Pion and Muon Masses*

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MESONIC x-rays have been shown by Koslov *et al.*[1] and Stearns *et al.*[2] to provide accurately defined limits to the values of the muon and pion masses. Direct precision mass measurements have been up to now several times more accurate than that determined from the x-ray limits. Barkas *et al.*[3] have obtained π^+ and π^- masses relative to the proton mass which have been converted to electron mass units m_e:

$$M_{\pi+}=(273.34\pm0.33)m_e, \tag{1}$$

$$M_{\pi-}=(272.8\ \pm0.45)m_e. \tag{2}$$

All errors are standard deviations.

Crowe and Phillips[4] have obtained a π^- mass from the π^- capture gamma-ray spectrum in hydrogen:

$$M_{\pi-}=(272.74\pm0.40)m_e. \tag{3}$$

Barkas *et al.*[3] also have obtained an accurate measure of the decay momentum p_0 for the $\pi^+-\mu^+$ decay, and deduce an accurate $\pi-\mu$ mass difference:

$$M_{\pi+}-M_{\mu+}=(66.41\pm0.10)m_e. \tag{4}$$

The purpose of this note is to point out that if one assumes that the masses of the positive and negative pions are identical, the mass difference combined with the mesonic x-ray limits gives a much narrower limit on the pion and muon masses than heretofore available. Table I shows the limits and the combination with the mass difference. The mesonic x-ray limits shown in this table come from determining for a given mesonic x-ray line the discontinuity in the absorption coefficient as one varies the Z of an absorber placed between the x-ray source and the poor-resolution NaI detector. The break occurs when the x-ray energy is between the K-absorption edges of the absorber material.

The measured values of the K-absorption edges for some of the heavy elements have appreciable errors, which in one particular case—the lead K edge used in the μ-mesonic x-ray measurement—gave a poorly defined limit. On re-examination of the errors of the limits, it was realized that it is better to use the measured energies[5] of the L edges and the K_α lines to arrive at the K edge. The mass limits in Table I have been recalculated with these revised data.

The value and the standard deviation for the pion and muon masses as obtained from the x-ray limits and the $\pi-\mu$ difference are calculated to be

$$M_\pi=(273.34\pm0.13)m_e, \tag{5}$$

$$M_\mu=(206.93\pm0.13)m_e. \tag{6}$$

The situation with regard to the other pion mass measurements is shown graphically in Fig. 1. There is no evidence of disagreement other than that attributed to the stated errors of the data.

If all the data are combined, the best light-meson mass values are

$$M_\pi=(273.25\pm0.12)m_e, \tag{7}$$

$$M_\mu=(206.84\pm0.12)m_e. \tag{8}$$

In conclusion, we should like to caution the reader with regard to the assumptions involved in arriving at these mean values, as well as to the genuine statistical uncertainties. We have assumed the exact equality of the positive and negative masses. The accuracy of the direct measurements does not itself justify such an assumption as to the accuracy of the final result. To our

TABLE I. Mesonic x-ray limits on the pion and muon masses. All errors are standard deviations.

Transition	K-absorption edges	Vacuum polarization	Mass limit
	(a) Upper and lower limits on the pion mass		
P: $4F-3D$	above Ce, 40.440±0.006 kev	+0.100 kev	$\geq(272.2\ \pm0.03)m_e$
Al: $4F-3D$	below Sb, 30.489±0.004 kev	+0.065±0.003 kev	$\leq(273.51\pm0.04)m_e$
K: $4F-3D$	below Hf, 65.347±0.003 kev	+0.190±0.010 kev	$\leq(273.52\pm0.04)m_e$
	(b) Upper and lower limits on the muon mass		
C: $2P-1S$	below Ir, 76.123±0.015 kev	+0.38 kev	$\leq(208.95\pm0.04)m_e$
P: $3D-2P$	above Pb, 88.015±0.002 kev	+0.325±0.016 kev	$\geq(206.77\pm0.04)m_e$
Si: $4F-3D$	above Cd, 26.713±0.006 kev	+0.045 kev	$\geq(206.47\pm0.04)m_e$

π	μ	Method
(c) Summary. The π-μ difference, Eq. (4), has been used to convert pion measurements to muon values and vice versa.		
272.2 ±0.03−273.51±0.04	205.8 ±0.10−207.10±0.11	π-mesonic x-rays
273.18±0.11−275.36±0.10	206.77±0.04−208.95±0.04	μ-mesonic x-rays
	$(273.18\pm0.11)m_e\leq M_\pi\leq(273.51\pm0.04)m_e$	
	$(206.77\pm0.04)m_e\leq M_\mu\leq(207.10\pm0.11)m_e$	

