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Question of Parity Conservation in Weak Interactions*

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The question of parity conservation in β decays and in hyperon and meson decays is examined. Possible experiments are suggested which might test parity conservation in these interactions.

RECENT experimental data indicate closely identical masses[1] and lifetimes[2] of the $\theta^+(\equiv K_{\pi 2}{}^+)$ and the $\tau^+(\equiv K_{\pi 3}{}^+)$ mesons. On the other hand, analyses[3] of the decay products of τ^+ strongly suggest on the grounds of angular momentum and parity conservation that the τ^+ and θ^+ are not the same particle. This poses a rather puzzling situation that has been extensively discussed.[4]

One way out of the difficulty is to assume that parity is not strictly conserved, so that θ^+ and τ^+ are two different decay modes of the same particle, which necessarily has a single mass value and a single lifetime. We wish to analyze this possibility in the present paper against the background of the existing experimental evidence of parity conservation. It will become clear that existing experiments do indicate parity conservation in strong and electromagnetic interactions to a high degree of accuracy, but that for the weak interactions (i.e., decay interactions for the mesons and hyperons, and various Fermi interactions) parity conservation is so far only an extrapolated hypothesis unsupported by experimental evidence. (One might even say that the present $\theta-\tau$ puzzle may be taken as an indication that parity conservation is violated in weak interactions. This argument is, however, not to be taken seriously because of the paucity of our present knowledge concerning the nature of the strange particles. It supplies rather an incentive for an examination of the question of parity conservation.) To decide unequivocally whether parity is conserved in weak interactions, one must perform an experiment to determine whether weak interactions differentiate the right from the left. Some such possible experiments will be discussed.

PRESENT EXPERIMENTAL LIMIT ON PARITY NONCONSERVATION

If parity is not strictly conserved, all atomic and nuclear states become mixtures consisting mainly of the state they are usually assigned, together with small percentages of states possessing the opposite parity. The fractional weight of the latter will be called $\mathfrak{F}^2$. It is a quantity that characterizes the degree of violation of parity conservation.

The existence of parity selection rules which work well in atomic and nuclear physics is a clear indication that the degree of mixing, $\mathfrak{F}^2$, cannot be large. From such considerations one can impose the limit $\mathfrak{F}^2 \lesssim (r/\lambda)^2$, which for atomic spectroscopy is, in most cases, $\sim 10^{-6}$. In general a less accurate limit obtains for nuclear spectroscopy.

Parity nonconservation implies the existence of interactions which mix parities. The strength of such interactions compared to the usual interactions will in general be characterized by $\mathfrak{F}$, so that the mixing will be of the order $\mathfrak{F}^2$. The presence of such interactions would affect angular distributions in nuclear reactions. As we shall see, however, the accuracy of these experiments is not good. The limit on $\mathfrak{F}^2$ obtained is not better than $\mathfrak{F}^2 < 10^{-4}$.

To give an illustration, let us examine the polarization experiments, since they are closely analogous to some experiments to be discussed later. A proton beam polarized in a direction z perpendicular to its momentum was scattered by nuclei. The scattered intensities were compared[5] in two directions A and B related to each other by a reflection in the x–y plane, and were found to be identical to within $\sim 1\%$. If the scattering originates from an ordinary parity-conserving interaction plus a parity-nonconserving interaction (e.g., $\boldsymbol{\sigma}\cdot\mathbf{r}$), then the scattering amplitudes in the directions A and B are in the proportion $(1+\mathfrak{F})/(1-\mathfrak{F})$, where $\mathfrak{F}$ represents the ratio of the strengths of the two kinds of interactions in the scattering. The experimental result therefore requires $\mathfrak{F} < 10^{-2}$, or $\mathfrak{F}^2 < 10^{-4}$.

The violation of parity conservation would lead to an electric dipole moment for all systems. The magnitude of the moment is

$$\text{moment} \sim e\mathfrak{F} \times (\text{dimension of system}). \qquad (1)$$

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[1] Whitehead, Stork, Perkins, Peterson, and Birge, Bull. Am. Phys. Soc. Ser. II, **1**, 184 (1956); Barkas, Heckman, and Smith, Bull. Am. Phys. Soc. Ser. II, **1**, 184 (1956).

