Pion Effects on Fermi Interactions

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Fermi interactions are examined from the standpoint that the interaction between bare nucleons and light fermions may have simple form. The relation of this proposal to the usual expression of Fermi interactions in terms of physical nucleons is evaluated. This relation is specified fairly definitely by current meson theories for allowed β-decay processes. The possibility of a simple universal Fermi interaction and the relation of associated allowed and forbidden β-decay matrix elements are examined. Although some interesting conclusions can be drawn in these cases, the effects do not appear observable at the present time.

1. INTRODUCTION

THE general theory of Fermi processes must, at present, be reconsidered in the light of newly observed Fermi processes in strange-particle decay. But before these new considerations appear, it is perhaps of interest to attempt to complete the study of the processes with respect to nonstrange particles. In particular, we would like here to study the effect of pion-nucleon interactions on the Fermi processes.[1] Our motivation is the possibility that Fermi interactions of nucleons in the absence of pions, i.e., the Fermi interactions of bare nucleons, may have a simple form. Consider the physical nucleon to be made up of a bare nucleon and a pion cloud. Assume that Fermi interactions see directly the bare nucleon. Direct interaction with the pion cloud is assumed negligible as are nucleon pair effects. Thus, the pion cloud is effective only in a geometrical way, i.e., it carries some of the total spin and charge of the physical nucleon (we shall not consider the role of the electromagnetic field analogous to this role of the pion field,[2] such effects should be of order α). It is proposed then that the light fermion fields measure properties (particularly charge and spin) of the bare particle in distinction to the usual situation, i.e., with the electromagnetic field, where we can only observe properties of the nucleon as a whole, the bare particle being only a concept of theoretical interest. We shall discuss two models of the physical nucleon and evaluate the relation between Fermi couplings to the bare particles and the same couplings as conventionally expressed in terms of the physical nucleons. We shall discuss possible application of these results to a universal Fermi interaction, to the relation between allowed and forbidden transitions in β decay, and to a universal Fermi interaction explanation of pion decay.

We can immediately discern some general properties of the proposal. The Fermi processes are usually discussed in terms of couplings of the form (e.g., β decay)

$$H=\sum_i g_i'\int dx(\bar{\psi}_e'(x)\Gamma_i'\psi_\nu'(x))(\bar{\psi}_p'(x)\Gamma_i'\psi_n'(x))+\text{c.c.},\tag{1.1}$$

where $(\bar{\psi}_a'\Gamma_i'\psi_b')$ for $i=1,\cdots,5$ are the covariant quantities: scalar, tensor, vector, axial vector, and pseudoscalar. The operators Γ_i' are usually limited to Dirac γ matrixes. As conventionally employed, the fields ψ_n', ψ_p' and operator Γ_i' are taken to refer to physical nucleons, those essentially involved in any phenomenological description of nuclei, where there is no explicit reference to the pion field. We want to consider the Hamiltonian H when it is written in terms of bare particles:

$$H=\sum_i g_i\int dx(\bar{\psi}_e(x)\Gamma_i\psi_\nu(x))(\bar{\psi}_p(x)\Gamma_i\psi_n(x))+\text{c.c.};\tag{1.2}$$

i.e., here ψ_n, ψ_p, Γ_i concern bare nucleons. (Unprimed operators should be considered as acting on bare particles in the following.) It is seen that matrix elements of (1.1) will involve

$$g_i'\langle p_0|\Gamma_i|n_0\rangle,\tag{1.3}$$

where $|n_0\rangle$ is the wave function of a bare neutron (this is just to say, for example, that $\langle p|\sigma'|n\rangle=\langle p_0|\sigma|n_0\rangle$, where $|n\rangle$, $|p\rangle$, and σ' are wave functions and operators for the nucleons as a whole). Meanwhile matrix elements of (1.2) will involve

$$g_i\langle p|\Gamma_i|n\rangle.\tag{1.4}$$

Now let us consider starting with a simple form for (1.2) and evaluating the conventional form (1.1) for H. The Γ_i's in (1.2) are assumed to be formed from γ matrixes (it is also convenient to include the appropriate isotopic spin operator). What will (1.1) look like in terms of the g_i's? Since the form of the light particle matrix element is completely unaffected, the relativistic transformation properties of the nucleon matrix element of (1.1) will be the same as in (1.2). Thus, corresponding to the scalar term in (1.2) there will only be scalar

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[1] T. Kotani *et al.*, Progr. Theoret. Phys. (Japan) **6**, 1007 (1951); R. J. Finklestein and S. A. Moszkowski, Phys. Rev. **95**, 1695 (1954). *Note added in proof.*—See also S. S. Gerschtein and Ia. B. Zel'dovich, Soviet Physics (JETP) **2**, 576 (1956); B. Stech, Z. Physik **145**, 319 (1956).

