately measured transmission lines to the plates of a fixed-frequency, tuned oscilloscope. The line lengths were chosen so that one introduced a 90° phase shift with respect to the other.

Each loop was fed from a separate power amplifier; both amplifiers were excited by the same signal generator. The phase of either loop could be adjusted slightly by detuning the condenser attached to it. In practice, all the neutron runs were made with the currents in the two loops out of phase by 90°. Alternate readings were taken with one loop first leading and then lagging. This was accomplished by interposing a toggle reversing switch in the line leading to one loop. The accuracy of control of the phase of the two currents was about five degrees.

PROCEDURE

The basic experimental data consist of measurements of the neutron and proton resonant frequencies. The separated loop technique is particularly advantageous in this experiment in that a single measurement gives the neutron resonant frequency averaged over the neutron path. The original separated loop technique involved the comparison of the beam intensities with the rf currents in phase and 180° out of phase in the two

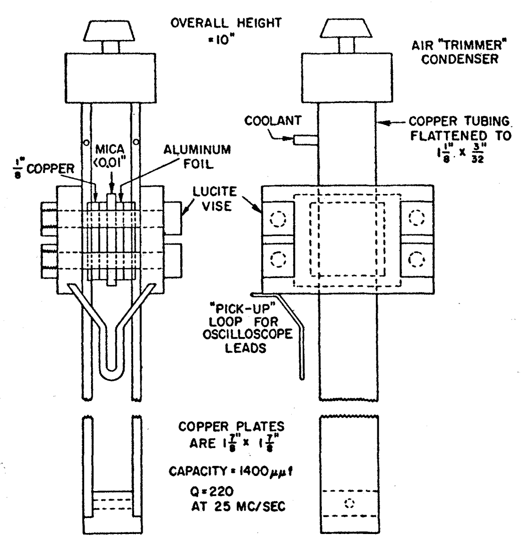

FIG. 7. Sketch of one of the neutron "flopping" loops. The loop was tuned for a resonance at 24.7 Mc/sec.

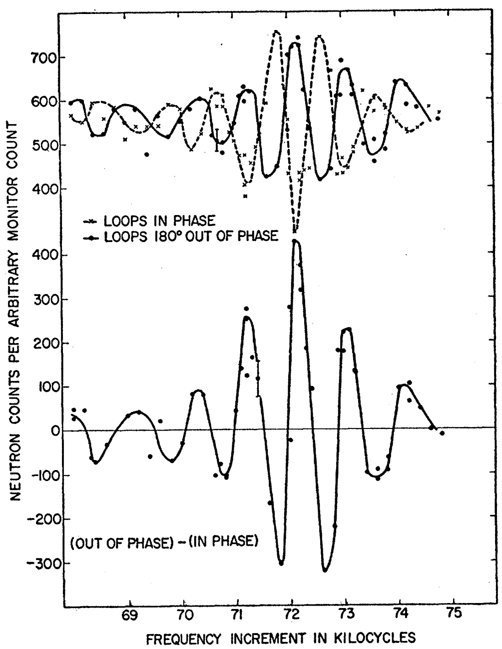

FIG. 8. Upper curves show neutron counts as a function of frequency with the loops in phase and 180° out of phase respectively. Lower curve shows the count difference between the two upper curves as a function of frequency.

loops. A plot of such differences appears as an absorption type of curve (see Fig. 8). If, however, the two loops are out of phase by 90° in one sense and the beam is compared to that with the two out of phase by 90° in the opposite sense, one obtains a dispersion type of curve with the crossover at the line center (see Fig. 9). Such a resonance curve permits an easier determination of the line center. During one typical run, we switched from absorption- to dispersion-type neutron resonance patterns, and found the line centers determined in each case to agree within a few cycles.

In practice, the frequency was set at a value near the line center and the phase and amplitude adjusted. A neutron count was then taken for 50 seconds. The reversing switch in the line feeding one loop was thrown and a similar count taken. A plot was then made of the count difference as a function of frequency. A typical sequence of neutron curves is shown in Fig. 9. The frequency differences between curves is indicative of the temperature drifts. The curve shown in Fig. 8 illustrates an extended curve. The center of the neutron line could be determined to within about ten cycles per second.

