Direct Interaction in Neutron Inelastic Scattering*

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The direct interaction contribution to the cross section for the inelastic scattering of neutrons leading to the excitation of a single particle of the target nucleus is derived by using the method of distorted waves. The interaction potential between the incident neutron and target nucleon is taken to be a three-dimensional delta function, and the incident and emergent neutrons are assumed to move in the complex potential of the "cloudy crystal ball" model. The wave functions for the target nucleons are described in terms of the single-particle model.

The theory is applied to the excitation of the first level in C^{12}. The angular distribution of the emergent neutrons is forward-peaked. The magnitude of the cross section is small, however, and the effect will most easily be observed at neutron energies which lie between the levels of the compound nucleus.

INTRODUCTION

According to the shell model of the nucleus,[1] neutron inelastic scattering can lead to the excitation of a single particle in the target nucleus. Such transitions can occur as a consequence of the direct interaction between the incident neutron and a nucleon in the target nucleus. At intermediate neutron energies ($E_n \gtrsim 14$ Mev) this process is of considerable importance[2] and has been discussed theoretically by Austern, Butler, and McManus.[3] These authors consider interactions with the surface nucleons and employ the impulse approximation to compute the cross section. The target nucleus is regarded as opaque, and as a result their Born approximation integrals are limited to regions outside of the nucleus. In this paper we shall consider inelastic scattering by direct interaction by a method which is valid not only at high energy but also in the low energy domain where the impulse approximation is less valid and where the nucleus is comparatively transparent.[4] As a consequence of the latter, the interaction may occur inside of the nucleus. Measurement of direct interaction effects will then provide a measure of the neutron-nucleon interaction *inside* the nucleus and may thus shed light on the shell model itself as well as related effects in nuclear reactions.

We turn now to the question of the observability of the direct interaction process. Inelastic scattering in this energy range can also proceed by formation of the compound nucleus and by interaction with rotational and surface modes described by Bohr and Mottelson[5] and calculated recently by Brink and by Hayakawa and Yoshida.[6] The compound-nucleus inelastic scattering will usually be the dominant mode. This scattering has a characteristic angular dependence in two limiting cases. If only a single level or sufficiently many levels of the compound nucleus are involved the angular distribution of the inelastically scattered neutrons will be symmetric about 90°.[7] If a few (but more than one) levels of the compound nucleus need to be considered, interference between the various modes of decay can lead to an asymmetrical distribution only if the levels differ in parity. On the other hand, the direct interaction process will usually not be symmetrical about 90°, but, for example, can be peaked in the forward direction.

This discussion implies that observation of the direct interaction process will be most readily made if the target nucleus is light. Then the levels of the compound nucleus are widely spaced and one can look between resonances for the direct interaction effects. In between resonances, the inelastic cross section via compound nucleus formation is reduced to the order of $g_s \pi \lambdabar^2 \times 4\Gamma_n(E)\Gamma_n(E')/D^2$, where $\lambdabar$ is the neutron wavelength divided by 2π, $\Gamma_n(E)$ is the neutron width at the incident energy E, E' is the final neutron energy, D is the distance between levels, and g_s is the usual statistical factor. This reduction is considerable and may be enough that the direct interaction process becomes dominant. Even if the two processes are of the same order of magnitude, measurements of the angular distribution of the inelastically scattered particles both

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[5] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 16 (1953).

[6] D. M. Brink, Proc. Phys. Soc. (London) **A68**, 994 (1955); S. Hayakawa and S. Yoshida, Progr. Theoret. Phys. (Japan) **14**, 1 (1955).

[7] W. Hauser and H. Feshbach, Phys. Rev. **87**, 366 (1952).

between resonances and in the resonance region will permit the separation of compound nuclear effects. In this connection we note it will be necessary to consider the interference between the two modes of excitation.

In the main body of this paper we have estimated the direct-interaction cross section for the inelastic scattering of neutrons. With only minor modification the theory can be applied to direct-interaction processes in inelastic proton scattering and to (n,p) and (p,n) reactions. We have employed the following crude model and approximations which we expect will yield results of correct qualitative character and order of magnitude. We have taken the interaction between the incident neutron and the struck nucleon to be a three-dimensional delta function, an assumption which permits important calculational simplifications. This interaction is not unreasonable physically, however, since we expect the incident neutron to move in the average field of all the nucleons in the nucleus, the particular force of interest being then a fluctuation away from this average which is felt only when the nucleons are very close.[8] We have not included exchange forces except those which, like the Majorana type, reduce to an ordinary force for a delta function interaction. The target nucleus has been described in terms of the single-particle model as follows. The nucleus has been considered to consist of a core of spin j_c together with an orbital nucleon of angular momentum l and spin s, which are combined to give a spin of j. The spin of the target nucleus J is composed of j_c and j. The interaction of the incident particle is taken with the orbital nucleon only, the core being regarded as inert.