[2] See Behrends, Finklestein, and Sirlin, Phys. Rev. **101**, 866 (1956).

terms in (1.1), but we may expect terms of form

$$\alpha g_S\left(\bar{\psi}_p'\left[\frac{\partial}{\partial x_\mu}\gamma_\mu\tau^+\right]'\psi_n'\right),$$

in addition to the usual

$$\beta g_S(\bar{\psi}_p'\tau^{+\prime}\psi_n')$$

[in the notation of (1.1)]. The new matrix element terms are such that they relative magnitude (v/c), where v is nucleon velocity. Our interest is then to determine the relation of the coupling constant $g'=\beta g_i$ of each term in the conventional form (1.1) of the interaction.

2. Infinite-Mass Model

In order to evaluate (1.4) we need some information about the physical nucleon in terms of the bare nucleon and meson cloud. Let us first consider the currently popular infinite-mass nucleon model. In this model we can only evaluate nuclear matrix elements such as those corresponding to allowed β-decay processes, i.e., for scalar and vector terms in (1.2) the nucleon matrix element becomes

$$g_{S,V}\langle p|\tau^+|n\rangle,$$

and for tensor and axial vector it is

$$g_{T,A}\langle p|\boldsymbol{\sigma}\tau^+|n\rangle.$$

In order to evaluate these matrix elements, only the total angular momentum and isotopic spin of the meson cloud and no detailed features, need to be described.[3] Thus, we can write

$$|m,n\rangle = a_{00}f_{00}(x)\chi^m\xi^n + a_{11}f_{11}(x)\boldsymbol{\sigma}\cdot\mathbf{r}\ \boldsymbol{\tau}\cdot\mathbf{t}\chi^m\xi^n + a_{10}[f_{10}(x)\boldsymbol{\sigma}\cdot\mathbf{r}+f_{01}\boldsymbol{\tau}\cdot\mathbf{t}]\chi^m\xi^n, \quad (2.1)$$

where $|m,n\rangle$ is the wave function of a physical nucleon with a z component of angular momentum m and of isotopic spin n; χ, ξ are spin $\frac{1}{2}$ functions of the bare nucleons for mechanical and isotopic spin, respectively; and $\mathbf{r}$, $\mathbf{t}$ are unit vectors in space and isotopic spin space, respectively. The a_{00} term corresponds to a pion cloud carrying zero angular momentum and isotopic spin (which includes the no-pion state), the a_{11} term corresponds to one unit of angular momentum and isotopic spin for the cloud, and the a_{10} terms to mixed total angular momentum and isotopic spin. The $f_i(x)$ are normalized functions containing the internal variables of the pion cloud. Spin and isotopic spin enter with complete symmetry in the infinite mass model. Letting $|a_{10}|^2=P_{10}$, $|a_{11}|^2=P_{11}$, $|a_{00}|^2=1-P_{11}-2P_{10}$ we then have the results for g'/g:

$$R=\frac{\langle p|\boldsymbol{\tau}|n\rangle}{\langle p_0|\boldsymbol{\tau}|n_0\rangle}=1-\frac{4}{3}P_{11}-\frac{4}{3}P_{10},$$
$$R_\sigma=\frac{\langle p|\boldsymbol{\tau}\cdot\boldsymbol{\sigma}|n\rangle}{\langle p_0|\boldsymbol{\tau}\cdot\boldsymbol{\sigma}|n_0\rangle}=1-\frac{8}{9}P_{11}-\frac{8}{3}P_{10}. \quad (2.2)$$

With the assumption that P_{10} is small, these quantities are plotted in Fig. 1.