The proton resonance, on the other hand, involved a point by point measurement, each one giving an average of the field in the vicinity of the center of the probe. Since field gradients were appreciable over distances of one centimeter it was necessary to take observations one half cm apart. The total distance between rf loops was approximately 110 cm so that 220 observations would be required for a single proton run. During the time required for such a run, the temperature of the magnet would change appreciably, thereby changing the value of the magnetic field.

As a compromise between accuracy of measurement and correcting for temperature drifts, the following procedure was adopted. First an extensive sweep of the neutron resonance curve had to be taken in order to identify unambiguously the central crossover of the pattern, and then a precise measurement was made of the crossover frequency. The signal generator was then switched over to the proton frequency, the rf loops removed, and the probe put in place at one end of the magnet. A sweep was then made with the probe, taking readings at intervals of 2.5 cm. This run took about 45 minutes. Alternate neutron and proton runs were made, with the positions of each probe setting displaced 0.5 cm from the corresponding position of the previous run. Thus, after five proton runs bracketed by six neutron runs, we had a sequence of proton points spaced 0.5 cm apart and could average them in time along with the neutron runs. Typical curves showing the variation with time of approximate magnet-yoke temperature and neutron resonance frequency are shown in Fig. 10. The frequency curve is a plot of neutron resonance frequency as a function of time. The points P_1, P_2, P_3, etc., indicate the values of neutron resonance frequency at the mean times of proton runs Nos. 1, 2, 3, etc. Since the drift in field during the time of the proton run was small compared to the neutron line width there was no ambiguity in the pattern near the central crossover. It was therefore necessary to take points only in that immediate vicinity. These points were spaced about 30 cycles apart and the center could usually be interpolated to within ten cycles. On all days during which runs were made, except one, two complete sets of runs were taken. For the second set, each proton

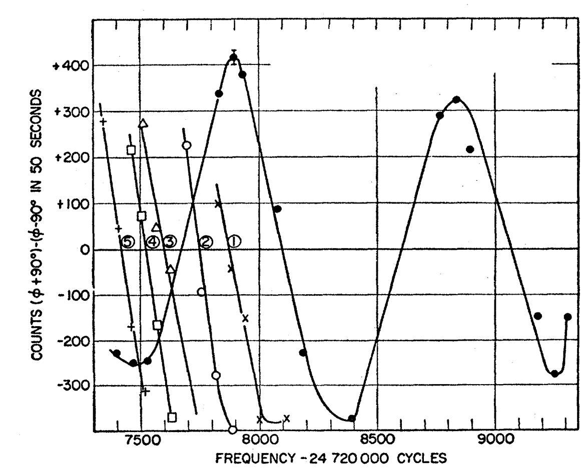

FIG. 9. Neutron resonance curve for 90° phase shift between loops. Curves designated (1), (2), (3), etc., are central sections of curves taken at later times during a series of runs.

probe position was displaced 0.25 cm from the first set. No significant difference in the average of resonant frequencies was observed.

LINE SHAPES

If all the neutrons in the beam traversed the identical average magnetic field, an absorption-type resonance curve would have the same general shape as Fig. 8. The width of the central peak would be determined by the time spent in the effective part of the magnetic field. For the velocity distribution of this experiment, the width of the central peak would be approximately 300 cycles. Suppose, now, that the neutrons of different trajectories traverse different values of the magnetic field. The resulting pattern would be a superposition of several curves. If the field variations were sufficiently great, the valleys of one curve might coincide with the peaks of another and thus wash out the pattern completely. From the curve of Fig. 8, we may conclude that the variation of average field as seen by the bulk of the neutrons is less than 150 cycles per second, although the field itself possesses the violent fluctuations shown in Fig. 5. The intensity of the side peaks relative to the central one depends upon the shape of the velocity spectrum. In our experiment, the polarized neutrons have velocities all close to the maximum or cutoff velocity (which is also, in this case, the most probable velocity). Consequently, our resonance pattern with its strong side-peaks resembles more the pattern to be expected from a partially monochromatic beam, than from a Maxwellian beam.

The proton resonance line was complicated by the variation of static magnetic field over the sample volume, the effects of a nonzero modulation frequency, and the perturbing fields of paramagnetic ions introduced into the sample to optimize the resonance line width and signal-to-noise ratio.