[2] Harris, Orear, and Taylor, Phys. Rev. **100**, 932 (1955); V. Fitch and K. Motley, Phys. Rev. **101**, 496 (1956); Alvarez, Crawford, Good, and Stevenson, Phys. Rev. **101**, 503 (1956).

[3] R. Dalitz, Phil. Mag. **44**, 1068 (1953); E. Fabri, Nuovo cimento **11**, 479 (1954). See Orear, Harris, and Taylor [Phys. Rev. **102**, 1676 (1956)] for recent experimental results.

[4] See, e.g., *Report of the Sixth Annual Rochester Conference on High Energy Physics* (Interscience Publishers, Inc., New York, to be published).

[5] See, e.g., Chamberlain, Segrè, Tripp, and Ypsilantis, Phys. Rev. **93**, 1430 (1954).

The presence of such electric dipole moments would have interesting consequences. For example, if the proton has an electric dipole moment $\cong e\times(10^{-16}$ cm), the perturbation caused by the presence of the neighboring $2p$ state of the hydrogen atom would shift the energy of the $2s$ state by about 1 Mc/sec. This would be inconsistent with the present theoretical interpretations of the Lamb shift. Another example is found in the electron-neutron interaction. An electric dipole moment for the neutron $\cong e\times(10^{-18}$ cm) is the upper limit allowable by the present experiments.

By far the most accurate measurement of the electric dipole moment was made by Purcell, Ramsey, and Smith. They gave[6] an upper limit for the electric dipole moment of the neutron of $e\times(5\times10^{-20}$ cm). This value sets the upper limit for $\mathfrak{F}^2$ as $\mathfrak{F}^2<3\times10^{-13}$, which is also the most accurate verification of the conservation of parity in strong and electromagnetic interactions. We shall see, however, that even this high degree of accuracy is not sufficient to supply an experimental proof of parity conservation in the weak interactions. For such a proof an accuracy of $\mathfrak{F}^2<10^{-24}$ is necessary.

QUESTION OF PARITY CONSERVATION IN β DECAY

At first sight it might appear that the numerous experiments related to β decay would provide a verification that the weak β interaction does conserve parity. We have examined this question in detail and found this to be not so. (See Appendix.) We start by writing down the five usual types of couplings. In addition to these we introduce the five types of couplings that conserve angular momentum but do not conserve parity. It is then apparent that the classification of β decays into allowed transitions, first forbidden, etc., proceeds exactly as usual. (The mixing of parity of the *nuclear states* would not measurably affect these selection rules. This phenomenon belongs to the discussions of the last section.) The following phenomena are then examined: allowed spectra, unique forbidden spectra, forbidden spectra with allowed shape, β-neutrino correlation, and $\beta-\gamma$ correlation. It is found that these experiments have no bearing on the question of parity conservation of the β-decay interactions. This comes about because in all of these phenomena no interference terms exist between the parity-conserving and parity-nonconserving interactions. In other words, the calculations always result in terms proportional to $|C|^2$ plus terms proportional to $|C'|^2$. Here C and C' are, respectively, the coupling constants for the usual parity-conserving interactions (a sum of five terms) and the parity-nonconserving interactions (also a sum of five terms.) Furthermore, it is well known[7] that without measuring the spin of the neutrino we cannot distinguish the couplings C from the couplings C' (provided the mass of the neutrino is zero). The experimental results concerning the above-named phenomena, which constitute the bulk of our present knowledge about β decay, therefore cannot decide the degree of mixing of the C' type interactions with the usual type.

The reason for the absence of interference terms CC' is actually quite obvious. Such terms can only occur as a pseudoscalar formed out of the experimentally measured quantities. For example, if three momenta $\mathbf{p}_1$, $\mathbf{p}_2$, $\mathbf{p}_3$ are measured, the term $CC'\mathbf{p}_1\cdot(\mathbf{p}_2\times\mathbf{p}_3)$ may occur. Or if a momentum $\mathbf{p}$ and a spin $\boldsymbol{\sigma}$ are measured, the term $CC'\mathbf{p}\cdot\boldsymbol{\sigma}$ may occur. In all the β-decay phenomena mentioned above, no such pseudoscalars can be formed out of the measured quantities.