There are numerous meson theoretical estimates of P_{11} (note that R_σ is just the ratio f_r/f of renormalized to unrenormalized pion coupling constants). In approximate treatments so far considered, $P_{10}=0$. The Chew Tamm-Damcoff theory[4] yields $P_{11}\approx 0.6$. The most recent intermediate-coupling calculation[5] yields $P_{11}\approx 0.7$. Exact sum rules over total sections derived via the Chew-Low-Miyazawa methods[6] yield[7] $P_{11}\approx 0.5$ and $P_{10}\approx 0.1$, deviating only slightly from the approximate treatments. Sachs' phenomenological treatment[3] yields $P_{11}+P_{10}=0.09$. If we want $g_T/g_S=\pm 1$ assuming $|g_T'/g_S'|=1.25$ from β decay, then $P_{11}=0.32$ or $P_{11}=0.88$. It is seen that for more likely values of P_{11} the ratio g_S/g_T might be much greater than one. More elaborate application of these results will be attempted on the next section.

3. APPLICATION TO UNIVERSAL FERMI INTERACTION

We wish to consider a universal theory[8,9] of the three Fermi processes

$$n\rightarrow p+e^-+\nu, \quad (3.1)$$

$$p+\mu^-\rightarrow n+\nu, \quad (3.2)$$

$$\mu^\pm\rightarrow e^\pm+2\nu, \quad (3.3)$$

and also, possibly, the pion decay.[10] We want to find first if some simple combination of coupling constants g_i can characterize all three processes. First let us state the problem exactly. The processes (3.1)–(3.3) are usually discussed in terms of couplings of form such as (1.1):

$$\sum_i g_i(\bar{\psi}_a\Gamma_i\psi_b)(\bar{\psi}_c\Gamma_i\psi_d)+\text{c.c.} \quad (3.4)$$

[3] For a discussion of the general features of this model, see R. G. Sachs, Phys. Rev. **87**, 1100 (1952).

[4] G. F. Chew, Phys. Rev. **95**, 1669 (1954).

[5] Friedman, Lee, and Christian, Phys. Rev. **100**, 1494 (1956).

[6] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1570 (1956); H. Miyazawa, Phys. Rev. **101**, 1564 (1956).

[7] Cini, Fubini, and Thirring, in *Proceedings of the Sixth Annual Rochester Conference on High-Energy Physics* [Interscience Publishers, Inc., New York (to be published)].

[8] See, for example, L. Michel, in *Progress in Cosmic-Ray Physics*, edited by J. G. Wilson (Interscience Publishers, Inc., New York, 1952) for early references.

[9] J. Tiomno and J. Wheeler, Revs. Modern Phys. **21**, 153 (1949); E. R. Caianello, Nuovo cimento **8**, 534, 749 (1951); **9**, 336 (1952); D. L. Pursey, Physica **18**, 1017 (1952); D. C. Peaslee, Phys. Rev. **91**, 1447 (1953); R. Finklestein and P. Kaus, Phys. Rev. **92**, 1316 (1953); E. J. Konopinski and H. Mahmoud, Phys. Rev. **92**, 1045 (1953); B. Stech and J. H. Jensen, Z. Physik **141**, 403 (1955); J. Tiomno, Nuovo cimento **1**, 226 (1955).

[10] M. Ruderman and R. Finklestein, Phys. Rev. **76**, 1458 (1949); M. Ruderman, Phys. Rev. **85**, 157 (1952); S. Treiman and H. Wyld, Phys. Rev. **101**, 1552 (1956).

One can make various assumptions of universality, for example, (*a*) setting coupling constants g_i associated with one foursome (*a*, *b*, *c*, *d*) of fermions, equal to the coupling constants associated with another foursome, or (*b*) suggesting that *all* pairs of fermions are coupled (various processes being forbidden by general selection rules). There are, unfortunately, ambiguities with respect to ordering the four fields in the Hamiltonian (3.4). One must make specific assumptions about the orderings in order to discuss a universality of type (*a*). Let us review briefly the results concerning these ambiguities. We follow Konopinski and Mahmoud[9] who showed that one can adopt the convention with complete generality that two "particles" (as distinguished from antiparticles) are created and two destroyed in all the Fermi couplings. For example, β decay proceeds as follows:

$$(n+\nu)\rightarrow(p+e^-). \tag{3.5}$$

We shall use parentheses to indicate use of this "normal ordering." Having adopted this convention, one makes a physical statement on deciding which of $\mu^\pm$ is the "particle" (relative to e^-). If one chooses μ^-, i.e.,

