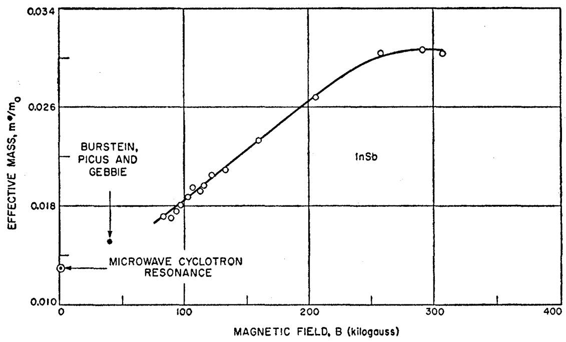

FIG. 2. Variation of effective mass with magnetic field in InSb from reflection experiments between $\lambda=10\,\mu$ and $\lambda=22\,\mu$. The magnetic field was perpendicular to the surface of the sample.

are planned to check this value and possible variation of effective mass with magnetic field.

A typical reflection trace for Bi is shown in Fig. 1(d), and m^* *versus* B is shown in Fig. 3. Two resonance peaks are observed when the magnetic field is along the [11$\bar{2}$0] direction; however, only one mass is observed along the [10$\bar{1}$0] direction. The mass values of Fig. 3 are identified with the values of $0.009m_0$ along [10$\bar{1}$0] and $0.008m_0$ and $0.016m_0$ along [11$\bar{2}$0] as calculated[6] from Shoenberg's de Haas-van Alphen data.[7] The observed increases in mass with B are again presumably due to changes in curvature of the bands with energy. The variation of the cyclotron resonance masses above and below these calculated masses probably represents transitions between Landau levels above and below the Fermi level at high and low magnetic fields, respectively. The de Haas-van Alphen masses correspond to those at the Fermi level.

Preliminary cyclotron resonance absorption effects have been observed in zinc and graphite. Further work is required before quantitative results can be obtained.

The samples of InSb and InAs were pure n-type material, with mobilities of $\sim$75 000 and $\sim$25 000, respectively, at 300°K, giving calculated values of $\omega\tau$ $\approx$100 at 10 μ. These materials were generously provided by T. C. Harman of Battelle Memorial Institute.

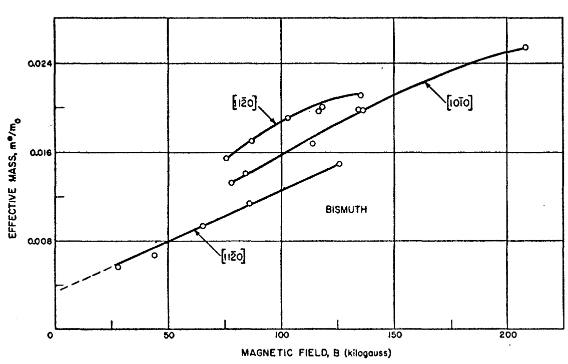

FIG. 3. Variation of effective masses with magnetic field in bismuth from reflection experiments. The magnetic field was perpendicular to the surface and parallel to the indicated crystal directions.

We wish to acknowledge technical assistance by H. Lipson and P. Kelley. We profited greatly from stimulating discussions with E. Burstein, G. S. Picus, H. Brooks, W. Paul, D. M. Warschauer, A. C. Beer, H. J. Zeiger, and K. J. Button.

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Magneto-band Effects in InAs and InSb in dc and High Pulsed Magnetic Fields*

S. ZWERDLING, R. J. KEYES, S. FONER, H. H. KOLM, AND BENJAMIN LAX
Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Massachusetts
(Received October 29, 1956)

In a magnetic field semiconductors and semimetals with small effective masses have quantized Landau levels whose energy separations can become comparable to the energy gap or overlap of the bands, respectively. Displacement of the lowest conduction and highest valence band levels in a semiconductor would increase the energy gap. This magneto-gap effect was first