In our experiment, the magnetic field at the probe was swept 350 times/sec, a rate whose period was comparable with the relaxation times encountered in the samples used. In such a case, the line shape for a sample placed in a homogeneous magnetic field is considered different from the static shape, the most important difference being the appearance of satellite lines, displaced an integral number of modulation frequency intervals away from the central line. The total pattern is, however, quite symmetric about the central line, and the center of symmetry is the appropriate resonance frequency for the proton moment in the homogeneous field. We observed satellite structure on several occasions, but there was never any doubt as to the identity of the principal resonance.

The resonance pattern observed when the magnitude of the static field exhibits spatial variation over the sample is characterized as a superposition of "homogeneous" patterns, each centered about a different value of the field, H_0, and weighted with a factor that

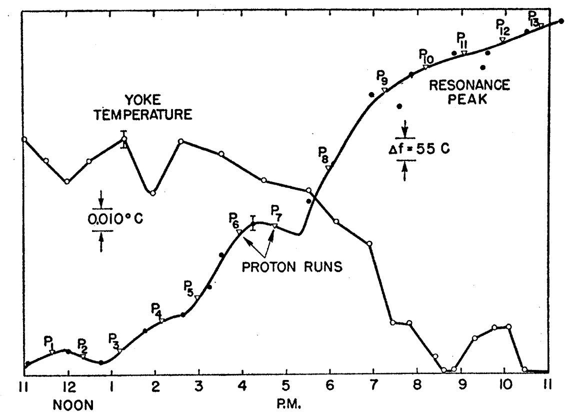

FIG. 10. Curve of changes in yoke temperature and average neutron frequency as a function of time for a typical run. Abscissae are in arbitrary units of temperature and frequency.

is proportional to the number of moments resonant in unit range of H_0. In the case of linear variation of field, the superposition is symmetrical about the field value at the center of symmetry of the probe sample, which value is the same as the field average over the region of sample. When the field exhibits an additional quadratic variation, the weighting factor is larger for the region of small absolute value of slope. The central resonance is no longer symmetrical; its maximum is no longer located at the midpoint in the range of variation of H_0, but is displaced toward the region of greater weight.

We cannot, even in the case of a probe sample in uniform field, state that the field "seen" by the protons is the same as that "seen" by the neutrons without considering the nature of the electrical environment of the proton as that particle finds itself in our probe sample. This is the question of so-called "shielding" effects.

Shielding is due primarily to diamagnetic circulation of charge in the molecule containing the resonating nucleus and to a second-order paramagnetic effect in the molecule. By making use of the calculations of Ramsey[7] and Newell,[8] one can obtain the shielding for the H_2 molecule. In addition, Thomas[9] and Gutowsky and McClure[10] have measured the shielding for H_2O relative to that for H_2. Consequently, one is able to give the neutron moment in terms of the true proton moment rather than in terms of a proton resonance in some standard molecule. It is important to recognize here that the neutron magnetic moment, in terms of the nuclear magneton, may be obtained directly—without the use of any calculated shielding corrections—by combining our uncorrected neutron-proton frequency ratio with recent measurements made at the National

[7] N. F. Ramsey, Phys. Rev. **78**, 699 (1950) and further references in N. F. Ramsey, *Nuclear Moments* (John Wiley and Sons, Inc., New York, 1953), pp. 71–77.
[8] G. F. Newell, Phys. Rev. **80**, 476 (1950).
[9] H. A. Thomas, Phys. Rev. **80**, 901 (1950).
[10] H. S. Gutowsky and R. E. McClure, Phys. Rev. **81**, 277 (1951).

Bureau of Standards.[11] The latter measurements were of the proton magnetic moment—in water and oil samples—in terms of the proton cyclotron frequency.

The analysis of shielding in our experiment is complicated somewhat by the presence of paramagnetic ions in the sample solution. These ions were introduced for the purpose of broadening the proton line to an extent comparable with that caused by variations in the external field. The increased relaxation gave one a maximum of signal to noise without significant broadening of the over-all line. The probe sample is now a magnetized paramagnetic medium and a shift in resonance line as well as a change in shape is to be expected.