$$(p+\mu^-)\rightarrow(n+\nu), \tag{3.6}$$

then under a universality such as (*b*) above one cannot prevent the process

$$(p+\mu^-)\rightarrow(p+e^-). \tag{3.7}$$

An intermediate boson field explanation of the Fermi processes also lead to (3.7) from (3.5) and (3.6). We shall in the following assume that μ^+ is the "particle",[11] i.e.,

$$(p+\nu)\rightarrow(n+\mu^+). \tag{3.8}$$

There still remains a further ambiguity of ordering, i.e., in β decay we could consider Hamiltonians:

$$\sum g_i(\bar\psi_p\Gamma_i\psi_n)(\bar\psi_e\text{-}\Gamma_i\psi_\nu), \tag{3.9}$$

or

$$\sum h_i(\bar\psi_p\Gamma_i\psi_\nu)(\psi_e\text{-}\Gamma_i\psi_n). \tag{3.10}$$

These formulations are simply related with[12]

$$h=Pg, \tag{3.11}$$

where P is a 5×5 matrix with $P^2=1$. This ambiguity has no great importance for any single process, but in relating the coupling constants of different processes. The conventional ordering for β decay is (3.9). The ambiguity resulting from analogous orderings in μ decay leads to the same results, (3.11), but the μ-decay observables are independent of which is used. Under the modified universality we want to consider, it is also seen that this ordering has no consequences on the relation of μ-decay and β-decay coupling constants. The equations are analogous for capture, (3.8), but here in case of a universality of coupling constants the transition probabilities in various cases will depend on the relative ordering of particles in the μ-capture and β-decay Hamiltonian. We adopt the *a priori* attractive assumption that the nucleons play the same role in β decay as in μ capture, i.e., specifically, we shall use the ordering analogous to (3.9) with a matrix element between the nucleon fields. Now this has little significance unless we make an assumption such as: the same coupling constants characterize the three processes. We would then arrive at an unambiguous example of the Tiomno-Wheeler[9] proposal that the pairs (pn), ($\mu\nu$), ($e\nu$) are connected by the same Fermi couplings.

We shall consider pion decay with this universal interaction in Sec. 6. Let us here try to fit the β-decay coupling constants to μ decay. Consider the quantities $A=(W_0^5/96\pi^3)^{-1}\lambda$, where λ is the decay constant, and spectrum shape ρ, for μ decay (neglecting the electron mass). We shall employ the numerical values

$$A/(g_T')^2=13$$

(the following is not sensitive to this number), where the prime on g refers to coupling constants measured in β decay and expressed in terms of (1.1); and[13]

$$\rho=0.64\pm0.09.$$

With the universal interaction assumed, we have[12]

$$(g_T/g_T')^2[g_S/g_T-g_P/g_T-2(g_V/g_T+g_A/g_T)]^2 = \tfrac{2}{3}\rho A/(g_T')^2\approx 6, \tag{3.12}$$

$$(g_T/g_T')^2[(g_S/g_T-6+g_P/g_T)^2+(g_V/g_T-g_A/g_T)^2 16] = [1-(4/3)\rho]A/(g_T')^2\approx 2. \tag{3.13}$$

Now consider modification of the universal Fermi interaction such that the bare nucleons interact in the same way as the light fermions. This modification leads to a change in each coupling constant separately so that

$$g_T'/g_T=g_A'/g_A=R_\sigma,\quad g_S'/g_S=g_V'/g_V=R, \tag{3.14}$$

where these ratios of the coupling constants g', effective in nuclear processes (3.1) and (3.2) to the bare constants which still apply to μ decay (3.3), are plotted in Fig. 1. For convenience in seeing the results the ratios

$$n^2=(g_S'/g_S)^2,\quad M=(g_T/g_T')(g_S'/g_S)$$

are plotted in Fig. 1. Noting experimentally that

$$|g_V'/g_S'|,\quad |g_A'/g_T'|\ll 1;\quad |g_S'/g_T'|\approx 0.8, \tag{3.15}$$

Eqs. (3.12) and (3.13) are approximately written

$$(1-g_P/g_S)^2\approx 9n^2, \tag{3.16}$$

$$[1-6(g_T'/g_S')M+g_P/g_S]^2\approx 3n^2. \tag{3.17}$$

[11] Stech and Jensen (reference 9) have discussed the question of which: $\mu^\pm$, is the "particle" under a particular symmetry condition. They find the "μ^- particle" assumption associated with reasonaly large ρ, i.e., $\rho=\frac{3}{4}$, in fair agreement with experiment (where ρ is the μ-meson spectrum shape parameter), while the "μ^+ particle" assumption leads to $\rho\approx 0$. These results are typical of the two assumptions. We do not adopt the thus seemingly attractive μ^- choice because it seems less reasonable on the general grounds mentioned above. See also reference 14.