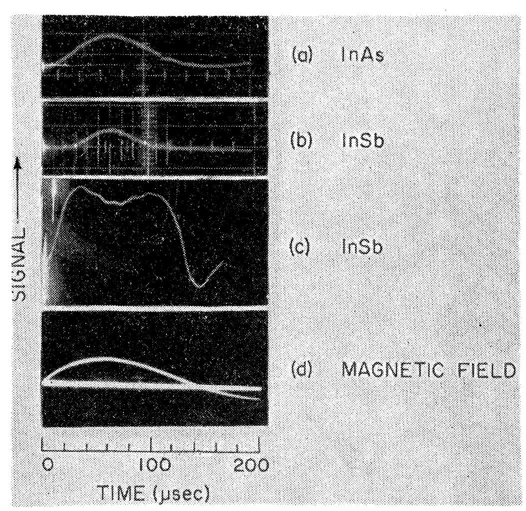

FIG. 1. Transmission signal in (a) InAs at $\lambda=3.55\,\mu$ and $B_{max}=150$ kilogauss, (b) InSb at $\lambda=5.99\,\mu$ and $B_{max}=220$ kilogauss, (c) InSb at $\lambda=7.55\,\mu$ and $B_{max}=220$ kilogauss, showing dip in trace due to onset of cyclotron resonance. The magnetic field trace *vs* time is shown in curve (d).

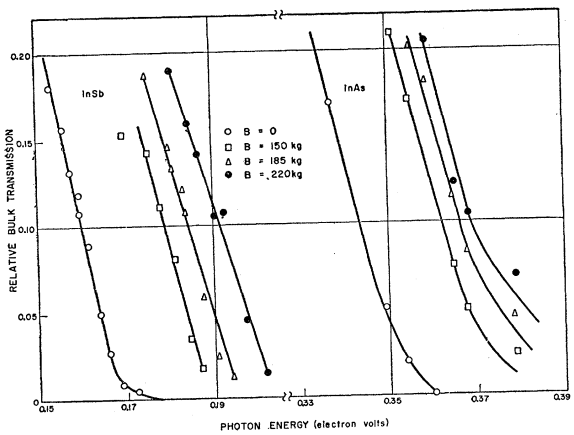

FIG. 2. Band edge curves for InSb (sample B) and InAs *vs* photon energy for three values of magnetic field.

observed by Burstein and co-workers[1] in InSb up to 60 000 gauss. We have made similar measurements with dc fields up to 35 000 gauss and also extended them to 220 000 gauss with pulsed fields, in both InSb and InAs.

Figure 1 shows photographic traces of the infrared transmission at the foot of the band edge of each semiconductor obtained with the pulsed system used for cyclotron resonance.[2] Correlating the signal amplitude of these and other traces at nearby wavelengths with the corresponding calibrated magnetic field traces, the magneto-band curves shown in Figs. 2 and 3 were obtained. Figure 2 shows the band edge of both materials as a function of the photon energy for three values of the magnetic field. Figure 3 shows the magneto-gap shift *versus* magnetic field near the short-wavelength region of each band edge. For InSb this reduces the possible error due to the onset of resonance absorption, which is detectable simultaneously with the magneto-gap transmission at its long-wavelength portion and with fields in the vicinity of 250 000 gauss, as shown in Fig. 1(c). The bending of the gap shift curve for InSb at high fields may be due partly to this. Curves for samples A and B in Fig. 3 differ, possibly due to difference in impurity concentration as evidenced by the difference in the band edge curve at $B=0$.

A dc magnet was used to obtain more accurate data up to 35 000 gauss. The average slope, $\Delta\epsilon/\Delta B$, found for InSb between 15 000 and 35 000 gauss was 1.7×10^{-7} ev/gauss, which agrees roughly with the value of 2.3 $\times10^{-7}$ ev/gauss obtained by Burstein *et al.*[1] The average slope obtained for InAs in this range of fields was 9.3×10^{-8} ev/gauss. These results were obtained with 200 μ thick samples, intrinsic at room temperature.

