the fraction of neutral K particles, produced in any way, which decay by the θ^0 scheme. Thus, although statistics and lifetime uncertainties do not permit one to say that the values of ρ_θ obtained from I_1 and I_3 disagree, the fact that the latter appears somewhat lower could conceivably arise from a real physical effect.

It appears clear that further elucidation of these questions will have to be carried out under the much more highly controllable conditions available with high-energy accelerators.

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Momentum of Nucleons in Various Nuclei from the High-Energy Photoeffect*

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The previously reported studies of the high-energy photoejection of neutrons and protons showed the basic validity of the "quasi-deuteron" model. Due to the motion of neutrons and protons in a complex nucleus one observes that the kinematics of the "quasi-deuteron" disintegration are modified from those of the photodisintegration of a free deuteron at rest. The modification of the kinematics was measured for Li, C, O, Al, and Cu in an attempt to compare the "average" momentum of the nucleons in these nuclei.

It is found that a Fermi momentum distribution (with $T=0°$) does not fit the observed data. A calculation employing only conservation of momentum and assuming a three-dimensional Gaussian momentum distribution for the neutrons and protons, gives an expression that fits the data quite well for Li, C, and O. The $1/e$ values of the Gaussians for Li, C, and O, respectively are 8, 19, and 19 Mev. These values have been corrected for the experimental angular resolution and for refraction at the nuclear surface.

I. INTRODUCTION

WHEN nuclei are bombarded by high-energy x-rays, neutrons and protons are observed in coincidence,[1] which is in agreement with the qualitative prediction of the "quasi-deuteron" model of Levinger.[2] On this model, high-energy x-rays interact with a pair of nucleons in a complex nucleus rather than with the entire nucleus.

A previous paper by the authors[3] on the photoejection of high-energy neutrons and protons gives some of the results of a series of experiments undertaken at the M.I.T. Synchrotron. The experiments previously reported indicate that the mechanisms involved in a complex nucleus, in the simultaneous photoejection of a neutron and proton, are the same mechanisms responsible for the photodisintegration of a free deuteron.

The kinematical relationships for a free deuteron (at rest) are modified in a complex nucleus by the initial momentum of the center of mass of the neutron-proton pair. Specifically, if the protons energy and angle are held fixed, the coincident neutrons are observed to have an angular spread about the angle at which one observes the neutrons from deuterium. In the preliminary measurements, the spread in neutron angles was found to be in semiquantitative agreement with what one would expect if the spread arose from the initial momentum of the center of mass of the neutron-proton pairs. Therefore, as part of the series of measurements, we undertook to study the momentum of nucleons in various nuclei; this article is a report on these measurements.

At the time these momentum studies were undertaken, the published data on the momentum of nucleons was somewhat discrepant and was limited essentially to data on Li, C, and O. The data available at that time are listed in Table I. More recently, Wilcox and Moyer[4] have used the quasi-elastic scattering of protons to study the momentum in Li, Be, and B, and Selove[5] has used the deuteron pickup process to study Be and C.

It is to be pointed out that the earliest experiments and the meson experiments were in general not motivated by a desire to determine momentum distribution,

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[1] M. Q. Barton and J. H. Smith, Phys. Rev. **95**, 573 (1954); Meyers, Odian, Stein, and Wattenberg, Phys. Rev. **95**, 576 (1954).

[2] J. S. Levinger, Phys. Rev. **84**, 43 (1951).

[3] Odian, Stein, Wattenberg, Feld, and Weinstein, Phys. Rev. **102**, 837 (1956).

[4] J. M. Wilcox and B. J. Moyer, Phys. Rev. **99**, 875 (1955).

[5] W. Selove, Phys. Rev. **101**, 231 (1956).

TABLE I. Early data on the momentum of nucleons.

