Hyperfine Structure of the Metastable Deuterium Atom*†

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The hyperfine separation $\Delta\nu(2S;\mathrm{D})$ of the metastable $2\,^2S_{\frac{1}{2}}$ state of the deuterium atom has been measured by an atomic-beam magnetic-resonance method that has been described previously. We found that $\Delta\nu(2S;\mathrm{D}) = 40\,924.439\pm0.020$ kc/sec. From this result and the ground-state separation $\Delta\nu(1S;\mathrm{D})$ we obtain $R(\mathrm{D}) \equiv \Delta\nu(2S;\mathrm{D})/\Delta\nu(1S;\mathrm{D}) = \frac{1}{8}(1.000\,034\,2\pm0.000\,000\,6)$. Comparison of this value with the corresponding quantity for hydrogen, $R(\mathrm{H}) = \frac{1}{8}(1.000\,034\,6\pm0.000\,000\,3)$, confirms within experimental error the theoretical prediction that $R(\mathrm{D}) = R(\mathrm{H})$.

INTRODUCTION

RECENTLY the hyperfine separation $\Delta\nu(2S;\mathrm{H})$ of the $2S$ state of the hydrogen atom has been determined.[1] The investigation resulted in the discovery of a small discrepancy ΔR, which has not yet been fully explained, between the experimental and theoretical values of the ratio $R(\mathrm{H}) \equiv \Delta\nu(2S;\mathrm{H})/\Delta\nu(1S;\mathrm{H})$.

For reasons discussed in Sec. 18 of HRK, contributions to R from nuclear-structure effects are expected to be of the order of a part in 10^{10}. Such an effect is too small to be observed with present techniques. A consideration of nuclear-structure effects alone leads to

$$R(\mathrm{D}) = R(\mathrm{H}). \quad (1)$$

An experimental measurement of $\Delta\nu(2S;\mathrm{D})$ serves two purposes, to test the prediction that $R(\mathrm{D}) = R(\mathrm{H})$ and to provide an additional determination of ΔR.

The relation $R(\mathrm{D}) = R(\mathrm{H})$ implies that the hfs anomaly between hydrogen and deuterium in the $1S$ state[2] is equal to that in the $2S$ state. However, the latest theoretical discussion[3] of the anomaly does not fully account for the observed effect; therefore it appeared worthwhile to test the prediction $R(\mathrm{D}) = R(\mathrm{H})$. A determination of $\Delta\nu(2S;\mathrm{D})$ by the method of HRK is described in the present article; all other quantities entering into (1) are known.

A theoretical value of R derived from the relativistic treatment of hfs of Breit[4] is given in HRK,

$$R_{\mathrm{theor}} = \tfrac{1}{8}[1+\tfrac{5}{8}\alpha^2+O(\alpha^4)]. \quad (2)$$

Deviations of order α^3 from R_{theor} presumably arise from quantum-electrodynamic effects, which are being evaluated.[5]

HYPERFINE STRUCTURE

The deuterium atom has nearly the same fine structure as the hydrogen atom. The Lamb shift is 1059 Mc/sec for D and 1058 Mc/sec for H. The hyperfine separation $\Delta\nu(2S;\mathrm{D})$ is however only about 0.23 that of hydrogen. Therefore the fractional precision of the final result for $\Delta\nu(2S;\mathrm{D})$ is less than that for $\Delta\nu(2S;\mathrm{H})$, because the magnetic-field dependence of the line most favorable for observation is increased markedly, and because the line width cannot readily be reduced in the ratio $\Delta\nu(\mathrm{D})/\Delta\nu(\mathrm{H})$.

In zero magnetic field there are two quantum states, $F=\frac{3}{2}$ and $F=\frac{1}{2}$. In a magnetic field each level splits into $2F+1$ sublevels, whose energies are given by the Breit-Rabi formula[6] and are plotted in Fig. 1. These states are denoted by the indices $1\cdots6$. The symbols α and β correspond to the notation of Lamb and Retherford[7] for $m_J=+\frac{1}{2}$ and $m_J=-\frac{1}{2}$ states, respectively. For low magnetic field ($H \ll \Delta W/\mu_0$), we expand the Breit-Rabi formula to obtain expressions for the resonant frequencies of interest:

$$\begin{aligned} \nu_{26} &= \Delta\nu + AH + BH^2, \\ \nu_{25} &= \Delta\nu + BH^2 - CH, \\ \nu_{36} &= \Delta\nu + BH^2 + CH, \\ \nu_{35} &= \Delta\nu - AH + BH^2, \end{aligned} \quad (3)$$

where $B=85.26$ (kc/sec) gauss^{-2}, $A=934$ (kc/sec) gauss^{-1}, and $C=0.654$ (kc/sec) gauss^{-1}. The approximate value 40 924 kc/sec was used for $\Delta\nu(2S;\mathrm{D})$ in computing B. Terms in H^3 and H^4 are negligible.