[12] See L. Michel, Proc. Phys. Soc. (London) **A63**, 514, 1371 (1951); Phys. Rev. **86**, 814 (1952).

[13] L. Lederman (private communication).

It is seen that the μ decay can be fitted for a variety of P_{11} merely by varying g_P (which has no effect on β decay). No simple combination such as $S\pm P\pm T$ seems possible.[14] We see, however, that the meson theories predict $n^2\lesssim 1/100$ and that the simple combination $S\pm\frac{1}{3}T$ will agree approximately with β decay. This suggests the combination

$$S+P+\tfrac{1}{3}T, \tag{3.18}$$

which is that associated with a universal theory in which fields of unlike fermions commute. One cannot accept simply (3.18), however, because the μ meson will then not decay. The μ-meson decay would then have to be due to small corrections to (3.18), which is an unhappy situation. Further speculation on simple relations between the g's must probably await some hints from more fundamental considerations.

4. Finite-Mass Model and First-Forbidden Transitions

Let us examine the general form of the nuclear matrix element when recoil or lowest order relativistic effects are considered. Denote spinors by u and associated two component spin functions by χ. We assume that we should associate with each nucleon of spin component m and momentum $\mathbf{k}$, in the usual nonrelativistic wave function, the corresponding spinor. Let

$$(\bar{u}_p\Gamma_i u_n)=(\chi_p,\Gamma_i'(\mathbf{k}_p/2M,\mathbf{k}_n/2M)\chi_n). \tag{4.1}$$

Then the matrix element for the emission of an electron and antineutrino of momentum $\mathbf{k}_e+\mathbf{k}_\nu=\mathbf{q}$ is

$$\mathfrak{M}=\sum_i g_i\left(\Psi_F(r)e^{i\mathbf{q}\cdot\mathbf{r}},\ \Gamma_i'\left(\frac{-i\nabla-\mathbf{q}}{2M},\frac{-i\nabla}{2M}\right)\Psi_I(r)\right), \tag{4.2}$$

where the variables explicitly shown concern the nucleon involved in the transition. We explicitly write the argument of Γ_i' as $k/2M$ because the arguments occur with this relative order of magnitude, i.e., $\frac{1}{2}v/c$. For allowed transitions $\exp(i\mathbf{q}\cdot\mathbf{r})=1$, and $k_n=k_p=0$ in the argument of Γ_i'. As is well known,[15] the forbidden-decay transitions thus arise from two sources: from $[\exp(i\mathbf{q}\cdot\mathbf{r})-1]$ and from finite velocities in the argument of Γ_i'. For a 2-Mev β decay, for example, one expects the following order of magnitude contributions (compared to allowed transitions, neglecting differences in selection rules)

$$\begin{aligned} O(\mathbf{q}\cdot\mathbf{r})&=A^{\frac{1}{3}}/100,\\ O(k/2M)&=1/10,\\ O(q/2m)&=1/1000.\end{aligned} \tag{4.3}$$

Let us again examine the change in coupling constant for a variety of nuclear matrix elements in going from bare nucleon to physical nucleon formalism. Let us consider allowed and first forbidden β-decay matrix elements and also nucleon anti-nucleon matrix elements in connection with π decay.