Element	Experimental method	Momentum distribution	By whom interpreted
C	(n,d) deuterium pickup, Hadley and York[b]	Chew-Goldberger distribution,[a] Fermi distribution with $KT=9$ Mev	Chew and Goldberger,[a] Heidman[c]
Li	$(p,2p)$ quasi-elastic scattering, Chamberlain and Segrè[d]	Fermi distribution, $E_{\mathrm{Max}}=20$ Mev, $T=0°$	Chamberlain and Segrè
C	$(p,2p)$ quasi-elastic scattering, Cladis *et al.*[f]	Gaussian distribution, $1/e=16$ Mev	Wolff[e]
C	(γ,π) photomeson production, Steinberger and Bishop[h]	Chew-Goldberger distribution	Lax and Feshbach[g]
C	(p,π) proton meson production, Richman and Wilcox[j]	Gaussian average, $E_{\mathrm{Av}}=19.3$ Mev	Henley and Huddlestone,[i] Henley[k]
C	(p,π) meson production Block *et al.*[l]	Gaussian average, $E_{\mathrm{Av}}=19.3$ Mev	Block *et al.*[l]

[a] G. F. Chew and M. L. Goldberger, Phys. Rev. **77**, 470 (1950). Their distribution is $\alpha/[\pi(\alpha^2+p^2)^2]$; $\alpha^2/2M=18$ Mev.
[b] J. Hadley and H. F. York, Phys. Rev. **80**, 345 (1950).
[c] J. Heidman, Phys. Rev. **80**, 171 (1950).
[d] O. Chamberlain and E. Segrè, Phys. Rev. **87**, 181 (1952).
[e] P. A. Wolff, Phys. Rev. **87**, 434 (1952).
[f] Cladis, Hess, and Moyer, Phys. Rev. **87**, 425 (1952).
[g] M. Lax and H. Feshbach, Phys. Rev. **81**, 189 (1951).
[h] J. Steinberger and A. S. Bishop, Phys. Rev. **78**, 494 (1950).
[i] E. M. Henley and R. Huddlestone, Phys. Rev. **82**, 754 (1951).
[j] C. Richman and H. A. Wilcox, Phys. Rev. **78**, 496 (1950).
[k] E. M. Henley, Phys. Rev. **85**, 204 (1952).
[l] Block, Passman, and Havens, Phys. Rev. **88**, 1239 (1952).

but that an understanding of the data required trying various momentum distributions to fit the data.

In the light of the data in Table I, it seemed worthwhile to perform an experiment which might answer the simple questions of whether the "average" momentum is the same in various nuclei, as would be predicted if the nuclear radius varies as $A^{\frac{1}{3}}$. It was hoped that the measurement of a single parameter, such as the full width at half-maximum of the neutron angular spread, might be sufficient to provide an answer to such a question.

Actually it was found worthwhile to understand the shapes of the experimental curves. This is discussed in Sec. IV.

II. EXPERIMENTAL PROCEDURE

The measurements were performed at the M.I.T. synchrotron with a 340-Mev bremsstrahlung beam. The arrangement of the apparatus is shown in Fig. 1. The characteristics, angular resolution, and efficiency of the neutron counter are discussed in some detail elsewhere.[6] The neutron counter was set at a 15-Mev bias, and it had no other energy discrimination. The neutron counter was on a rolling table that pivoted about the midpoint of the target. It always subtended a 10° angle (midway along its axis).

The proton counter was the same as that previously described.[3,6] The absorbers in the proton counter were chosen so that protons from about 120 to about 140 Mev would be detected. The proton telescope subtended a 10° angle centered about 76° in the laboratory. This energy and angle setting of the proton counter were held fixed throughout all the measurements reported here. They correspond to 90° in the center-of-mass system of a deuteron at rest being disintegrated by a photon of about 260 Mev.

[6] Christie, Feld, Odian, Stein, and Wattenberg, Rev. Sci. Instr. **27**, 127 (1956).

The electronics (see reference 6) included two parallel coincidence circuits fed by a prompt and a delayed pulse from the neutron counter to enable the simultaneous determination of the accidental rate correction.

The targets employed in this work were water, heavy water, lithium, carbon, aluminum, and copper. All the targets had an average energy loss of about 9 Mev for 130-Mev protons, except the lithium which had about a 5-Mev average loss. All targets were larger than the beam. The targets were placed at an angle to the bremsstrahlung beam so that the beam's interception of the target would subtend an angle of about 7° at the back of the proton telescope.