POSSIBLE EXPERIMENTAL TESTS OF PARITY CONSERVATION IN β DECAYS

The above discussion also suggests the kind of experiments that could detect the possible interference between C and C' and consequently could establish whether parity conservation is violated in β decay. A relatively simple possibility is to measure the angular distribution of the electrons coming from β decays of oriented nuclei. If θ is the angle between the orientation of the parent nucleus and the momentum of the electron, an asymmetry of distribution between θ and $180°-\theta$ constitutes an unequivocal proof that parity is not conserved in β decay.

To be more specific, let us consider the allowed β transition of any oriented nucleus, say Co^{60}. The angular distribution of the β radiation is of the form (see Appendix):

$$I(\theta)d\theta=(\text{constant})(1+\alpha\cos\theta)\sin\theta d\theta, \qquad (2)$$

where α is proportional to the interference term CC'. If $\alpha\neq0$, one would then have a positive proof of parity nonconservation in β decay. The quantity α can be obtained by measuring the fractional asymmetry between $\theta<90°$ and $\theta>90°$; i.e.,

$$\alpha=2\left[\int_0^{\pi/2}I(\theta)d\theta-\int_{\pi/2}^{\pi}I(\theta)d\theta\right]\Big/\int_0^{\pi}I(\theta)d\theta.$$

It is noteworthy that in this case the presence of the magnetic field used for orienting the nuclei would automatically cause a spatial separation between the electrons emitted with $\theta<90°$ and those with $\theta>90°$. Thus, this experiment may prove to be quite feasible.

It appears at first sight that in the study of γ-radiation distribution from β-decay products of oriented nuclei one can form a pseudoscalar from the spin of the oriented nucleus and the γ-ray momentum $\mathbf{p}_\gamma$. Thus it may seem to offer another possible experimental test of parity conservation. Unfortunately, the nuclear levels have definite parities, and electromagnetic inter-

[6] E. M. Purcell and N. F. Ramsey, Phys. Rev. **78**, 807 (1950); Smith *et al.* as quoted in N. F. Ramsey, *Molecular Beams* (Oxford University Press, London, 1956).
[7] C. N. Yang and J. Tiomno, Phys. Rev. **79**, 495 (1950).

actions conserve parity. (Any small mixing of parities characterized by $\mathfrak{F}^2 < 3\times 10^{-15}$ would not affect the arguments here.) Consequently the γ rays carry away definite parities. Thus the observed probability function must be an even function of $\mathbf{p}_\gamma$. This property eliminates the possibility of forming a pseudoscalar quantity. It is therefore not possible to use such experiments as a test of parity conservation.

In β-γ-γ' triple correlation experiments one can, by some rather similar but more complicated reasoning, prove that a measurement of the three momenta cannot supply any information on the question of parity conservation in β decay.

In β-γ correlation experiments the nature of the polarization of the γ ray could provide a test. To be more specific, let us consider the polarization state of γ rays emitted parallel to the β ray. If parity conservation holds for β decay, the γ ray will be unpolarized. On the other hand, if parity conservation is violated in β decay, the γ ray will in general be polarized. However, this polarization must be circular in nature and therefore may not lend itself to easy experimental detection. (The usual ways of measuring polarization through Compton effect, photoelectric effect, and photodisintegration of the deuteron are all incapable of detecting circular polarization. This is because circular polarization is specified by an axial vector parallel to the direction of propagation. From the observed momenta in these detection techniques such an axial vector cannot be formed.) For other directions of γ-ray propagation, elliptical polarization will result if parity is not conserved. This effect will thus be more difficult to detect.