Consider a positive energy nucleon of small momentum $\mathbf{k}$ spin direction m and isotopic spin n. We adopt a very simple one-meson model for a physical nucleon of finite mass:

$$\begin{aligned}|\mathbf{k},m,n\rangle=A_0a^*(\mathbf{k},m,n)+\int d\mathbf{k}'f(k')\\ \times\left(\chi^{m'},\left[\frac{(\mathbf{k}+\mathbf{k}')\cdot\boldsymbol{\sigma}}{E_{k'}+M}-\frac{\mathbf{k}\cdot\boldsymbol{\sigma}}{2M}\right]\chi^m\right)\\ \times(\xi^{n'},\tau_\alpha\xi^n)a^*(\mathbf{k}+\mathbf{k}',m',n')b^*(-\mathbf{k}',\alpha),\end{aligned} \tag{4.4}$$

where $b^*(-\mathbf{k}',\alpha)$ is the creation operator for a meson of momentum $-\mathbf{k}'$ and charge α. We need only consider nucleon matrix elements between nucleons of the same momentum [i.e., the q terms in the argument of Γ' in (4.2) are neglected according to (4.3), although they introduce new selection rules]. We can set $\exp(i\mathbf{q}\cdot r)=1$ since terms $[\exp(i\mathbf{q}\cdot\mathbf{r})-1]$ times various allowed operators will have the same coupling constants as the allowed operators. We shall ignore contributions of

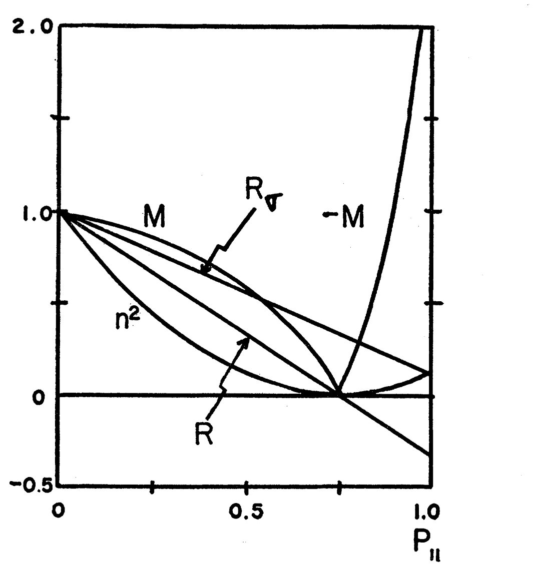

FIG. 1. Pion field radiative corrections in the infinite mass nucleon model. Symbols are defined in Sec. 2.

[14] It is such *a priori* attractive combinations which give promising predictions for μ decay upon adoption of the "μ^- particle" assumption discussed above and in reference 11. For this reason this has been the line of approach adopted in much of the work mentioned in references 2, 9, and 11. We now see that (3.18) would, for example, be a preferred combination. What μ decay would be associated with (3.18) in the "μ^- particle" assumption? The expressions (3.12), (3.13) apply to the "μ^+ particle" assumption. If one considers the corresponding expressions for the "μ^- particle" assumption (reference 12) in relation to Fig. 1, one finds that they cannot be satisfied because the small value of n makes μ decay too rapid compared to β decay.

[15] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **60**, 308 (1941); E. J. Konopinski, Revs. Modern Phys. **15**, 209 (1943); J. Blatt and V. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), pp. 705, 726.

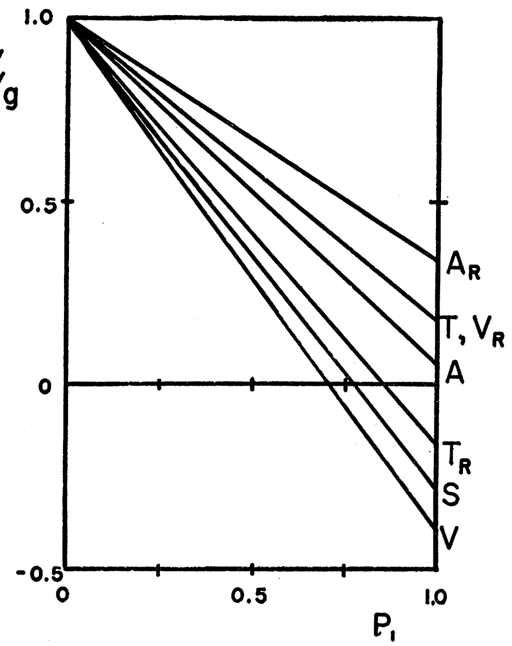

FIG. 2. Pion field radiative corrections in a finite mass nucleon model as discussed in Sec. 4.