The nonlinearity of the gap shift curve at low fields (up to $\sim$100 000 gauss) for both semiconductors cannot be reconciled with the simple Landau model, according to which the pertinent levels can be represented by the expression $\hbar eB/2m^*$. Even if the valence band is neglected and the low-field cyclotron resonance masses are used in the Landau formula, the theory predicts

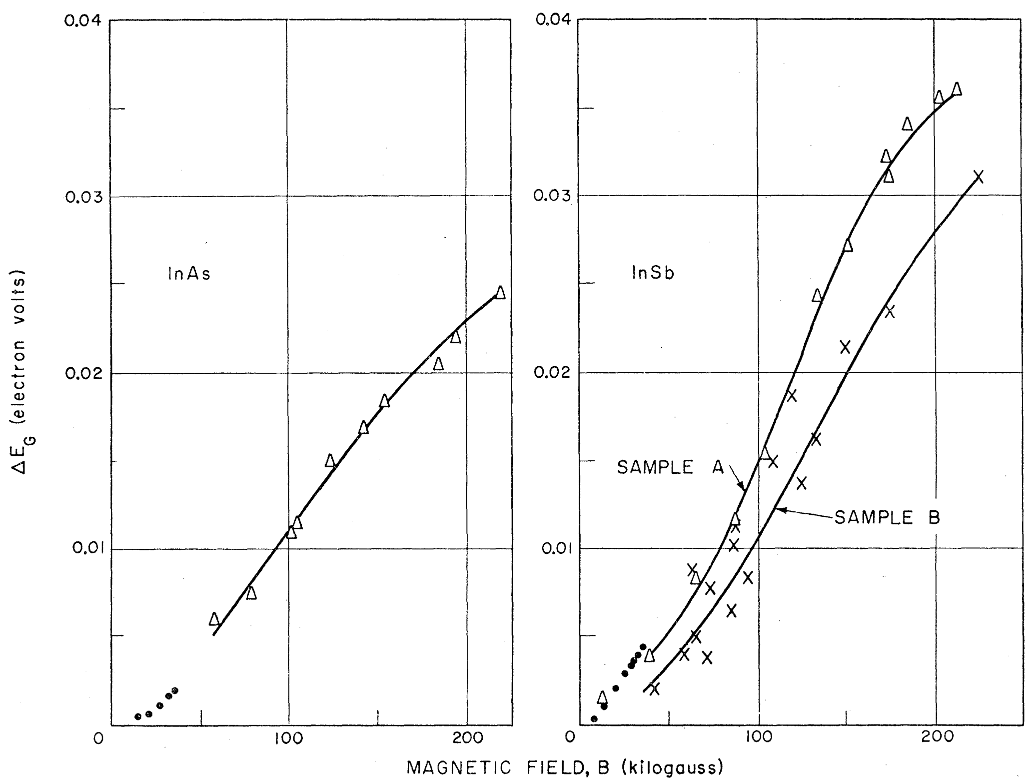

FIG. 3. Magneto-gap shift as a function of magnetic field. Solid lines connect data obtained from pulsed magnetic field measurements; dots represent dc field measurements.

larger gap shifts than those of Fig. 3. The maximum slope values at $B \approx 100\,000$ gauss give apparent mass values of $\sim 0.023 m_0$ for InSb and $\sim 0.041 m_0$ for InAs from the relation $\Delta\epsilon/\Delta B = \hbar e/2m^*$.

The mass increase with energy[2,3] suggests a possible energy momentum relation $\epsilon = p^2/2m^* + \alpha p^4 + \cdots$, containing higher order terms. Using this as a first approximation for the Hamiltonian and following the prescription of Luttinger and Kohn,[4] we obtain by perturbation theory[5]

$$\epsilon_n \approx L + 4\alpha m^{*2} L^2 + \cdots, \quad L = (n+\tfrac{1}{2})\hbar e B/m^* + \hbar^2 k_z^2/2m^*.$$

This suggests a decrease in the rate of gap change with B, since from cyclotron resonance α is negative. Hence, other effects such as possible perturbation of the Landau levels by ionized donors should be considered. The impurity Bohr orbit is smaller than the Landau orbit at low magnetic fields and larger at high fields. Perhaps this accounts for the larger slope of the magneto-gap shift at higher fields. The nonlinear magneto-gap shift should modify the theory of other magneto-band effects, such as the longitudinal magnetoresistance,[6] freeze-out,[7] density of states and effect of magnetic field on electron scattering.[6]

Obviously the change in the energy gap affects the electron density and also the scattering,[6] leading to longitudinal magnetoresistance. Analogous magneto-band effects should occur in semimetals such as Bi or Zn with overlapping bands. In high fields of the order of 300 000 gauss, one may create an energy gap of several hundredths ev with B along an appropriate axis.