All allowed transitions from any α to any β state are observable by the present method. The best determination of $\Delta\nu(2S;\mathrm{D})$ may be made from a measurement of the frequency of the center of the closely spaced doublet (2,5) (3,6), together with one of the frequency of a line with linear field dependence, here chosen to be (3,5). Any other field-dependent line would serve equally well for the determination of the field. The doublet, however, has the unique property that its central frequency has no term linearly dependent on field.

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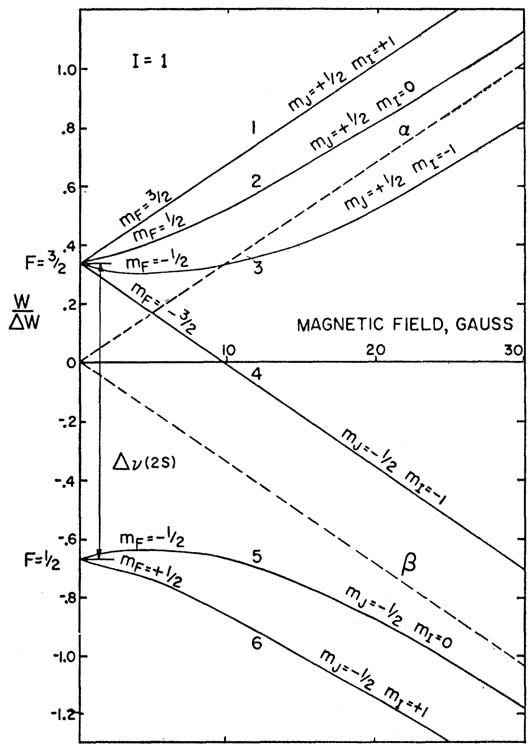

FIG. 1. Zeeman splitting of the hyperfine structure of the $2S$ state of deuterium. The dashed lines show the energy levels when hfs is ignored.

The doublet splitting of $2g_I\mu_0H/h$ is approximately 200 cps at the field used in this experiment. Such a doublet is unresolved even at the highest resolutions attained in atomic-beam spectroscopy. Actually the mean frequency $\nu_\pi\equiv\frac{1}{2}(\nu_{25}+\nu_{36})$ of the two lines is determined here.

A possible uncertainty in the result is introduced because the matrix elements of the rf perturbation between the two pairs of states have the ratio $(1+x)/(1-x)$, where $x=(g_J-g_I)\mu_0H/h\Delta\nu$. Hence the transition probabilities are unequal and have the same ratio at low rf amplitudes. The parameter x has a value of 1 when $H=14.6$ gauss. In this experiment the effect of the differential probability is very small since $x=0.012$.

METHOD AND APPARATUS

Both the method and the apparatus used in the present experiment were the same as those described previously (HRK). The frequency at which transitions occur in D is less by a factor of 4.3 than that at which they occur in H. Even though the hyperfine separation in D differs from that in H, the components of the β level of the $2S_{\frac{1}{2}}$ state cross the components of the e level of the $2P_{\frac{1}{2}}$ state at almost the same magnetic field in D as in H. The decay length of atoms in the β states is very short for a considerable range of fields about 575 gauss. Consequently it was not necessary to change the fields in the polarizing and analyzing magnets.

The *fractional quenching* ϕ has been defined in Sec. 14 of HRK in terms of observable quantities. It is proportional and nearly equal to the ratio of the number of metastable atoms quenched by the rf to the number of metastable atoms in the beam.

rf AND dc FIELDS

The Collins 32V-3 transmitter,[8] operated in the vicinity of 20.46 Mc/sec and followed by a frequency doubler, was used to generate rf power for inducing transitions.

The two lines adjacent to the doublet under observation are separated from it by about 160 kc/sec, and the line (1,6) is separated from the doublet by about 320 kc/sec at the magnetic field used in these experiments. Therefore the rf signal had to be free from side bands, since such components of the rf signal might have induced transitions in lines other than the one under investigation and thus have led to a distortion or shift of the line. No side bands were observed within the frequency range of the whole spectrum $\Delta F=-1$, $\Delta m_F=0, \pm1$.

The choice of field in the transition region is determined by two requirements. The magnetostatic field must be so low that the term BH^2 in Eq. (3) can be determined accurately, even in the face of inevitable inhomogeneity in the field. On the other hand, the field must be sufficiently high so that overlap of the adjacent lines (2,6) and (3,5) does not distort the shape of the doublet under observation. Evidently these two requirements are not consistent, and a compromise must be made.