QUESTION OF PARITY CONSERVATION IN MESON AND HYPERON DECAYS

If the weak interactions, such as the β-decay interactions or the decay interactions of mesons and hyperons, do not conserve parity, parity mixing will occur in all interactions by means of second-order processes. To examine this effect let us consider, for example, the decay of the Λ^0:

$$\Lambda^0 \rightarrow p + \pi^-.$$

The assumption that parity is not conserved in this decay implies that the Λ^0 exists virtually in states of opposite parities. It could therefore possess an electric dipole moment of a magnitude

$$\text{moment} \sim e\mathcal{G}^2 \times (\text{dimension of } \Lambda^0), \tag{3}$$

where $\mathcal{G}$ is the coupling strength of the decay interaction of the Λ^0. ($\mathcal{G}^2 \lesssim 10^{-12}$.) The electric dipole moment of the Λ^0 is therefore $\lesssim e\times(10^{-25}$ cm).

Clearly the proton would have an electric dipole moment of the same order of magnitude. The existence of such a small electric dipole moment is, as we have seen, completely consistent with the present experimental information. Another way of putting this is to observe that by comparing Eq. (3) with Eq. (1), one has

$$\mathfrak{F} \sim \mathcal{G}^2.$$

Since all the weak interaction including β interactions are characterized by coupling strengths $\mathcal{G}^2 < 10^{-12}$, a violation of parity in weak interactions would introduce a parity mixing characterized by an $\mathfrak{F}^2 < 10^{-24}$. This is outside the present limit of experimental knowledge, as we have discussed before.

If the weak interactions violate parity conservation, parity would be defined and measured in strong and electromagnetic interactions only, just as strangeness is. Furthermore it is important to notice that with the conservation of strangeness, as with every conservation law, there is an element of arbitrariness introduced into the parity of all systems. The parity of all strange particles would be defined only up to a factor of $(-1)^S$, where S is the strangeness. The parity of the Λ^0 (relative to the nucleons) is therefore a matter of definition. But once this is defined, the parity of other strange particles would be measurable from the strong interactions.

POSSIBLE EXPERIMENTAL TESTS OF PARITY CONSERVATION IN MESON AND HYPERON DECAYS

To have a sensitive unequivocal test of whether parity is conserved in weak interactions, one must decide whether the weak interactions differentiate between the right and the left. This is possible only if one produces interference between states of opposite parities. The mere observation of two decay products of opposite parities originating from a "particle" cannot provide conclusive evidence that parity is not conserved. Such indeed is the state of affairs of the present $\theta - \tau$ puzzle.

As we have discussed before, these interference terms are possible only if the observed quantities can form a pseudoscalar such as $\mathbf{p}_1 \cdot (\mathbf{p}_2 \times \mathbf{p}_3)$. The observation of Λ^0 decays in association with their production does provide such a possible pseudoscalar and hence a possible test of whether parity is conserved in the Λ^0 decay interaction. Let us consider the experiment

$$\pi^- + p \rightarrow \Lambda^0 + \theta^0, \quad \Lambda^0 \rightarrow p + \pi^-. \tag{4}$$

Let $\mathbf{p}_{\text{in}}$, $\mathbf{p}_\Lambda$, and $\mathbf{p}_{\text{out}}$ be, respectively, the momenta in the laboratory system of the incoming pion, the Λ^0, and the decay pion. We define a parameter R as the projection of $\mathbf{p}_{\text{out}}$ in the direction of $\mathbf{p}_{\text{in}} \times \mathbf{p}_\Lambda$. The value of R ranges from approximately -100 Mev/c to approximately $+100$ Mev/c. Switching from a right-handed convention for vector products (which we use) to a left-handed convention means a switch of the sign of R. Parity conservation in the weak decay interaction of Λ^0 can therefore be experimentally checked by investigating whether $+R$ and $-R$ have equal probabilities of occurrence.

To see more clearly the meaning of the parameter R, one transforms $\mathbf{p}_{\text{out}}(\rightarrow\mathbf{p}')$ into the center-of-mass system of Λ^0. The new vector $\mathbf{p}'$ has a constant magnitude $\cong 100$ Mev/c. The frequency distribution of this vector $\mathbf{p}'$ can then be plotted on a spherical surface. Taking the z axis for this sphere to be in the direction of $\mathbf{p}_{\text{in}}\times\mathbf{p}_{\Lambda}$, one can prove the following two symmetries:

(*a*) The frequency distribution on the sphere remains unchanged under a rotation through 180° around the z axis. This symmetry follows from parity conservation in the strong reaction producing the Λ^0. It does not depend on the nature of the weak interaction.