Certain of the approximations which have been made for higher energies are not made here, however. The effect of the nucleus upon the incident neutron is included through the use of the method of distorted waves[9] in which the empirically known nuclear potential is used to describe the elastic scattering, and the interaction between the incident neutron and the target nucleon is considered throughout the entire nucleus.

INELASTIC SCATTERING

We consider a neutron of wave number k, energy E, incident upon a target nucleus of mass number A. The Hamiltonian for the system is

$$H=H_0+T(\mathbf{r}_0)+\sum_i V(\mathbf{r}_0,\mathbf{r}_i). \qquad (1)$$

Here, H_0 is the Hamiltonian of the target nucleus, $\mathbf{r}_i$ are the coordinates of the ith constituent nucleon, $\mathbf{r}_0$ are the coordinates of the incident neutron, $T(\mathbf{r}_0)$ is its kinetic energy, and $V(\mathbf{r}_0,\mathbf{r}_i)$ is the interaction energy between the incident neutron and the ith nucleon in the target nucleus.

We expand the wave function describing the system, $\Psi(\mathbf{r}_0,\mathbf{r})$, where $\mathbf{r}$ represents the coordinates $\mathbf{r}_1, \cdots, \mathbf{r}_A$, in a set of product functions

$$\Psi(\mathbf{r}_0,\mathbf{r})=\sum v_n(\mathbf{r}_0)\psi_n(\mathbf{r}), \qquad (2)$$

where

$$H_0\psi_n(\mathbf{r})=\epsilon_n\psi_n(\mathbf{r}), \qquad (3)$$

and

$$(\psi_n,\psi_m)=\delta_{mn}.$$

The ϵ_n are the energies of the various states of the target nucleus arranged according to increasing energy. The function v_0 in Eq. (2) gives the nonexchange elastic scattering, v_1 gives the nonexchange inelastic scattering with excitation of the first level of the target nucleus, etc. We now insert expansion Eq. (2) into the Schrödinger equation for the system and obtain, after premultiplication by ψ_k and integration over the coordinates of the target nucleons, the following set of equations:

$$[T+\epsilon_k-E+(k|\sum_i V|k)]v_k+\sum_{n\neq k}(k|\sum_i V|n)v_n=0, \qquad (4)$$

where

$$(k|\sum_i V|n)=(\psi_k, \sum_i V(\mathbf{r}_0,\mathbf{r}_i)\psi_n).$$

The complete solution of these equations is of course impossible. We shall outline, however, the form of the solution in sufficient detail to indicate the origin of the average potential for v_0 and the perturbing potential that gives rise to the excitation of higher states in the target nucleus. For this purpose it is sufficient to discuss only the equations for v_0 and v_1. The generalization to the more complete system is obvious. Consider, therefore,

$$(\mathcal{H}_0-E_0)v_0=-V_{01}v_1, \qquad (5a)$$

$$(\mathcal{H}_1-E_1)v_1=-V_{10}v_0, \qquad (5b)$$

where $\mathcal{H}_k=T+(k|\sum V_i|k)$, $V_{kl}=(k|\sum V_i|l)$, and $E_k=E-\epsilon_k$. We may solve the second Eq. (5b) to obtain

$$v_1=\frac{1}{E_1+i\epsilon-\mathcal{H}_1}V_{10}v_0, \qquad (6)$$

where the $i\epsilon$ insures that we are considering only outgoing waves. Substituting in Eq. (5a), we obtain

$$\left[\mathcal{H}_0+V_{01}\frac{1}{E_1+i\epsilon-\mathcal{H}_1}V_{10}-E_0\right]v_0=0, \qquad (7)$$

so that the effective potential for v_0 (and therefore the empirical potential had we included all the effects from the higher states) is

$$V_{\text{eff}}=V_{00}+V_{01}\frac{1}{E_1+i\epsilon-\mathcal{H}_1}V_{10}. \qquad (8)$$

We could now introduce the solution of Eq. (7) for v_0 to obtain v_1. Another procedure, and the one we shall use, assumes that the effective potential for v_1 is given approximately by Eq. (8). We do expect some energy

[8] Lane, Thomas, and Wigner, Phys. Rev. **98**, 693 (1955).
[9] N. F. Mott and H. S. Massey, *The Theory of Atomic Collisions* (Oxford University Press, London, 1950), second edition, p. 144 ff.