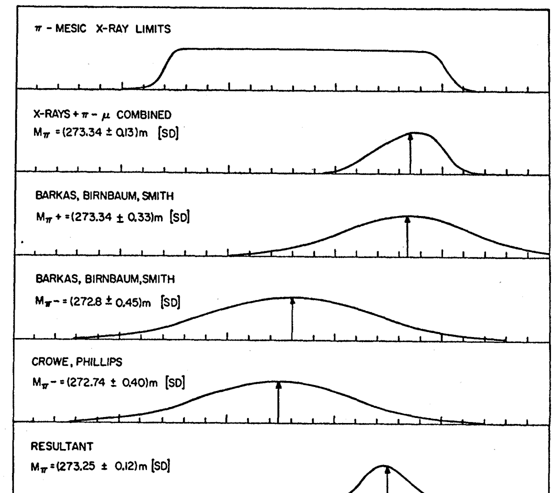

FIG. 1. Graphical summary of the data on the pion and muon mass values.

knowledge, the $\pi-\mu$ mass difference is the only appreciable source of error in the derivation of masses from the x-ray limits. However, even if the error of the $\pi-\mu$ difference were doubled, the final error would be raised only to $\pm 0.15\ m_e$. Errors in the K edges, and uncertainties in and corrections to the mesonic x-ray levels due to vacuum polarization, finite nuclear size, pion-nucleon interactions, etc., contribute a negligible error.

* A more detailed discussion will appear in Cohen, Crowe, and DuMond, *The Fundamental Constants of Physics* (Interscience Publishers, Inc., New York, to be published).

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Mass Difference of $\Sigma^\pm$ and Their Anomalous Magnetic Moments*

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THE idea that the various hyperons constitute definite isotopic multiplets is finding increasing use in the explanation of the interactions of strange particles. One would then expect that the relatively small mass difference between components of the same multiplet is due to interactions which are electromagnetic in origin. In another connection Feynman and Speisman[1] and Peterman[2] have shown that the mass difference between neutron and proton can be understood in terms of electromagnetic self-energies, if their anomalous moments are taken into account. Unlike the proton-neutron case, it will turn out that, since $\Sigma^\pm$ are both charged, a much larger mass difference is possible for comparable values of the anomalous moments.

The mass measurements on the Σ hyperon[3] indicate that

$$m(\Sigma^-)-m(\Sigma^+)=(16.2\pm 5.5)\text{ electron masses.}$$

If we assume that the Σ is a Dirac particle, we find that the observed mass difference requires that the sum of the magnetic moments of the Σ^+ and Σ^- is positive and of the order of 3 to 4 nucleon magnetons.

The self-energy contribution to the mass of a fermion of charge e and anomalous moment μ is given by (for notation see reference 1)

$$\Delta m=\frac{e^2}{(2\pi)^4 i}\frac{m}{E(p)}\bar{u}(p)\int d^4k \times\left\{\gamma_\nu-\frac{\mu}{4m}(\gamma_\nu k-k\gamma_\nu)G(k)\right\} \times\{p-k-m\}^{-1}\left\{\gamma_\nu+\frac{\mu}{4m} \times(\gamma_\nu k-k\gamma_\nu)G(k)\right\}\frac{C(k)}{k^2}u(p), \quad (1)$$

where $G(k)$ and $C(k)$ are invariant cut-off factors for the divergent integrals. Performing the indicated integration leads to an expression of the form

$$\Delta m/m=(\alpha/4\pi)\{2I_0\mp 3\mu I_1+\tfrac{3}{4}\mu^2 I_2\}, \quad (2)$$

where I_0, I_1, and I_2 are positive functions of the mass of the particle and of the cutoffs; the $\mp$ signs correspond to positively or negatively charged particles, respectively. We have chosen two typical forms of cutoff:

Type	$C(k)$	$G(k)$
(A)	$\Lambda^2/(\Lambda^2-k^2)$	$\lambda^2/(\lambda^2-k^2)$
(B)	$\Lambda^4/(\Lambda^2-k^2)^2$	$\lambda^2/(\lambda^2-k^2)$.

From (2), the mass difference of the charged Σ hyperons, can be written in the form

$$m(\Sigma^-)-m(\Sigma^+) =(\alpha m/4\pi)\{3I_1-\tfrac{3}{4}I_2(\mu^+-\mu^-)\}(\mu^++\mu^-). \quad (3)$$

We note that the sum of the anomalous moments has to be positive in order to explain the observed mass