A preliminary observation of magneto-band effect on the direct transition in Ge has also been made up to 35 000 gauss.[8] This should exceed the shift of the indirect transition because theory indicates a small mass of about[9] $0.034 m_0$ in the conduction band at $k=0$. Fine structure, probably associated with transitions from both degenerate valence bands, was also observed.

Additional experiments are now in progress to investigate the effects of impurity concentration on the magneto-gap shift in both InSb and InAs.

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Thermal Equilibrium in Nuclear Magnetic Cooling of Metals*

C. Kittel

Department of Physics, University of California, Berkeley, California

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RECENTLY Kurti, Robinson, Simon, and Spohr[1] have reported the attainment of a temperature of 20 μdeg K by nuclear magnetic cooling of metallic copper from an electronic cooling stage at 0.01°K, about 1% of the nuclear entropy being removed during isothermal magnetization at the latter temperature. The purpose of the present letter is to discuss the rate at which the conduction electrons and lattice phonons initially at 0.01°K come into thermal equilibrium with the nuclear spins at 20 μdeg. We shall see by a solution of the relaxation equations that equilibrium may be established in a time of the order of 1 sec, despite a nuclear relaxation time of the order of 10^5 sec at the lower temperature. Casimir[2] was the first to realize that the relaxation time does not tell the whole story in spin-lattice equilibrium problems.

We first consider the equilibrium between the conduction electron (e) and nuclear spin (s) systems; the phonons do not add significantly to the entropy. We write for the electronic heat capacity

$$C_e = (\pi^2/2)Nk(T/T_F); \quad (T_F \cong 10^5\ ^\circ\text{K}), \tag{1}$$

and for the nuclear heat capacity of spins on a face-centered cubic lattice, after Van Vleck and Waller,[3]

$$C_s = 2.40 Nk(\tau_m/T)^2, \quad (T \gg \tau_m), \tag{2}$$

where τ_m is the magnetic interaction temperature. Now $\delta Q_e = -\delta Q_s$, so that for small temperature variations

$$\frac{\delta T_s}{\delta T_e} = -\frac{C_e(T_e)}{C_s(T_s)} = -\frac{\pi^2}{4.8}\frac{T_s^2 T_e}{T_F \tau_m^2}; \tag{3}$$

this ratio is of the order of 10^{-5} immediately after nuclear demagnetization, if one takes $\tau_m \approx 10^{-6}$ °K as found by the Oxford workers. At any stage of the cooling process the temperatures may be found by integration of (3). The final change ΔT_s to equilibrium is, approximately,

$$\Delta T_s = \pi^2 T_e^2(0) T_s^2(0)/(9.6 T_F \tau_m^2) \approx 10^{-7}\ ^\circ\text{K}, \tag{4}$$

so that in the isolated specimen the conduction electrons are cooled to essentially the temperature of the nuclear spins after demagnetization, under the particu-

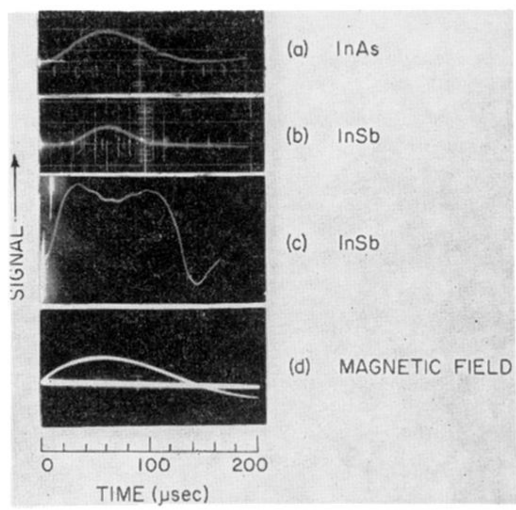

FIG. 1. Transmission signal in (a) InAs at $\lambda = 3.55\ \mu$ and $B_{\text{max}} = 150$ kilogauss, (b) InSb at $\lambda = 5.99\ \mu$ and $B_{\text{max}} = 220$ kilogauss, (c) InSb at $\lambda = 7.55\ \mu$ and $B_{\text{max}} = 220$ kilogauss, showing dip in trace due to onset of cyclotron resonance. The magnetic field trace *vs* time is shown in curve (d).