dependence for V_{eff}, but over the energy ranges of significance here this should not be important. We can therefore rewrite Eqs. (5) in the following form:

$$(T+V_{\text{eff}}-E_0)v_0=0, \tag{9a}$$

$$(T+V_{\text{eff}}-E_1)v_1=-\Omega_{10}v_0, \tag{9b}$$

where

$$\Omega_{10}=V_{10}+(V_{11}-V_{00})\frac{1}{E_1+i\epsilon-\mathcal{H}_1}V_{10} -V_{01}\frac{1}{E_1+i\epsilon-\mathcal{H}_1}V_{10}\frac{1}{E_1+i\epsilon-\mathcal{H}_1}V_{10}. \tag{10}$$

It is clear that the general solution of the complete set of Eqs. (4) may be put into form Eq. (9) where, of course, Ω and V_{eff} will not be given by Eqs. (8) and (10). Note that V_{eff} and $\mathbf{\Omega}$ are not Hermitian as would be expected. At this point we must note the physical meaning of the quantities appearing in Eqs. (9).

V_{eff} is fairly well known empirically. We take the simple square-well form[4]

$$\begin{aligned} V_{\text{eff}}&=-V_0(1+i\zeta), \quad r<R\\ &=0, \quad r>R, \end{aligned} \tag{11}$$

where $V_0=42$ Mev, $\zeta=0.05$, and $R=(1.26A^{\frac{1}{3}}+0.7)\times10^{-13}$ cm.

The quantity Ω_{10} in Eqs. (9b) and (10) is a matrix element of an operator Ω. This effective interaction operator is the remainder of the interaction term in Eq. (1) after the average interaction with the target nucleus, V_{eff}, has been subtracted. We therefore expect Ω to be of very short range, i.e., to be important only when two particles, the incident neutron and a constituent nucleon of the target nucleus, are so close together that the field of all other $(A-1)$ particles in the nucleus may be disregarded. This suggests the form

$$\Omega=g\sum_i \delta(\mathbf{r}_i-\mathbf{r}_0). \tag{12}$$

This omits any many-body forces which may be expected to be present in the nucleus.

With the assumptions contained in Eqs. (11) and (12), we now have a completely defined set of Eqs. (9a) and (9b) for determining the cross section for the excitation of the first excited state of the target nucleus. We must next evaluate the matrix element Ω_{10} which is only a function of $\mathbf{r}_0$. The angular momentum state of the single target nucleon, whose transition is the result of the inelastic scattering, is given by $\psi(j,m)$ in the ground state and $\psi(j',m')$ in the first excited state. The target nucleus has the angular momentum quantum numbers (J,M) and (J',M') in the ground and first excited states, respectively. We shall assume only one term of Eq. (12) has any effect (this approximation will be discussed later). Its matrix element may be readily evaluated by means of the Racah tensor formalism.[10] We first note that

$$\delta(\mathbf{r}-\mathbf{r}_0)=\sum(-)^m Y(L,-m;\Omega)\times Y(L,m;\Omega_0)\delta(r-r_0)/r^2, \tag{13}$$

where the Y's are the usual spherical harmonics. Then the matrix element is given by

$$\begin{aligned}(J'M'|\delta(\mathbf{r}-\mathbf{r}_0)|JM) &=\textstyle\sum_L(-)^{J'+M'-L}(2L+1)^{-\frac{1}{2}}\\ &\quad\times(J'J-M'M|J'JLM-M')\\ &\quad\times(J'\|Y_L\|J)Y(L,M-M';\Omega_0)R(r_0)R'(r_0),\end{aligned} \tag{14}$$

where $R(r_0)$ and $R'(r_0)$ are the radial functions for the orbital nucleon in the ground and excited states, respectively. To continue further, it is necessary to evaluate the reduced matrix element of Y_L which in turn depends upon the model. We assume that

$$\mathbf{J}=\mathbf{j}_c+\mathbf{j} \quad\text{and}\quad \mathbf{J}'=\mathbf{j}_c+\mathbf{j}', \tag{15}$$

where j_c is the spin of the core. Inserting these eigennumbers, we obtain

$$\begin{aligned}(j'j_cJ'\|Y_L\|jj_cJ) &=(-)^{j_c+L-j-J'}[(2J+1)(2J'+1)]^{\frac{1}{2}}\\ &\quad\times W(j'J'jJ;j_cL)(j'\|Y_L\|j).\end{aligned} \tag{16}$$