We used Fe^{++} to broaden our line and in such concentrations as seemed desirable from the work of Bloembergen, Purcell, and Pound[12] on the variation of relaxation time with paramagnetic concentration. Dickinson[13] has measured the line shift for proton resonance in a solution of $FeCl_2$ for various sample geometries. Using his results, we calculate the line shift ΔH induced by the ferrous ions to be less than one part per million. In addition, a series of measurements made with paramagnetic solutions ranging from 0.01 molar to 0.1 molar revealed no appreciable change in proton resonance frequency.

RESULTS

A number of preliminary runs were made to work out the details of the technique and locate some of the sources of error. Later the final runs were made. All except one were made in sequences of eleven neutron runs bracketing 10 proton runs. The results are summarized in Table I. The data are clearly divided into two groups. Between the two the only relevant changes, so far as we know, were the movement and readjustment of the magnet system, and replacement of the water sample. The results in the first set are distributed about their mean value with an average deviation from the mean of 18 parts in 7 million. The second set has an average deviation from the mean of 11 parts in 7 million. The difference between the two averages is about 160 in 7 million or 830 cycles in the equivalent proton frequency. Clearly this indicates that there is some source of systematic error between the two sets of data, which we are unable to explain.

The sources of error which we consider significant are:

1. For the neutron resonance. (a) Statistics of neutron count; (b) Frequency measurement; (c) Errors in positioning the current loops; (d) Temperature drifts; (e) Departures from 90° phase shifts in loop currents.

2. For the proton resonance. (a) Detection of the null point of the resonance; (b) Stability of the calibration of the signal generator; (c) Accuracy of placement of the proton probe with respect to the location of the neutron beam; (d) Integration of the plot of proton resonance *vs* position along the magnet; (e) Diamagnetic and paramagnetic corrections; (f) Asymmetries in the line shape.

For the neutron resonances, it is apparent from Fig. 9 that the errors due to temperature and statistics are not greater than about ten cycles. The frequency measurement error, as discussed previously, is very small compared to all others. Slight differences in positioning the current loops for successive runs imply differences in the region over which the field is averaged by the neutrons. However, the field was shimmed in such a way that it was fairly uniform over the region about each loop and very close to the over-all average value. The error introduced by this is probably not greater than three cycles. A departure from 90° in phase between the two loops would be definitely detectable by the oscilloscope pattern if it was great enough to cause an error of 25 cycles. Considering all the above errors, it appears that the over-all error involved in the neutron resonance should average out to appreciably less than 25 cycles.

The nature of the experiment is such that we make essentially one measurement of neutron resonance but must make a large number of measurements of the proton resonance spread over space and time and then perform a numerical averaging process. In detection of the null point of the proton resonance in the region where the field was fairly uniform, the repeatability of readings was to about 20 cycles. There were about five points out of the 223 at which the uncertainty of setting might have been as high as 250 cycles. Averaged over the whole curve, these points contribute little to the total error. The calibration error of the signal generator ranged from a few cycles near the point at which it was frequently checked, to a maximum of about 20 at the extremes. The location of the proton probe with respect to the neutron beam is a possible source of systematic error. The accidental errors are extremely small since the carriage from which the probe was mounted had very little play. The location of the probe was checked with the aid of a cadmium stop mounted over the probe coil. The ability of the cadmium to stop the bulk of the neutrons seems to indicate the correctness of location of the probe. The field gradient was studied in a vertical direction and showed that an error of one mm in height would have made an error in the final result of 13 (rather than 160) in the last two figures given in the result.

There is undoubtedly some systematic error in the numerical integration of the proton resonance curve. The greatest uncertainty occurs in the region of the sharp "spikes" in the field as shown in Fig. 5. Since the source of the "spikes" is undoubtedly associated with the sheared edge of the stainless steel shims close to the

[11] Hipple, Sommer, and Thomas, Phys. Rev. **76**, 1877 (1949); **80**, 487 (1950); **82**, 697 (1951).
[12] N. Bloembergen, *Nuclear Magnetic Relaxation* (Martinus Nijhoff, The Hague, 1948), pp. 97–101.
[13] W. C. Dickinson, Phys. Rev. **81**, 717 (1951).

plane of the pole faces, one should expect that the field fluctuations in the plane of the beam, about 7 mm distant, should not be large over the length of the probe, 5 mm. In order to determine the error introduced by the plot at 5-mm intervals, a special plot was made at 2-mm intervals. The correction, when applied to the entire curve, amounted to about 40 cycles. This amount was therefore added to each average of the field as measured with 5-mm intervals.