The single and coincidence counting rates were then measured as a function of neutron angle. First a setting of 78° for the neutron counter was run, then two other angles, and then back to 78° to provide a cyclic method of taking the measurements.

Runs were made for a fixed number of monitor units. However, to remove all experimental drifts except

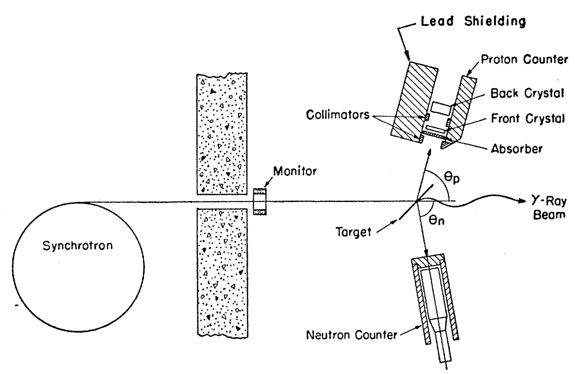

FIG. 1. The geometrical arrangement of the apparatus. The proton angle was kept fixed; the angle of the neutron counter was varied. The cross-hatching surrounding both counters indicates lead shielding.

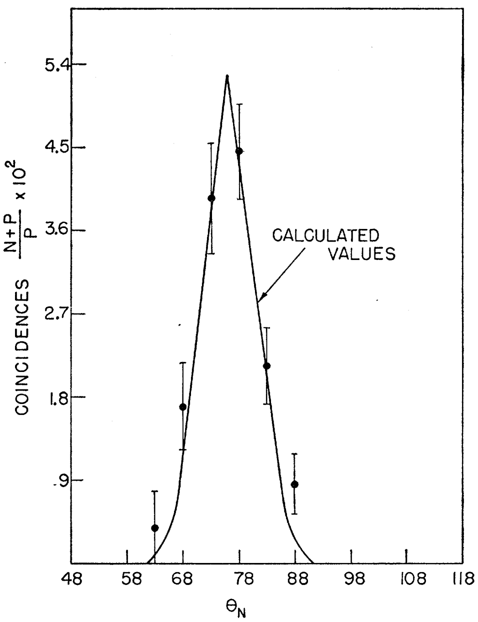

FIG. 2. Neutron-proton coincidences from deuterium. This shows the angular resolution due to the geometry employed. The ratio of coincidences to total single protons is plotted as a function of the angle of the neutron counter; the solid curve is the calculated distribution.

those in the neutron counter, the data are reported as a ratio of neutron-proton coincidences to proton counts. The deuterium measurements required a heavy-water light-water subtraction. All counters were frequently checked by a radioactive standard.

III. EXPERIMENTAL RESULTS

The neutron-proton coincidence rate as a function of neutron angle for deuterium is shown in Fig. 2. The width of this curve arises from the finite angular resolution of the geometrical arrangement (not from the relative momentum of the neutron and proton). The calculated curve was obtained by Christie[6] by taking into account the finite size of the intersection of the bremsstrahlung beam with the target. Figure 2 has a full width at half-maximum of about 11°; this can be considered the angular resolution of the experimental setup employed in these measurements.

In Figs. 3 through 7 are shown the data from Li, C, O, Al, and Cu. The ratio of neutron+proton coincidences to single proton counts are plotted as a function of neutron angle. The data have been corrected for the accidental counting rate. These corrections contribute appreciably to the experimental uncertainty especially at smaller angles in the heavier elements. (An electronic failure occurred in the accidental rate measuring circuit during the aluminum runs, and the corrections were calculated from the singles counting rates in this case.)

In the heavier elements the measurements could not be made at sufficiently large and small angles to determine if the curves went to zero.

The lines drawn through the data are arbitrary. All of these curves are appreciably broader than the instrumental resolution.

In the case of lithium, the narrowest curve is obtained, and it is about 20° broader at half-maximum than the experimental resolution. The Li curve is in agreement with that obtained by Barton and Smith[1] who employed a similar (not identical) geometrical arrangement. The broadening of the width of these curves is attributed to the finite momentum of the center of mass of the neutron-proton pair in the nucleus.