Finally, from

$$\mathbf{j}=\mathbf{l}+\mathbf{s} \quad\text{and}\quad \mathbf{j}'=\mathbf{l}'+\mathbf{s}, \tag{17}$$

we have

$$\begin{aligned}(l'sj'\|Y_L\|lsj)&=(-)^{s+L-j'}(4\pi)^{-\frac{1}{2}}[(2j+1)(2j'+1)\\ &\quad\times(2l+1)(2l'+1)]^{\frac{1}{2}}(l'l00|l'lL0)W(l'j'lj;sL).\end{aligned} \tag{18}$$

Combining these results, we obtain

$$\begin{aligned}\Omega_{10}&=g(-)^{j_c-j-j'+M'}i^{l'-l+1}(4\pi)^{-\frac{1}{2}}[(2J+1)(2J'+1)]^{\frac{1}{2}}\\ &\quad\times\textstyle\sum_L i^L(2L+1)^{-\frac{1}{2}}(J'J-M'M|J'JLM-M')\\ &\quad\times W(J'j'Jj;j_cL)Z(l'j'lj;\tfrac{1}{2}L)\\ &\quad\times Y(L,M-M';\Omega_0)R(r_0)R'(r_0),\end{aligned} \tag{19}$$

where the function Z is defined by Biedenharn, Blatt, and Rose.[11]

We comment on the neglect of interaction with each particle in the core. The effect of these terms is to a great extent cancelled by the results of the antisymmetrization of the orbital wave function with that of the core. The matrix element Ω_{10} must still be of form Eq. (19) except for a numerical factor whose value would depend upon rather model dependent properties of the ground state and first excited state wave functions of the target nucleus. However, since g is not known we shall in the remaining equations combine g and this numerical factor to form a new constant g'.

We finally wish to expand the right-hand side of (9a) in spherical harmonics. For this purpose we write

$$v_0(\mathbf{r}_0)=\sum_{L'=0}^{\infty} f_{L'}(r_0)Y(L',0), \tag{20}$$

[10] G. Racah, Phys. Rev. **62**, 438 (1942).

[11] Biedenharn, Blatt, and Rose, Revs. Modern Phys. **24**, 249 (1952).

where we have omitted the spin dependence in view of the spin-independent nature of Eqs. (11) and (12). Combining this with Eq. (19) and employing the expansion for a product of angular momentum states, we obtain (dropping the subscript on r_0)

$$\Omega_{10}v_0=(g'/4\pi)(-)^{j_c-j-j'+M'}\sum_L\sum_{L'}\sum_{L''}i^L(2L'+1)^{\frac{1}{2}}$$
$$\times(2L''+1)^{-\frac{1}{2}}(J'J-M'M|J'JLM-M')$$
$$\times(L'L00|L'LL''0)(L'L0M-M'|L'LL''M-M')$$
$$\times W(J'j'Jj;j_cL)Z(l'j'lj;\tfrac{1}{2}L)R(r)$$
$$\times R'(r)f_{L'}(r)Y(L'',M-M'). \quad (21)$$

The solution of Eq. (9a) then reduces to the solution of the equation

$$\left[\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}-\frac{L''(L''+1)}{r^2}+\frac{2m}{\hbar^2}(E_1-V_{\text{eff}})\right]h_{L''L'}(r)$$
$$=-R(r)R'(r)f_{L'}(r) \quad (22)$$

with the required asymptotic dependence

$$h_{L''L'}(r)\underset{r\to\infty}{\longrightarrow}H_{L''L'}\frac{1}{r}e^{i[k'r-(L''+1)\pi/2]}, \quad (23)$$

where k' is the wave number of the emergent neutron. The asymptotic dependence of v_1 may therefore be given in terms of $H_{L''L'}$ which, in turn, determines the inelastic scattering amplitude.

It should be noted that the analysis so far gives us only the nonexchange inelastic scattering. There are also contributions from processes in which the incident neutron and the struck target nucleon exchange places. We should only like to point out that for the interaction potential Eq. (12) there is essentially no effect. To prove this, it is sufficient to consider the interaction between the incident particle and the target nucleon, placing the parent nucleus in a passive role. For interaction Eq. (12) there can be no scattering in the space antisymmetric states, i.e., in the triplet spin states. As a consequence, the amplitude of the exchange and nonexchange scattering must be equal so that the amplitude for scattering in the space symmetric state, singlet spin state, is doubled and the cross section quadrupled. This, however, must be multiplied by the probability of finding the particles in the singlet state which is just (1/4), reducing the cross section to that for nonexchange scattering.