(*b*) If parity is conserved in the decay interaction of Λ^0, the frequency distribution on the sphere is unchanged under a reflection with respect to the production plane of Λ^0.

To prove statement (*a*), one need only consider the invariance of the production process under a reflection with respect to the production plane defined by $\mathbf{p}_{\text{in}}$ and $\mathbf{p}_{\Lambda}$. This reflection is the resultant of an inversion and a rotation through 180° around the z axis (which is normal to the production plane). The state of polarization of Λ^0 is thus invariant under a 180° rotation around the z axis, leading to the stated symmetry.[8]

Statement (*b*) follows[8] directly from the assumption that the weak interaction as well as the strong interaction conserves parity. A reflection with respect to the production plane must then leave the whole process invariant.

The frequency distribution of R is just the projection of the distribution on the sphere onto the z axis. An asymmetry between $+R$ and $-R$ therefore implies parity nonconservation in Λ^0 decay. However, if the spin of Λ^0 is unpolarized, no asymmetry[9] can obtain even if parity is not conserved in Λ^0 decay. To obtain a polarized Λ^0 beam, the experiment is therefore best done at a definite nonforward angle of production of Λ^0 and at a definite incoming energy.

The above discussions apply also to any other strange particle decay if (1) the particle has a nonvanishing spin and (2) it decays into two particles at least one of which has a nonvanishing spin, or it decays into three or more particles. Thus the above considerations can be applied also to the decays of $\Sigma^{\pm}$ and *maybe* also to $K_{\mu 2}^{\pm}$, $K_{\mu 3}^{\pm}$ and $K_{\pi 3}^{\pm}$ ($\equiv\tau^{\pm}$).

In the decay processes

$$\pi\rightarrow\mu+\nu, \tag{5}$$

$$\mu\rightarrow e+\nu+\nu, \tag{6}$$

starting from a π meson at rest, one could study the distribution of the angle θ between the μ-meson momentum and the electron momentum, the latter being in the center-of-mass system of the μ meson. If parity is conserved in neither (5) nor (6), the distribution will not in general be identical for θ and $\pi-\theta$. To understand this, consider first the orientation of the muon spin. If (5) violates parity conservation, the muon would be in general polarized in its direction of motion. In the subsequent decay (6), the angular distribution problem with respect to θ is therefore closely similar to the angular distribution problem of β rays from oriented nuclei, which we have discussed before. (Entirely similar considerations can be applied to $\Xi^-\rightarrow\Lambda^0+\pi^-$ and $\Lambda^0\rightarrow p+\pi^-$.)

REMARKS

If parity conservation is violated in hyperon decay, the decay products will have mixed parities. This, however, does not affect the arguments of Adair[10] and of Treiman[11] concerning the relationship between the spin of the hyperons and the angular distribution of their decay products in certain special cases.[12]

One may question whether the other conservation laws of physics could also be violated in the weak interactions. Upon examining this question, one finds that the conservations of the number of heavy particles, of electric charge, of energy, and of momentum all appear to be inviolate in the weak interactions. The same cannot be said of the conservation of angular momentum, and of parity. Nor can it be said of the invariance under time reversal. It might appear at first sight that the equality of the life times of $\pi^{\pm}$ and of those of $\mu^{\pm}$ furnish proofs of the invariance under charge conjugation of the weak interactions. A closer examination of this problem reveals, however, that this is not so. In fact, the equality of the life times of a charged particle and its charge conjugate against decay through a weak interaction (to the lowest order of the strength of the weak interaction) can be shown to follow from the invariance under proper Lorentz transformations (i.e., Lorentz transformation with neither space nor time inversion). One has therefore at present no experimental proof of the invariance under charge conjugation of the weak interactions. In the present paper, only the question of parity nonconservation is discussed.