Shifts due to paramagnetic effects have been considered in the discussion of line shapes.

Since the proton line width at most points was approximately 500 cycles, the error introduced because of asymmetry will be very small compared to this. An asymmetry effect was observed during the preliminary runs. This was manifest by a shift of the null point to the low-frequency side if the modulation amplitude were raised. The modulation was reduced to a level low enough so this effect was not over 50 cycles.

The root mean square of the errors, as discussed above, amounts to about 70 cycles in the proton frequency. The average deviation from the mean of each individual set of runs was comparable to this. It is difficult for us to understand the cause of the spread between the two sets as given above. We therefore recommend the average of the two sets with an estimated maximum error equal to the difference between the two sets. In support of the average value, we may refer to five preliminary runs made with nearly the same parameters as the final runs with the exception of: (1) 180° phase shift between the current loops, (2) a higher value of the 350-cycle modulation field. These runs give an average ratio of 0.6850578 with an average deviation of 14 in the last two places. From the known systematic error, this value should be high by roughly 20 in the same two places.

The ratio of neutron to proton resonant frequencies which we finally obtain is

$$\nu_n/\nu_p=0.685057 \pm 0.000017 \text{ as an estimated maximum error.}$$

This may be compared with the last published value, that determined by Bloch and his associates[3]:

$$\nu_n/\nu_p=0.685001\pm0.000030.$$

These values have not been corrected for molecular shielding. To obtain the neutron moment in terms of the proton moment, we must consider the effects of the diamagnetic field, δH. Synthesis of the results of Ramsey,[7] and Newell,[8] (see the section on "Line Shapes") yields $\delta H/H_0=26.2\times10^{-6}$ with an estimated error of about one percent. Our corrected ratio is, then

$$\mu_n/\mu_p=0.685039\pm0.000017.$$

TABLE I. Summary of observations.

Run no.	Curve Nos.	ν_n (cycles/sec)	ν_p (cycles/sec)	Ratio
1	(1– 5)	24 727 562	36 096 153	0.6850470
	(6–10)	24 727 314	36 095 596	505
2	(1– 5)	24 726 432	36 094 258	517
	(6–10)	24 726 056	36 093 690	520
3	(1– 5)	24 723 920	36 090 775	482
4	(1– 5)	24 726 546	36 094 667	471
	(6–10)	24 726 076	36 093 923	482
				0.6850492 av dev=18
5	(1– 5)	24 724 798	36 091 115	660
	(6–10)	24 725 296	36 091 843	660
6	(1– 5)	24 729 000	36 097 415	629
	(6–10)	24 728 956	36 097 221	654
				0.6850650 av dev=11

The neutron magnetic moment may be evaluated in terms of the nuclear magneton independently of the molecular shielding correction. By combining our ν_n/ν_p with the ratio of the proton precession to cyclotron frequencies as observed by Sommer, Thomas, and Hipple,[14] and the sign-measurement of Rogers and Staub,[15] we obtain

$$\mu_n=-1.913148\pm0.000066.$$

It is clear from the foregoing discussion that the neutron resonance measurement possesses greater precision than does the proton. Indeed, with some additional effort we believe that we can measure the center of a neutron resonance pattern to an accuracy of one part in ten million. It is in the rather clumsy method of field-averaging that we use, and in our drifting magnet temperature, that our uncertainties lie. Ideally, we would alternate neutron-beam with proton-beam experiments, using for the latter a proton-rich sample flowing through a tube in the magnet gap. Experiments in which spins in a flowing sample have been oriented and then farther along the tube their coherent motion detected, have received some attention in recent years. We hope that this technique will soon be advanced to such a point that a measurement of the neutron-proton moment ratio accurate to considerably better than one part per million can be achieved.

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[14] Sommer, Thomas, and Hipple, Phys. Rev. **82**, 697 (1951).
[15] Rogers and Staub, Phys. Rev. **76**, 980 (1949).