We return to Eq. (23) and obtain the cross section as

$$\sigma(\theta)=(g'/2\pi)^2(m/\hbar^2)^2(k'/k)(2J'+1)\Big|\sum_M\sum_{M'}\sum_{L'}\sum_{L_1'}\sum_L\sum_{L_1}\sum_{L''}\sum_{L_1''}(-)^{2M'}\exp[\tfrac{1}{2}i\pi(L+L_1''-L_1-L'')]$$
$$\times[(2L'+1)(2L_1'+1)]^{\frac{1}{2}}[(2L''+1)(2L_1''+1)]^{-\frac{1}{2}}(J'J-M'M|J'JLM-M')(J'J-M'M|J'JL_1M-M')$$
$$\times(L'L00|L'LL''0)(L_1'L_100|L_1'L_1L_1''0)(L'L0M-M'|L'LL''M-M')(L_1'L_10M-M'|L_1'L_1L_1''M-M')$$
$$\times W(J'j'Jj;j_cL)W(J'j'Jj;j_cL_1)Z(l'j'lj;\tfrac{1}{2}L)Z(l'j'lj;\tfrac{1}{2}L_1)H_{L''L'}H_{L_1''L_1'}^*$$
$$\times Y(L'',M-M')Y^*(L_1'',M-M')\Big|. \quad (24)$$

This expression can be simplified somewhat by the methods discussed in reference 11. We obtain finally

$$\sigma(\theta)=\left(\frac{g'^2}{8\pi^{5/2}}\right)\left(\frac{m}{\hbar^2}\right)^2\left(\frac{k'}{k}\right)(2J'+1)\Bigg|\sum_{L'=0}^{\infty}\sum_{L_1'=0}^{\infty}\sum_L\sum_{L''}\sum_{L_1''}\sum_n(-)^L(2n+1)^{-\frac{1}{2}}\exp[\tfrac{1}{2}i\pi(L_1''-L''+L_1'-L'-n)]$$
$$\times(L'L00|L'LL''0)(L_1'L00|L_1'LL_1''0)(L_1''L''00|L_1''L''n0)W^2(J'j'Jj;j_cL)Z^2(l'j'lj;\tfrac{1}{2}L)$$
$$\times Z(L_1'L_1''L'L'';Ln)H_{L''L'}H_{L_1''L_1'}^*Y(n,0)\Bigg|, \quad (25)$$

where the sums on L, L'', L_1'', and n are limited by the vector inequalities

$$|L'-L|\le L''\le L'+L,\quad |L'-L|\le L_1''\le L'+L,$$
$$|l-l'|\le L\le l+l',$$
$$|L_1''-L''|\le n\le L_1''+L'',\quad |L_1'-L'|\le n\le L_1'+L',$$

and $L''+L'+L$, $L_1''+L_1'+L$, $l+l'+L$, and $L_1'+L'+n$ are all even. The function $Y(n,0)$ is

$$Y(n,0)=[(2n+1)/4\pi]^{\frac{1}{2}}P_n(\cos\theta).$$

The total inelastic cross section is given by 4π times the $n=0$ term in Eq. (25). We obtain

$$\sigma=\left(\frac{g'}{2\pi}\right)^2\left(\frac{m}{\hbar^2}\right)^2\left(\frac{k'}{k}\right)(2J'+1)\Bigg|\sum_{L'=0}^{\infty}\sum_L\sum_{L''}$$
$$\times(L'L00|L'LL''0)^2W^2(J'j'Jj;j_cL)$$
$$\times Z^2(l'j'lj;\tfrac{1}{2}L)|H_{L''L'}|^2\Bigg|. \quad (26)$$

With Eqs. (25) and (26), the problem of evaluating the inelastic cross section requires the solution of differential Eq. (22) subject to the asymptotic condition of Eq. (23). The equation is solved in terms of a Green's function by

$$h_{L''L'}(r)=\int_0^\infty G_{L''}(r|r')w_{L'}(r')r'^2dr', \quad (27)$$

where

$$w_{L'}(r)=R(r)R'(r)f_{L'}(r)$$

and $G_{L''}(r|r')$ satisfies the differential equation

$$\left[\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}-\frac{L''(L''+1)}{r^2}+\frac{2m}{\hbar^2}(E_1-V_{\text{eff}})\right]\times G_{L''}(r|r')=-\delta(r-r'). \quad (28)$$

We distinguish two solutions of the homogeneous form of (28), $F_{L''}(r)$ and $I_{L''}(r)$, where F and I are regular and irregular at the origin, respectively. We note that asymptotically

$$F_{L''}(r)\underset{r\to\infty}{\longrightarrow}\frac{1}{k'r}\exp[i\eta_{L''}(k')]\times\cos[k'r-\tfrac{1}{2}\pi(L''+1)+\eta_{L''}(k')]. \quad (29)$$

We choose the irregular solution which has the following asymptotic form:

$$I_{L''}(r)\underset{r\to\infty}{\longrightarrow}\frac{1}{k'r}\exp[ik'r-\tfrac{1}{2}i\pi(L''+1)+2i\eta_{L''}(k')]. \quad (30)$$

The quantity $\eta_{L''}$ is a complex phase shift. We note that the homogeneous form of Eq. (28) is identical with the equation satisfied by $f_{L''}$, except that the wave number k is replaced by k'. Therefore, $\eta_{L''}(k')$ is the scattering phase shift as given by the "cloudy crystal ball" model[4] evaluated at the energy of the emergent neutron.

