larger gap shifts than those of Fig. 3. The maximum slope values at $B \approx 100\,000$ gauss give apparent mass values of $\sim 0.023 m_0$ for InSb and $\sim 0.041 m_0$ for InAs from the relation $\Delta\epsilon/\Delta B = he/2m^*$.

The mass increase with energy[2,3] suggests a possible energy momentum relation $\epsilon = p^2/2m^* + \alpha p^4 + \cdots$, containing higher order terms. Using this as a first approximation for the Hamiltonian and following the prescription of Luttinger and Kohn,[4] we obtain by perturbation theory[5]

$$\epsilon_n \approx L + 4\alpha m^{*2} L^2 + \cdots, \quad L = (n+\tfrac{1}{2})heB/m^* + \hbar^2 k_z^2/2m^*.$$

This suggests a decrease in the rate of gap change with B, since from cyclotron resonance α is negative. Hence, other effects such as possible perturbation of the Landau levels by ionized donors should be considered. The impurity Bohr orbit is smaller than the Landau orbit at low magnetic fields and larger at high fields. Perhaps this accounts for the larger slope of the magneto-gap shift at higher fields. The nonlinear magneto-gap shift should modify the theory of other magneto-band effects, such as the longitudinal magnetoresistance,[6] freeze-out,[7] density of states and effect of magnetic field on electron scattering.[6]

Obviously the change in the energy gap affects the electron density and also the scattering,[6] leading to longitudinal magnetoresistance. Analogous magneto-band effects should occur in semimetals such as Bi or Zn with overlapping bands. In high fields of the order of 300 000 gauss, one may create an energy gap of several hundredths ev with B along an appropriate axis.

A preliminary observation of magneto-band effect on the direct transition in Ge has also been made up to 35 000 gauss.[8] This should exceed the shift of the indirect transition because theory indicates a small mass of about[9] $0.034 m_0$ in the conduction band at $k=0$. Fine structure, probably associated with transitions from both degenerate valence bands, was also observed.

Additional experiments are now in progress to investigate the effects of impurity concentration on the magneto-gap shift in both InSb and InAs.

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Thermal Equilibrium in Nuclear Magnetic Cooling of Metals*

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RECENTLY Kurti, Robinson, Simon, and Spohr[1] have reported the attainment of a temperature of 20 μdeg K by nuclear magnetic cooling of metallic copper from an electronic cooling stage at 0.01°K, about 1% of the nuclear entropy being removed during isothermal magnetization at the latter temperature. The purpose of the present letter is to discuss the rate at which the conduction electrons and lattice phonons initially at 0.01°K come into thermal equilibrium with the nuclear spins at 20 μdeg. We shall see by a solution of the relaxation equations that equilibrium may be established in a time of the order of 1 sec, despite a nuclear relaxation time of the order of 10^5 sec at the lower temperature. Casimir[2] was the first to realize that the relaxation time does not tell the whole story in spin-lattice equilibrium problems.

We first consider the equilibrium between the conduction electron (e) and nuclear spin (s) systems; the phonons do not add significantly to the entropy. We write for the electronic heat capacity

$$C_e = (\pi^2/2)Nk(T/T_F); \quad (T_F \cong 10^5\ ^\circ\mathrm{K}), \tag{1}$$

and for the nuclear heat capacity of spins on a face-centered cubic lattice, after Van Vleck and Waller,[3]

$$C_s = 2.40Nk(\tau_m/T)^2, \quad (T \gg \tau_m), \tag{2}$$

where τ_m is the magnetic interaction temperature. Now $\delta Q_e = -\delta Q_s$, so that for small temperature variations

$$\frac{\delta T_s}{\delta T_e} = -\frac{C_e(T_e)}{C_s(T_s)} = -\frac{\pi^2}{4.8}\frac{T_s^2 T_e}{T_F \tau_m^2}; \tag{3}$$

this ratio is of the order of 10^{-5} immediately after nuclear demagnetization, if one takes $\tau_m \approx 10^{-6}$ °K as found by the Oxford workers. At any stage of the cooling process the temperatures may be found by integration of (3). The final change ΔT_s to equilibrium is, approximately,

$$\Delta T_s = \pi^2 T_e^2(0) T_s^2(0)/(9.6 T_F \tau_m^2) \approx 10^{-7}\ ^\circ\mathrm{K}, \tag{4}$$

so that in the isolated specimen the conduction electrons are cooled to essentially the temperature of the nuclear spins after demagnetization, under the particu-

lar conditions of the experiment discussed. If $T_e(0)$ had been 1°K, the final temperature would have been near 1°K.