The increase in the spread occurring between lithium and carbon is considered to be definite evidence that the average momentum of nucleons in carbon is greater than that in lithium. Some reservations are necessary regarding the observed increase in spread in going from

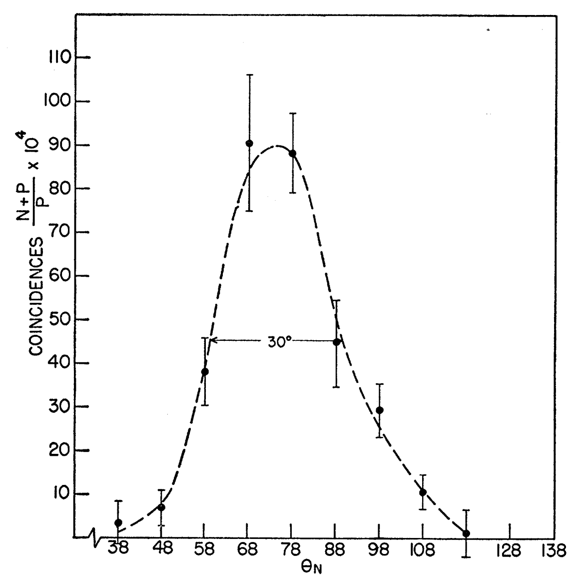

FIG. 3. Neutron-proton coincidences from lithium. The ratio of coincidences to total proton counts is plotted as a function of the angle of the neutron counter. The curve is drawn so as to connect the experimental points.

carbon to aluminum and copper. As the curves for Al and Cu were not taken at sufficiently large and small angles to establish that they went to zero, a background might be present which should be subtracted. (E.g., neutrons internally scattered in the nucleus could be present.) If there were a background, the widths indicated in the Al and Cu figures would be too large. Some data on scattering within the nucleus have already been obtained in our search for p-p coincidences[7] and in other experiments to be described in a later publication. However a knowledge of the angular distribution of scattered neutrons would be needed to correct the Al and Cu data. Obviously curves on Cu and Al that extend to larger and smaller angles need to be obtained

[7] Weinstein, Odian, Stein, and Wattenberg, Phys. Rev. 99, 1621 (1955).

before definite quantitative statements are made about the average momentum of the nucleons in the heavier elements relative to the average momentum in carbon.

IV. INTERPRETATION OF DATA

We attempted to be more specific about what kind of "average" was being indicated by the widths at half-maximum. A simple calculation was performed employing only conservation of momentum and a Fermi distribution. It was found that a Fermi momentum distribution leads to an experimental distribution of the type shown in Fig. 8; this does not bear much resemblance to the curves of Figs. 3 through 7.

A (three-dimensional) Gaussian momentum distribution was then tried employing only conservation of momentum. The neutrons and protons were assumed to each have a distribution of the form

$$p^2dp\,\exp(-p^2/2ME_g), \qquad (1)$$

with E_g the same for both neutrons and protons. The calculation can then be carried through analytically with no approximations and is given in the appendix. The result is that the neutron+proton coincidences should have an angular distribution given by,

$$\text{constant}\left[\frac{E_G}{E'}+\cos^2\psi\right]\exp\left(-\frac{E'}{2E_g}\sin^2\psi\right)d(\cos\psi). \qquad (2)$$

Here E' is essentially the energy of the proton; it differs from the exact energy by a small relativistic correction and the binding energy of a nucleon in the nucleus. (See Appendix.) E_g is the parameter of the Gaussian distribution (i.e., the $1/e$ energy); ψ is the deviation of the angle of observation from the angle at which one observes the neutron from the photodisintegration of a deuteron at rest; $d(\cos\psi)$ is the differential solid angle which is constant in the experiments.

The experimental data were first compared with this calculated distribution on the assumption that the term E_g/E' is small; specifically, $E_g/E'\approx 0.2$ where $E'=152$ Mev. The observed neutron-proton coincidences (N_{np}) were divided by $\cos^2\psi$ and plotted on semilog paper against $\sin^2\psi$ (i.e., $\ln[(N_{np})/\cos^2\psi]=-(E_p/2E_g)\sin^2\psi$). The results are shown in Fig. 8, and it appears that this simple theory fits the data quite well.