The Green's function may now be expressed in terms of $F_{L''}$ and $I_{L''}$ as follows:

$$G_{L''}(r|r')=ik'\exp[-i\eta_{L''}(k')]\times\begin{cases}F_{L''}(r')I_{L''}(r), & r'\le r\\ F_{L''}(r)I_{L''}(r'), & r'\ge r\end{cases} \quad (31)$$

$$\underset{r\to\infty}{\longrightarrow}\frac{i}{r}F_{L''}(r')\exp[ik'r-\tfrac{1}{2}i\pi(L''+1)+i\eta_{L''}(k')]. \quad (32)$$

We may now obtain the general expression for $H_{L''L'}$ by inserting the above result into Eq. (27). We find

$$H_{L''L'}=i\exp[i\eta_{L''}(k')]\int_0^\infty F_{L''}(r')R(r')\times R'(r')f_{L'}(r')r'^2dr'. \quad (33)$$

Note that at sufficiently large distances

$$F_{L''}(r)\to[\cos\eta_{L''}j_{L''}(k'r)-\sin\eta_{L''}n_{L''}(k'r)]\exp(i\eta_{L''}),$$

where j_L and n_L are spherical Bessel and Neumann functions, respectively. As k' approaches zero it is clear the $F_{L''}$ and therefore $H_{L''L'}$ will go to zero as $(k')^{L''}$.

The total cross section and the angular distribution are, of course, sensitive to $H_{L''L'}$. Near threshold, $(k'\to 0)$ the $L''=0$ term will make the principal contribution. In this case Eq. (26) gives

$$\sigma=\left(\frac{g'}{2\pi}\right)^2\left(\frac{m}{\hbar^2}\right)^2\left(\frac{k'}{k}\right)(2J'+1)\sum_L\left(\frac{1}{2L+1}\right)\times W^2(J'j'Jj;j_cL)Z^2(l'j'lj;\tfrac{1}{2}L)|H_{0L}|^2, \quad (34)$$

so that near threshold $\sigma\sim k'$. Similarly, we may see directly from Eq. (25) that the angular distribution near threshold will be spherical. Both of these results are, of course, expected.

Simplifications will also occur when the energies E or E' fall within a resonance of the "cloudy crystal ball" model. Then $F_{L''}$ or $f_{L'}$ or possibly both will be large within the nucleus and one may expect that $H_{L''L'}$ would be correspondingly large. If both E and E' are within such a resonance, one would expect that the terms in Eqs. (25) and (26) involving only a particular L'' and L' to be dominant. The angular distribution in this case will be for the most part symmetric about 90°. This, however, will not hold when one of the energies falls outside the resonant region. Then many terms in Eq. (25) will contribute to the angular distribution which will then depend upon the details of the situation under investigation.

We conclude this section by listing one case for which Eq. (25) simplifies greatly. If $J=J'=0$, i.e., if both the ground and excited states of the target nucleus have zero spin, then

$$\sigma(\theta)=\left(\frac{g'}{2\pi}\right)^2\left(\frac{m}{\hbar^2}\right)^2\left(\frac{k'}{k}\right)|\sum_n i^nH_{nn}Y(n,0)|^2. \quad (35)$$

BORN APPROXIMATION

For purposes of qualitative understanding it is useful to solve Eq. (9b) by the Born approximation rather than using the wave functions given by the "cloudy crystal ball." We of course realize that such a calculation can only serve as a guide and as a consequence we do not give any of the details. The cross section for inelastic scattering in the Born approximation is

$$\sigma(\theta)=\left(\frac{g'}{2\pi}\right)^2\left(\frac{m}{\hbar^2}\right)^2\left(\frac{k'}{k}\right)(2J'+1)\times\sum_L W^2(J'j'Jj;j_cL)Z^2(l'j'lj;\tfrac{1}{2}L)|f_L|^2, \quad (36)$$