[8] This proof for statement (*a*) is correct only if Λ^0 exists as a single particle with a definite parity in the strong interactions, (as discussed in the last section); i.e. if Λ^0 does not exist as two degenerate states Λ_1^0 and Λ_2^0 of opposite parity, as has been suggested [T. D. Lee and C. N. Yang, Phys. Rev. **102**, 290 (1956)]. [It is to be emphasized, that if parity is indeed not conserved in the weak interactions, there would be (at present) no necessity to introduce the complication of two degenerate states of opposite parity at all.] On the other hand, statement (*b*) is correct even if Λ^0 exists as two degenerate states Λ_1^0 and Λ_2^0 of opposite parity. *To summarize, violation of the symmetry stated in* (*a*) *implies the existence of the parity doublets* Λ_1^0 *and* Λ_2^0 *with a mass difference less than their widths. Violation of the symmetry stated in* (*b*) *implies the nonconservation of parity in* Λ *decay.* See also footnote 12 and T. D. Lee and C. N. Yang, Phys. Rev. (to be published).

[9] Also the interference may accidentally be absent if the relative phase between the two parities in the decay product is 90°. This, however, cannot be the case if time-reversal invariance is preserved in the decay process.

[10] R. K. Adair, Phys. Rev. **100**, 1540 (1955).

[11] S. B. Treiman, Phys. Rev. **101**, 1216 (1956).

[12] The existence of Λ_1^0 and Λ_2^0 of opposite parity may affect these relationships. This is similar to the violation of symmetry (*a*) discussed in footnote 8. See T. D. Lee and C. N. Yang, Phys. Rev. (to be published).

The conservation of parity is usually accepted without questions concerning its possible limit of validity being asked. There is actually no *a priori* reason why its violation is undesirable. As is well known, its violation implies the existence of a right-left asymmetry. We have seen in the above some possible experimental tests of this asymmetry. These experiments test whether the present elementary particles exhibit asymmetrical behavior with respect to the right and the left. If such asymmetry is indeed found, the question could still be raised whether there could not exist corresponding elementary particles exhibiting opposite asymmetry such that in the broader sense there will still be over-all right-left symmetry. If this is the case, it should be pointed out, there must exist two kinds of protons p_R and p_L, the right-handed one and the left-handed one. Furthermore, at the present time the protons in the laboratory must be predominantly of one kind in order to produce the supposedly observed asymmetry, and also to give rise to the observed Fermi-Dirac statistical character of the proton. This means that the free oscillation period between them must be longer than the age of the universe. They could therefore both be regarded as stable particles. Furthermore, the numbers of p_R and p_L must be separately conserved. However, the interaction between them is not necessarily weak. For example, p_R and p_L could interact with the same electromagnetic field and perhaps the same pion field. They could then be separately pair-produced, giving rise to interesting observational possibilities.

In such a picture the supposedly observed right-and-left asymmetry is therefore ascribed not to a basic non-invariance under inversion, but to a cosmologically local preponderance of, say, p_R over p_L, a situation not unlike that of the preponderance of the positive proton over the negative. Speculations along these lines are extremely interesting, but are quite beyond the scope of this note.

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APPENDIX

If parity is not conserved in β decay, the most general form of Hamiltonian can be written as

$$\begin{aligned}H_{\text{int}}= &(\psi_p^\dagger\gamma_4\psi_n)(C_S\psi_e^\dagger\gamma_4\psi_\nu+C_S'\psi_e^\dagger\gamma_4\gamma_5\psi_\nu)\\ &+(\psi_p^\dagger\gamma_4\gamma_\mu\psi_n)(C_V\psi_e^\dagger\gamma_4\gamma_\mu\psi_\nu+C_V'\psi_e^\dagger\gamma_4\gamma_\mu\gamma_5\psi_\nu)\\ &+\tfrac{1}{2}(\psi_p^\dagger\gamma_4\sigma_{\lambda\mu}\psi_n)(C_T\psi_e^\dagger\gamma_4\sigma_{\lambda\mu}\psi_\nu\\ &+C_T'\psi_e^\dagger\gamma_4\sigma_{\lambda\mu}\gamma_5\psi_\nu)+(\psi_p^\dagger\gamma_4\gamma_\mu\gamma_5\psi_n)\\ &\times(-C_A\psi_e^\dagger\gamma_4\gamma_\mu\gamma_5\psi_\nu-C_A'\psi_e^\dagger\gamma_4\gamma_\mu\psi_\nu)\\ &+(\psi_p^\dagger\gamma_4\gamma_5\psi_n)(C_P\psi_e^\dagger\gamma_4\gamma_5\psi_\nu+C_P'\psi_e^\dagger\gamma_4\psi_\nu),\end{aligned}\quad\text{(A.1)}$$