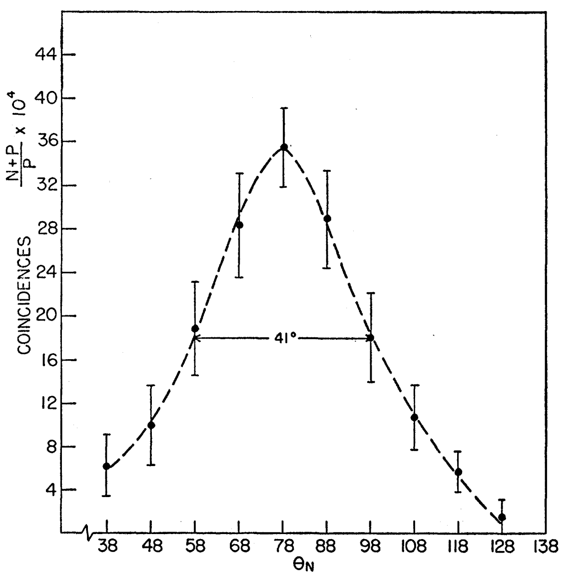

FIG. 4. Neutron-proton coincidences from carbon as a function of neutron angle.

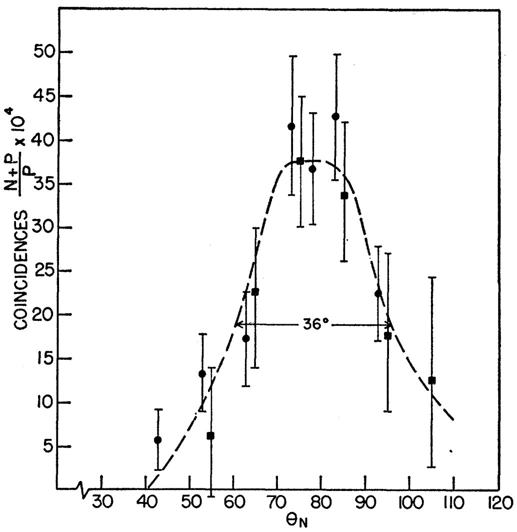

FIG. 5. Neutron-proton coincidences from oxygen as a function of neutron angle.

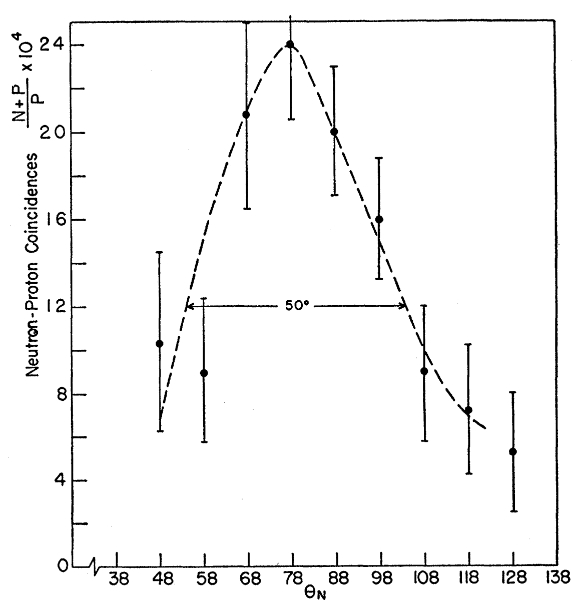

FIG. 6. Neutron-proton coincidences from aluminum as a function of neutron angle.