where

$$f_L=\int RR'j_L(|\mathbf{k}-\mathbf{k}'|r)r^2dr. \quad (37)$$

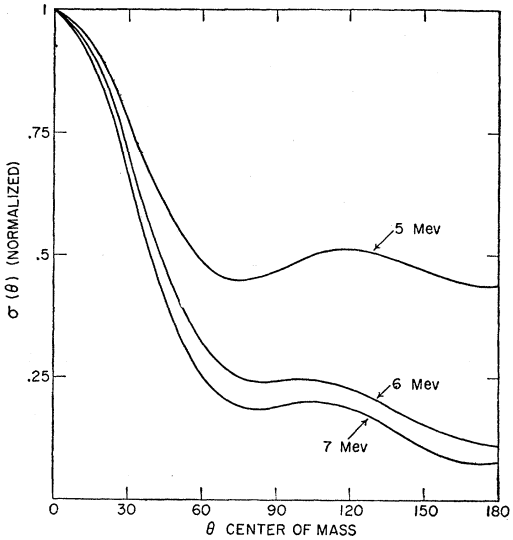

FIG. 1. Normalized angular distributions in center-of-mass system of neutrons inelastically scattered from C^{12} with excitation of 4.43-Mev level for incident energy of 5, 6, and 7 Mev.

Here $\mathbf{k}$ is the wave number of the incident neutron, $\mathbf{k}'$ of the outgoing neutron.

From this formula we can deduce some simple results. The angular distribution near threshold is flat, but as the energy increases it becomes rapidly anisotropic. Even when J' is not equal to zero, the angular distribution is not zero near zero deflection since even here k does not equal k'. Since the argument of j_L in Eq. (37) will generally be relatively large for neutrons undergoing 180° deflection, we will generally find that the distribution is peaked in the forward direction. Similarly we find that for relatively large excitation energies the magnitude of the cross section is reduced since again $\mathbf{k}-\mathbf{k}'$ is large. Again, if the change in the spin of the target nucleus is large so that L cannot assume small values, the cross section is reduced. When these two factors combine, the over-all effect is to reduce the cross section by an order of magnitude. For example, the inelastic cross section for excitation of a level 4.4 Mev above the ground state and involving a spin change of 2 is roughly (1/10) the cross section for the excitation of a level 1 Mev above the ground state and involving no spin change, all other factors being equal.

APPLICATION TO CARBON

As we pointed out in the introduction, the most favorable set of circumstances for the observation of this effect at low energies will occur in light nuclei. Here, the levels of the compound nucleus are sufficiently widely spaced so that in between them their contribution to the inelastic cross section will be a minimum. We have therefore evaluated the cross section for the direct process for a light nucleus, namely, C^{12}. Unfortunately, recent evidence indicates that there are several resonances in the energy region near the first level so that this nucleus may not be a good target for testing our results.

The chief problem which arises in the application of the preceding analysis to a specific nucleus is the determination of the functions $R(r)$ and $R'(r)$. In the present calculations, we have assumed that the 4.43-Mev level arises from the transition of a single particle across the split $1p$ shell.[12] To obtain specific representations of $R(r)$ and $R'(r)$, we have used wave functions of a square well with radius equal to the nuclear radius, and with a depth adjusted to give the observed binding energy of the ground and first excited state for a δ-function spin-orbit potential located at the well edge. For C^{12} we use $J=0$, $J'=2$, $j_c=\frac{3}{2}$, $j=\frac{3}{2}$, $j'=\frac{1}{2}$, $l=l'=1$. The elastic scattering cross section calculated with the parameters of Eq. (11) fits the experimental data only very roughly. However, since we are mainly interested in qualitative results we have not attempted to adjust the parameters of the well to obtain a better fit, although this procedure should be followed in a more quantitative study of inelastic scattering. The computed (normalized) inelastic angular distributions for three different incident energies are shown in Fig. 1.

The total cross section is given in Fig. 2, and shows a characteristic rise with energy. The scale is, of course, relative since we have no way of evaluating g'. However, it is useful to make a rough estimate. We take for g the integral over the singlet nucleon-nucleon potential, *viz.*,

$$g=4\pi\int V_s(r)r^2dr.$$

We take unity for the factor arising out of interactions with "core" particles, minimizing the resultant cross section. The constant $(mg'/2\pi\hbar^2)^2$ then turns out to be about 100 mb, indicating that under favorable circumstances, e.g., spin change zero and low excited level, the cross section for the direct process can be consider-