Before equilibrium is attained the difference in populations n of the nuclear spin system decays by the equation

$$dn/dT=-(n-n_0)/T_1. \quad (5)$$

Here T_1 is the spin-conduction electron relaxation time for the Korringa process; the excess populations are pictured in terms of a fictitious magnetic field which would give an interaction temperature τ_m. We write, with b a constant of the order of one,

$$n=b\tau_m/T_s; \quad n_0=b\tau_m/T_e; \quad T_1T_e=\alpha; \quad (6)$$

and find, from Eq. (5),

$$\frac{dT_e}{dt}=\frac{4.8T_F\tau_m^2}{\pi^2\alpha}\left(\frac{1}{T_e}-\frac{1}{T_s}\right). \quad (7)$$

Using (3) we find, in an approximation valid for the Oxford experiment over most of the approach to equilibrium,

$$T_e(t)=T_e(0)\{1-t[4.8T_F\tau_m^2/\alpha\pi^2T_e(0)T_s(0)]\}. \quad (8)$$

Thus the time t_0 to establish approximate equilibrium between the nuclear and conduction electron systems may be written as

$$t_0\cong T_1(T_s)C_e(T_e)/C_s(T_s), \quad (9)$$

where the temperatures are taken at the time $t=0$. For copper $T_eT_1\cong 1$ sec deg according to Redfield[4]; for $T_e=0.01$ deg and $T_s=10^{-5}$ deg, we have $t_0\cong 1$ sec. Equilibrium under these conditions between the conduction electron and nuclear spin systems may be established quite rapidly. We may say that because of the low entropy of the conduction electrons, only a very small proportion of the nuclear spins have to relax to produce equilibrium; this consideration explains why $t_0\ll T_1$.

Order-of-magnitude considerations suggest that under the conditions of the experiment discussed, the phonons equilibrate with the conduction electrons in a time less than the time t_0 required for the conduction electrons to equilibrate with the nuclear spins. At low temperatures the average time between electron-phonon (low-angle) collisions will be $\tau_1\approx(\Theta/T_p)^3\tau_0$, where τ_0 is the conductivity relaxation time at the Debye temperature Θ, and T_p is the phonon temperature. The collision rate of conduction electrons is $\approx N_{\text{eff}}/\tau_1$, where $N_{\text{eff}}\approx NT_e/T_F$. In the average collision the conduction electron absorbs energy $\approx k(T_p-T_e)$; thus the phonon energy changes at the rate

$$\frac{dU_p}{dt}\approx 234Nk\left(\frac{T_p}{\Theta}\right)^3\frac{dT_p}{dt}\approx\frac{NkT_e}{\tau_0T_F}\left(\frac{T_p}{\Theta}\right)^3(T_e-T_p). \quad (10)$$

We may define a characteristic equilibration time t_{ep} for the electron-phonon equilibrium by considering the extreme case in which T_e is maintained at 10^{-5} °K. Then

$$t_{ep}\approx 234\tau_0T_F/T_e\approx 0.1 \text{ sec}, \quad (11)$$

if one uses $\tau_0\approx 3\times10^{-14}$ sec for Cu. This grossly overestimates the equilibration time, and we may safely say that T_p follows T_e closely. Finally, the *electron* spin temperature should equilibrate to the translational temperature T_e in the microsecond range (in specimens of normal purity) by the Elliott spin-orbit mechanism.

The rather high value of τ_m reported in the Oxford work might be caused by nuclear quadrupole interactions with lattice strains; paramagnetic impurities; or an anomalously high value[4] of the Fröhlich-Nabarro or Ruderman-Kittel indirect exchange interactions.

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Errata

Application of the Rayleigh-Schrödinger Perturbation Theory to the Hydrogen Atom, RICHARD E. TREES [Phys. Rev. **102**, 1553 (1956)]. Contrary to the statement made in footnote 6, the method of A. Dalgarno and J. T. Lewis can be used to sum the second-order perturbation. I am also indebted to Dr. Charles Schwartz for a communication showing a method of evaluating this perturbation rigorously.

Neutron-Induced Charged Particle Reactions in Potassium-39, MARY JEAN SCOTT AND RALPH E. SEGEL [Phys. Rev. **102**, 1557 (1956)]. The first sentence of the note added in proof should read: "Penning, Maltrud, Hopkins, and Schmidt [Bull. Am. Phys. Soc. Ser. II, **1**, 162 (1956)] have recently investigated the β decay of Cl^{39}." instead of ". . . the β decay of A^{39}."

Galactic Radio Emission and the Energy Released in Nuclear Collisions of Primary Cosmic-Ray Protons, G. R. BURBIDGE [Phys. Rev. **103**, 264 (1956)]. A typographical error appeared in the seventh line from the end of the article: It should read "Thus, for example, if $E=500$ Mev . . ." instead of "Thus, for example, if $E=5000$ Mev . . ."