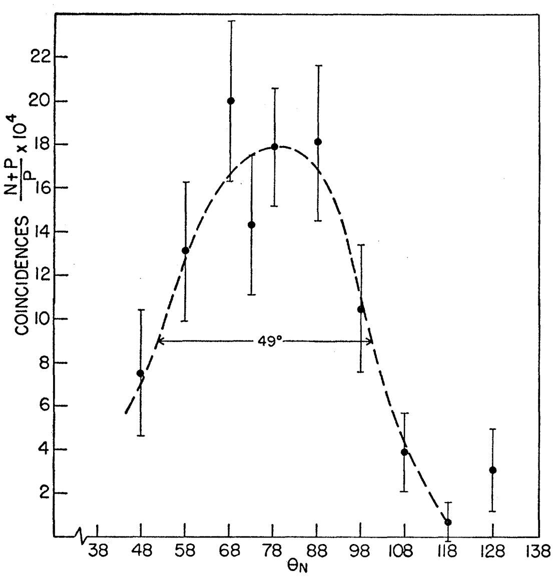

FIG. 7. Neutron-proton coincidences from copper as a function of neutron angle.

Using the approximate results of Fig. 9, we fitted the data to expression (2) and also corrected for the finite angular resolution of the detecting equipment. The values of E_G obtained are: for Li, 9.0±1.0 Mev; for C, 19.7±1.5 Mev; and for O, 19.7±2.5 Mev. The errors given are obtained from the slopes of the most extreme straight lines that will still fit the corrected data.

A three-dimensional Gaussian is the momentum distribution one would get for the lowest state of an harmonic oscillator potential. The next highest state of an harmonic oscillator has a momentum distribution which is dominated by a Gaussian factor. The data certainly are not good enough to distinguish between such similar distributions.

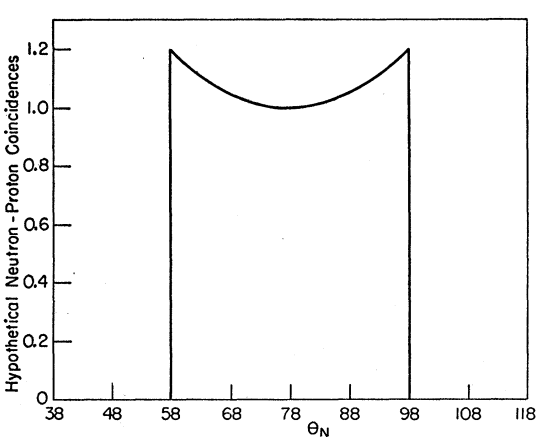

FIG. 8. The hypothetical angular distribution of neutron-proton coincidences as a function of neutron angle predicted by a crude theory assuming a Fermi momentum distribution. The shape of this curve is to be compared with the data of Figs. 3 to 7.

The theory as given in the appendix is very naive and neglects conservation of energy, the effect of the bremsstrahlung distribution, scattering before escape from the target nucleus, and scattering (or refraction) by the potential well. Investigation of these has not led to any better fit of the data. If one includes refraction at the nuclear surface of a potential well which is 15 Mev deep, the value of E_G is decreased by about 1 Mev. However, there is some question of the legitimacy of adding to the momentum spread a nuclear surface refraction effect. A nuclear surface is a somewhat questionable construct for a nucleus with five residual nucleons.

These questions and the legitimacy of the (classical) momentum calculation deserve further study. It would

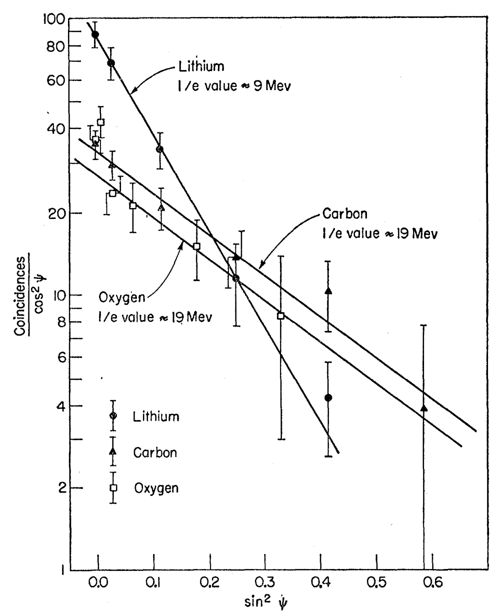

FIG. 9. Experimental data for Li, C, and O fitted to theoretical expression. This is a plot of coincidences/$\cos^2\psi$ *versus* $\sin^2\psi$; on this semilogarithmic plot, one should obtain a straight line if the theoretical expression is correct. From the slope of the straight line one can immediately obtain the $1/e$ value of the Gaussian (see text).

be nice to have a theory that did not start in momentum space and then switch over to ordinary space. One possibility would be to use other than plane waves.