relative order $(k/2M)^2$ that come up, as they are small and do not yield new selection rules. There are then eight positive-energy nucleon matrix elements of interest. Using (4.4), we express the physical nucleon matrix element in terms of the corresponding nonrelativistic bare nucleon elements:

$$\begin{aligned}
S:&\quad \langle p|n\rangle=[1-(4/3-\bar{\alpha})P_1](\chi_p,\chi_n),\\
V:&\quad \langle p|\beta|n\rangle=[1-(4/3+\bar{\alpha})P_1](\chi_p,\chi_n),\\
T:&\quad \langle p|\boldsymbol{\sigma}|n\rangle=[1-(8/9-\bar{\alpha})P_1](\chi_p,\boldsymbol{\sigma}\chi_n),\\
A:&\quad \langle p|\beta\boldsymbol{\sigma}|n\rangle=[1-(8/9+\bar{\alpha})P_1](\chi_p,\boldsymbol{\sigma}\chi_n),\\
T_R:&\quad \langle p|\gamma_5\boldsymbol{\sigma}|n\rangle=[1-(1+\bar{\beta})P_1](\chi_p,(i\boldsymbol{\sigma}\times\mathbf{k}/M)\chi_n),\\
V_R:&\quad \langle p|\beta\gamma_5\boldsymbol{\sigma}|n\rangle=[1-(1-\bar{\beta})P_1](\chi_p,(\mathbf{k}/M)\chi_n),\\
P_R:&\quad \langle p|\gamma_5|n\rangle=O(k/M)^3,\\
A_R:&\quad \langle p|\beta\gamma_5|n\rangle=[1-(1-2\bar{\beta})P_1](\chi_p,(\mathbf{k}\cdot\boldsymbol{\sigma}/M)\chi_n).
\end{aligned} \tag{4.5}$$

The isotopic spin variables are not explicitly shown. The relativistic matrix elements $\langle p|\Gamma|n\rangle$ mean, of course, $(\bar{\psi}\Gamma\psi)$. Thus usual transition from relativistic to nonrelativistic forms would involve, of course, setting $P_1=0$. The constants are

$$\bar{\alpha}=\tfrac{1}{3}[k'/(E_{k'}+M)]_{\mathrm{Av}}^2,\quad \bar{\beta}\approx\tfrac{1}{3}[E_{k'}/(E_{k'}+M)]_{\mathrm{Av}},$$

where the averaging means the expectation value of the quantity with respect to the second term of (4.4), P_1 is the normalization of the second term of (4.4), and $|A_0|^2=1-P_1$. The brackets, which are just g'/g, are plotted in Fig. 2.

Of interest is the relationship between the coupling constants g' of the relativistic first forbidden operators and associated allowed operators: T_R/T, V_R/V, A_R/A. The considerations of Mahmoud and Konopinski[16] leading to selection of S and T or V and A for β decay on the basis of the allowed shapes taken by first-forbidden spectra are, for example, affected only in that the parameters relating the matrix elements for relativistic and nonrelativistic first-forbidden operators are changed. One finds that this change would not lead them to different conclusions. As remarked earlier we now have different coefficients for the two types of forbidden operators, nonrelativistic and relativistic. Here, in principle, we could measure the coupling constants and thus deduce the relevant properties of the physical nucleon, i.e., P_1, and the bare β-decay coupling constants. In practice each coupling constant is multiplied by a different nuclear matrix element so that only within our knowledge of the nuclear matrix elements could the changed coupling constants be measured.

5. Pion Decay

There are three important difficulties arising in a universal Fermi interaction of π decay[10] wherein an intermediate pair of nucleons annihilates to produce a mu meson and neutrino via (3.2) or (3.8). The first difficulty is the electron decay which will occur in the presence of pseudoscalar coupling. The second difficulty shown recently by Treiman and Wyld[10] is that the radiative electron decay process

$$\pi\rightarrow e+\nu+\gamma \tag{5.1}$$

may occur through the tensor interaction. Thirdly, the axial vector interaction, which must be principally responsible for the process, is known to be so weak in β decay[17] that it is difficult to understand the rapidity of the pion decay. Our analysis of these processes would require an understanding of the details of the complicated decay process. We may note that it seems unlikely in the extreme that these difficulties would disappear as a result of such an investigation. Let us merely attempt here to study, as an example, the effective strength of the axial vector coupling in pion decay. We assume that the same coupling constants g apply universally as discussed in Sec. 3. Thus we observe constants g' in β decay and constants g'' in annihilation of a nucleon pair to produce μ or e. Let us evaluate the g''. This is done very crudely because we have no accurate model of relativistic nucleons and we do not know the details of the pion decay process, e.g., the interaction between nucleon pairs. As an exercise to see what might happen, we calculate with the model of (4.4), charge-conjugation a for the antinucleon. Thus we adopt again the one-pion assumption and, among all types of processes, look only at annihilation of *slow*