where $\sigma_{\lambda\mu}=-\frac{1}{2}i(\gamma_\lambda\gamma_\mu-\gamma_\mu\gamma_\lambda)$ and $\gamma_5=\gamma_1\gamma_2\gamma_3\gamma_4$. The ten constants C and C' are all real if time-reversal invariance is preserved in β decay. This however, will not be assumed in the following.

Calculation with this interaction proceeds exactly as usual. One obtains, e.g., for the energy and angle distribution of the electron in an allowed transition

$$\begin{aligned}N(W,\theta)dW\sin\theta d\theta=&\frac{\xi}{4\pi^3}F(Z,W)pW(W_0-W)^2\\ &\times\left(1+\frac{ap}{W}\cos\theta+\frac{b}{W}\right)dW\sin\theta d\theta,\end{aligned}\quad\text{(A.2)}$$

where

$$\begin{aligned}\xi=&(|C_S|^2+|C_V|^2+|C_S'|^2+|C_V'|^2)|M_{\text{F.}}|^2\\ &+(|C_T|^2+|C_A|^2+|C_T'|^2+|C_A'|^2)|M_{\text{G.T.}}|^2,\end{aligned}\quad\text{(A.3)}$$

$$\begin{aligned}a\xi=&\tfrac{1}{3}(|C_T|^2-|C_A|^2+|C_T'|^2-|C_A'|^2)|M_{\text{G.T.}}|^2\\ &-(|C_S|^2-|C_V|^2+|C_S'|^2-|C_V'|^2)|M_{\text{F.}}|^2,\end{aligned}\quad\text{(A.4)}$$

$$\begin{aligned}b\xi=&\gamma[(C_S^*C_V+C_SC_V^*)+(C_S'^*C_V'+C_S'C_V'^*)]|M_{\text{F.}}|^2\\ &+\gamma[(C_T^*C_A+C_A^*C_T)+(C_T'^*C_A'+C_A'^*C_T')]\\ &\times|M_{\text{G.T.}}|^2.\end{aligned}\quad\text{(A.5)}$$

In the above expression all unexplained notations are identical with the standard notations. (See, e.g., the article by Rose.[13])

The above expression does not contain any interference terms between the parity-conserving part of the interactions and the parity-nonconserving ones. It is in fact directly obtainable by replacing in the usual expression the quantity $|C_S|^2$ by $|C_S|^2+|C_S'|^2$, and $C_SC_V^*$ by $C_SC_V^*+C_S'C_V'^*$, etc. This rule also holds in general, except for the cases where a pseudoscalar can be formed out of the measured quantities, as discussed in the text.

When a pseudoscalar can be formed, for example, in the β decay of oriented nuclei, interference terms would be present, as explicitly displayed in Eq. (2). In an allowed transition $J\to J-1$ (no), the quantity α is given by

$$\alpha=\beta\langle J_z\rangle/J,$$

$$\begin{aligned}\beta=\text{Re}&\left[C_TC_T'^*-C_AC_A'^*+i\frac{Ze^2}{\hbar cp}(C_AC_T'^*+C_A'C_T^*)\right]\\ &\times|M_{\text{G.T.}}|^2\frac{v_e}{c}\frac{2}{\xi+(\xi b/W)},\end{aligned}\quad\text{(A.6)}$$

where $M_{\text{G.T.}}$, ξ, and b are defined in Eqs. (A.3)–(A.5), v_e is the velocity of the electron, and $\langle J_z\rangle$ is the average spin component of the initial nucleus. For an allowed transition $J\to J+1$ (no), α is given by

$$\alpha=-\beta\langle J_z\rangle/(J+1).\quad\text{(A.7)}$$

The effect of the Coulomb field is included in all the above considerations.

[13] M. E. Rose, in *Beta- and Gamma-Ray Spectroscopy* (Interscience Publishers, Inc., New York, 1955), pp. 271–291.