V. CONCLUSIONS

The shape of the experimental curves indicates that a Fermi momentum distribution (with $T=0°$) does not fit the data for any of the elements studied. A Gaussian-type distribution seems to fit the lighter elements quite well. This would be in agreement with the interpretation of the data of Hofstadter *et al.*[8] which requires nucleon density distributions which do not

[8] See K. W. Ford and D. L. Hill, *Annual Review of Nuclear Science* (Annual Reviews, Inc., Stanford, 1955), Vol. 5, p. 25.

cut off sharply at the edge of the nucleus. Such density distributions, if carried over to light nuclei, have little resemblance to a square well density distribution and the concomitant Fermi momentum distribution.

Lithium clearly has a mean momentum that is appreciably smaller than the mean momentum in carbon and oxygen. Carbon and oxygen have the same mean momentum within our experimental accuracy. However, considering the assumptions and the naivete of the theory employed, one should probably take the absolute values less seriously than the relative values.

It is important to note that in the process employed in these measurements, the momentum being studied is that of the center of mass of a neutron and proton pair. The kinematical deviations measured give no information on the relative momentum of the neutron and the proton. The experiments listed in Table I referred to measurements involving a single nucleon. For this reason one should not necessarily expect agreement between our results and those of Table I, and the more recent results of Wilcox and Moyer[4] and of Selove.[5]

These recent measurements seem to be more extensive and accurate than those in Table I. Wilcox and Moyer also find that the momentum of lithium is relatively low. They did not give a quantitative momentum distribution for lithium. They found that their Be data would be fitted by a Gaussian with $E_G=20$ Mev.

Selove's interpretation of his own data requires that the higher momentum components are more frequent than would be given by a Gaussian momentum distribution. Selove believes these higher momentum components arise from the interaction of two nucleons. If this is the case, such momentum components should not appear in the kinematical deviation measurements of the quasi-deuteron disintegration. More accurate measurements of the wings of our curves might be useful in quantitatively evaluating this question. However the higher momentum components required to interpret pickup data have not been required in analyzing the data from quasi-elastic p-p scattering experiments.

In comparing our results with those of others, it is also to be noted that others have not included the effect on their data of refraction at the nuclear surface.

ACKNOWLEDGMENTS

The authors have discussed these data and their interpretation with many people and are grateful to them all for the time given to us. We wish especially to thank Professor Feld, Professor Drell, Professor Villars, and Professor Weisskopf of M.I.T., Professor Selove of Harvard, and Professor J. B. Smith[9] of Illinois. We are indebted to J. Bjorken for his assistance in the calculations, and to E. Christie and W. Rankin for assistance during the experimental measurements.

VI. APPENDIX

The calculation which follows is based on the following assumptions: (1) The neutrons and protons both have a Gaussian momentum distribution. (2) There is no correlation in the motion of the neutrons and protons. (3) The observed spread in neutron angles is due to the momentum of the center of mass of a random neutron-proton pair. (4) Conservation of energy can be neglected.

Figures 10(a) and 10(b) show the conservation of momentum for a deuteron at rest (a free deuteron) and for a deuteron in motion (quasi-deuteron). The experimental situation was that the proton's momentum, $\mathbf{K}_P$, was held constant (i.e., both the angle and the energy were fixed). For the deuteron at rest,

$$\mathbf{K}_P+\mathbf{K}_N=\mathbf{k};$$

for the deuteron in motion,

$$\mathbf{K}_P+\mathbf{K}_N'=\mathbf{k}+\mathbf{P}.$$

$\mathbf{K}_N$ and $\mathbf{k}$ are the momenta of the neutron and photon, respectively. $\mathbf{K}_N'$ is the momentum of the neutron from the photodisintegration of a neutron-proton pair in motion. $\mathbf{K}_N'$ makes an angle ψ with $\mathbf{K}_N$, and it is desired to obtain a distribution as a function of ψ.