[16] H. Mahmoud and E. J. Konopinski, Phys. Rev. **88**, 1266 (1952).
[17] R. Sherr and R. H. Miller, Phys. Rev. **93**, 1076 (1954).

nucleon pairs. The matrix elements we want are simply

$$\langle\bar{p}|\Gamma_i|n\rangle = -\langle p|\gamma_5\Gamma_i|n\rangle.$$

Referring to (4.5), we find that the results for g''/g for P and A are given by the brackets g'/g appropriate to S and V.

6. CONCLUSIONS

The *a priori* attractive suggestion that Fermi interactions have some simple form with respect to bare nucleons has been examined. It is found very difficult to test experimentally. With regard to a universal Fermi interaction, no direct verification seems likely. The suggestion does not lead to immediate simplifications, although it is indicated that the universal couplings that should be considered are different from those which have received most attention recently. With regard to forbidden β-decay processes, we can in principle observe the effects of this suggestion. But these effects depend theoretically on models for both a recoiling nucleon and for the nucleus.

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Interaction of *P*- and *S*-Wave Pions with Fixed Nucleons

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A method is given of replacing pion scattering parts in a Feynman diagram by experimentally observed quantities. In this paper momenta of nucleons are neglected, although nucleon pair creation is not. It is assumed that only *P*- and *S*-wave pions interact with nucleons. No interaction is assumed between pions. Two examples are given: (1) The anomalous magnetic moment of the proton is rigorously expressed in terms of pion-nucleon scattering amplitudes, or, alternatively, in terms of the renormalized coupling constant and the total cross sections for pions. (2) The internucleon potential is also expressed by means of scattering quantities. In this case the number of virtual pions exchanged between the two nucleons is limited to two, although the number of pions emitted and absorbed by the same nucleon is not limited.

I. INTRODUCTION

THE static model of the pion-nucleon interaction has proved to be quite powerful in correlating certain experiments. As far as the low-energy scattering of *P*-wave pions by a nucleon is concerned, this theory is very successful. Experiments show that δ_{33}, the phase shift for the state with $J=\frac{3}{2}$ and $I=\frac{3}{2}$ is very large, while the other three phase shifts are small. This comes from the simple fact that the pion-nucleon interaction for the $\frac{3}{2}$–$\frac{3}{2}$ state is attractive while for all other states it is repulsive. Thus almost every method, the Tamm-Dancoff approximation,[1] the Tomonaga intermediate coupling approximation,[2] or the Chew-Low method,[3] gives satisfactory agreement with experiment if the field reaction is taken into account.

Granted that this scattering problem has been solved, how can other quantities like the anomalous magnetic moment or nuclear forces be calculated with similar accuracy? The purpose of this paper is to describe a method to express these quantities in terms of scattering quantities.

As an example, suppose that one wants to calculate the anomalous magnetic moment of the proton. One draws a Feynman diagram as in Fig. 1. The shaded area contains a number of virtual pions emitted and absorbed by the nucleon. The sum over the virtual interactions represented by this shaded area is identical to the graph which appears also in pion-nucleon scattering. Let us call this contribution a scattering part, which means the sum of all Feynman graphs with two external free nucleon lines and two external (free or virtual) pion lines. This scattering part is equal to the *S*-matrix element if the two pion lines are free. The difference here is that the pions are virtual and do not satisfy the energy relation $k_0^2=k^2+\mu^2$ as real pions do. This difficulty is overcome, however, in the static approximation.

We make the following assumptions:

(1) The static approximation is applicable; that is, the momenta of the nucleons and antinucleons (if any) can be neglected.

(2) There is no interaction between pions.

(3) The pion-nucleon interaction does not contain higher derivatives of the field. For *P* waves it is sufficient to assume the usual pseudovector coupling, although we do not use its explicit form. The *S*-wave interaction is unknown. We assume only the regularity of the interaction (see Sec. V). *D* and higher waves are assumed to have no interaction with the nucleon.

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