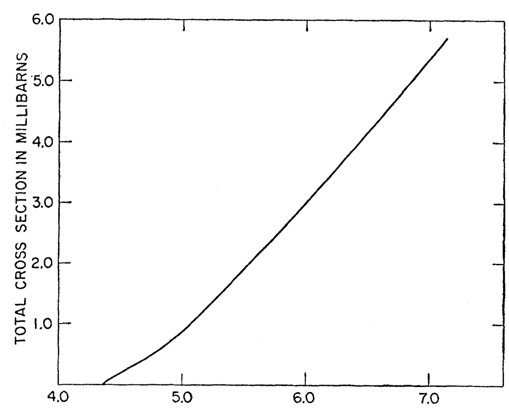

FIG. 2. Total cross section for excitation of 4.43-Mev level in C^{12} as a function of energy in center-of-mass system using integrated singlet nucleon-nucleon potential for interaction constant.

[12] D. R. Inglis, Revs. Modern Phys. 25, 390 (1953).

able. With this value the scale for the abscissa in Fig. 2 is, as shown, of the order of millibarns, and we see that the cross section for this case is very small. On the other hand, a similar calculation by Margolis and Pollack[13] for a zero to zero transition, with the excited level 1 Mev above ground, gives a cross section of several tens of millibarns, very close to the maximum of 100 mb.

[13] B. Margolis (private communication).

ACKNOWLEDGMENTS

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Neutrons from the D-D Reactions*

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An investigation has been made of the neutron spectra resulting from D-D reactions for a range of incident deuteron energies of 4.75 to 7.33 Mev, using a pulsed-beam time-of-flight method. In addition to monoenergetic neutrons from the reaction $D(d,n)He^3$, a continuum of neutrons is observed corresponding to $D(d,np)D$. The yield of the latter reaction has been measured at about 0.5-Mev intervals at zero degrees, and at an energy of 6.3 Mev an angular distribution has been obtained for both reactions. At 6.3-Mev deuteron energy and at zero degrees the yield of the continuum for neutron energies above 0.95 Mev is 17% of the yield of monoenergetic neutrons. The yield from both reactions is strongly forward in the laboratory reference frame, and for the breakup process increases rapidly with increase in energy above threshold.

INTRODUCTION

THE exothermic reaction $D(d,n)He^3$ is commonly used as a source of monoenergetic neutrons for energies above 2.5 Mev. The neutrons from this reaction are the only ones produced in a D-D process up to the bombarding energy for which breakup of one of the deuterons becomes energetically possible. Above that energy a continuum of neutrons is to be expected due to the three-body process $D(d,np)D$ in addition to the monoenergetic group due to $D(d,n)He^3$. Such a continuum has indeed been observed for 14-Mev deuterons,[1] although at that energy breakup of both deuterons is energetically possible. It seemed worthwhile, therefore, to make a detailed examination of the neutron spectra from the D-D reaction at energies above the threshold for breakup, that is, above a laboratory energy of 4.45 Mev. Such data should be useful as input data for other experiments which use energetic D-D neutrons, and are pertinent to the physics of the reactions of light nuclei.

METHOD AND APPARATUS

Neutron spectra were obtained by a straightforward application of the pulsed-beam time-of-flight techniques previously described,[2] using the pulsed deuteron beam output of the large Los Alamos electrostatic accelerator. The gas target was 6 cm long, filled with D_2 gas at a pressure of 60 cm of Hg, and was sealed off from the vacuum system by a nickel foil 1.5 μ thick. Design of the target was such as to minimize the amount of material in the immediate vicinity of the gas which might scatter neutrons. Although this was the first time a pulsed deuteron beam was used no special problems were encountered. A high neutron background filled the target area even when there was no beam on target, due presumably to the deflected beam striking low-Z material, but the shielding around the detector was very effective against this background. The detector used was the same as the one previously described[2] and was mounted at a distance of 1.5 meters from the target. The average target current was about 0.05 μa and the running time for a spectrum was about ten minutes.

RESULTS

Spectra have been obtained as a function of angle at the primary deuteron energy of 6.3 Mev at ten degree intervals up to 40 degrees, and at zero degrees data have been taken in approximately 0.5-Mev intervals from 4.75 Mev to 7.33 Mev.

Figure 1 shows the time spectra obtained at zero degrees with and without deuterium in the target for a deuteron energy of 6.3 Mev. These spectra were recorded in the same way as those described previously.[2] Although the raw data are in the form of a 100-channel

* This work performed under the auspices of the U. S. Atomic Energy Commission.
[1] Bogdanov, Vlasov, Kalinin, Rybakov, and Sidorov (private communication); J. Exptl. Theoret. Phys. (U.S.S.R.) (to be published).
[2] L. Cranberg and J. S. Levin, Phys. Rev. 103, 343 (1956).