$\mathbf{P}$ is the momentum of the center of mass of the neutron and proton in the nucleus, namely,

$$\mathbf{P}=\mathbf{p}+\mathbf{q},$$

where $\mathbf{p}$ and $\mathbf{q}$ are, respectively the initial momentum of the proton and neutron; the appropriate second variable for a transformation is the relative momentum

$$\mathbf{Q}=\mathbf{p}-\mathbf{q}.$$

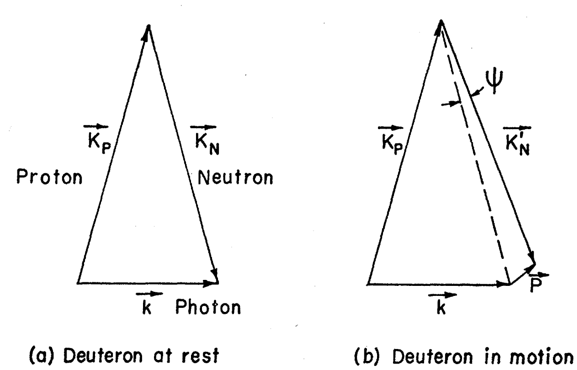

FIG. 10. Conservation of momentum diagram. (a) is for a free deuteron at rest; (b) is for a neutron-proton pair whose center of mass has momentum P.

[9] A report on similar measurements performed concurrently at the University of Illinois by J. B. Smith and associates will be submitted for publication in a forthcoming issue of *The Physical Review*. The experimental data on lithium reported above is in agreement with the results of the more extensive measurements of the group at the University of Illinois. However, there appears to exist a discrepancy in the $1/e$ value of the momentum obtained from the independent interpretations of the data.

The probability for p to be between p and $p+dp$ and q to be between q and $q+dq$ is assumed to be

$$NZ16\pi^2\left(\frac{1}{2\pi ME_G}\right)^3 \exp\left(-\frac{p^2}{2ME_G}\right)p^2dp \times\exp\left(-\frac{q^2}{2ME_G}\right)q^2dq.$$

In terms of P and Q, this becomes

$$NZ2\pi^2\left(\frac{1}{2\pi ME_G}\right)^3 \exp\left(-\frac{P^2}{4ME_G}\right)P^2dP \times\exp\left(-\frac{Q^2}{4ME_G}\right)Q^2dQ. \quad (3)$$

The observations being made are kinematical and are affected by the motion of the center of mass, P. The relative momentum affects the matrix element and the cross section for the process. Therefore an integral over the Q distribution occurs as a multiplicative constant of the P distribution. In treating the Q distribution as a purely multiplicative constant, the center of mass and relative momenta are assumed independent. They might not be independent on certain models which include correlation between neutrons and protons in the nucleus, or more generally, models in which interactions between more than two nucleons are included.

Transforming to cylindrical coordinates P_Z and R with the Z axis parallel to K_N, one obtains for (3)

$$C\exp\left(-\frac{P_Z{}^2+R^2}{4ME_G}\right)RdRdP_Z,$$

where C includes all multiplicative constants. In this coordinate system, $\tan\psi=R/(K_N+P_Z)$, and substituting for R gives

$$C\exp\left(-\frac{(K_N+P_Z)^2\tan^2\psi+P_Z{}^2}{4ME_g}\right) \times(K_N+P_Z)^2\sin\psi\sec^3\psi d\psi dP_Z. \quad (4)$$

Expression (4) is the appropriate expression to use if the neutron's energy is being measured. In our measurements there was no energy discrimination; consequently we integrate over all values of P_Z and obtain

$$C'[2ME_g+K_N{}^2\cos^2\psi]\exp\left(-\frac{K_N{}^2}{4ME_g}\sin^2\psi\right)d(\cos\psi),$$

where C' is a new constant. Substituting

$$E'=\frac{K_N{}^2}{2M}=E\left(1+\frac{E}{2M}\right),$$

where E is the kinetic energy in the laboratory, we obtain Eq. (2) of the text.

The E' used in Eq. (2) of Sec. IV was taken as 152 Mev.