It is desirable to arrange the magnetostatic field in the rf quenchers so that it forms an angle of about 60° with the direction of the rf magnetic field which induces transitions. The σ lines (2,6) and (3,5) may then be observed together with the π doublet (2,5) (3,6) with no change in the externally imposed parameters except in the frequency and amplitude of the rf field.

In actual fact the constant field in the rf quenchers was almost the same as that used by HRK, where the relative direction of the rf and static fields was found to be satisfactory and the static field was reasonably homogeneous. A comparison of Eq. (3) with Eq. (4) of HRK indicates that the quadratic term in D is fractionally about 17 times as great as the corresponding quantity in H. Therefore, to obtain the same fractional precision in $\Delta\nu$(D) as in $\Delta\nu$(H), the magnetic fields must be determined with a precision 17 times as great. For example, an error of 0.001 gauss in the determination of the field of 0.170 gauss causes an absolute error of 29 cps in $\Delta\nu$(D), or fractionally, 0.75 ppm; the same error in the field gives a fractional error of only about 0.05 ppm in $\Delta\nu$(H). Nevertheless it was not believed to be worthwhile to reduce the field because of the danger of developing an overlap error.

[8] Manufactured by Collins Radio Company, Cedar Rapids, Iowa.

LINE SHAPE AND INTENSITY

Line Width

Since the mass of the deuterium atom is twice that of the hydrogen atom, the deuterium atoms should have an average velocity 0.707 that of hydrogen atoms produced in a source at the same temperature. The width of the resonance line for D might therefore be expected to be 0.707 that for H. Since the metastable atoms recoil in the exciter, a unidirectional beam entering the exciter leaves it with some distribution in recoil angle about some mean angle which depends on the velocity distribution in the incident beam. The mean recoil angle is smaller than in H, and the range of velocities corresponding to a given range of recoil angles is greater than in H, because of the increased mass of deuterium. In the adjustment of the apparatus the beam of normal atoms is directed into the exciter in such a way that the ratio of the flux of metastable atoms incident at the detector to the background signal is a maximum. The range of recoil angles is determined by the aperture in the second rf quencher. Hence the velocity distribution under observation is broader in D than in H. The width of the resonance line as extrapolated from measurements on H will therefore be increased. It is not possible to make an exact calculation of this increase, especially since the apparatus is subject to empirical adjustments which have an effect on the line width. Typical observations on H were taken with a line width of 5.5 kc/sec. The line width observed for D, 4.8 kc/sec, is greater than the width of 3.9 kc/sec extrapolated from H. The discrepancy is considered to be reasonable.

It will be noted that the rf resonances shown in Fig. 2 have very nearly the same half-widths even though they are observed at rf amplitudes which differ by a factor of three. As the rf amplitude is reduced below the value at which the rf quenching is a maximum, a decrease in the half-width is to be expected since the transition probability is then greater for atoms of low velocity. Ramsey[9] gives an example of such a reduction. In the present case, where the distribution in velocity is narrower than a Maxwellian distribution and where the total range in rf amplitude includes both those greater and less than the optimum amplitude, no large change in widths is to be expected. Without a detailed knowledge of the velocity distribution it is not possible to make a precise calculation of line width as a function of rf amplitude.

Line Intensity

The rf quenching depends on both the rf amplitude and the frequency. We define the *peak quenching* as the maximum quenching at the center of a resonance line as the rf amplitude is varied.

If the states 1, 2, and 3 have equal populations, and if all α atoms have the same velocity, a peak quenching of 66% is predicted. On the other hand, if the velocity distribution were Maxwellian, the peak quenching would be about 50%. The observed peak quenching is 45% (see Fig. 2).

Several effects may cause an observed peak quenching less than the predicted value. The peak quenching is sensitive to the rf amplitude, and an adjustment of the amplitude to a value which deviates on either side from its optimum value will give a reduced peak quenching. The beam has a large cross-sectional area, and the rf amplitude is not wholly constant over the beam. Hence it is not possible to adjust the amplitude to its optimum value over the whole beam cross section, and the rf quenching will, for this reason alone, be less than its maximum value.

The rf amplitude is always the same in the two quenchers. If the direction of the static field with respect to the direction of the rf field (whose direction in space is fixed by mechanical arrangements) were different in the two quenchers, the effective amplitude of the rf field in the two quenchers would be different, with a consequent reduction in the peak quenching. Even if this effect is significant, it does not introduce a systematic error in the calculation of BH^2.

PROCEDURE

To determine the magnetic field, the field-dependent line (3,5) was traversed in 6 to 8 steps at predetermined frequencies. The quenching of the field-independent line

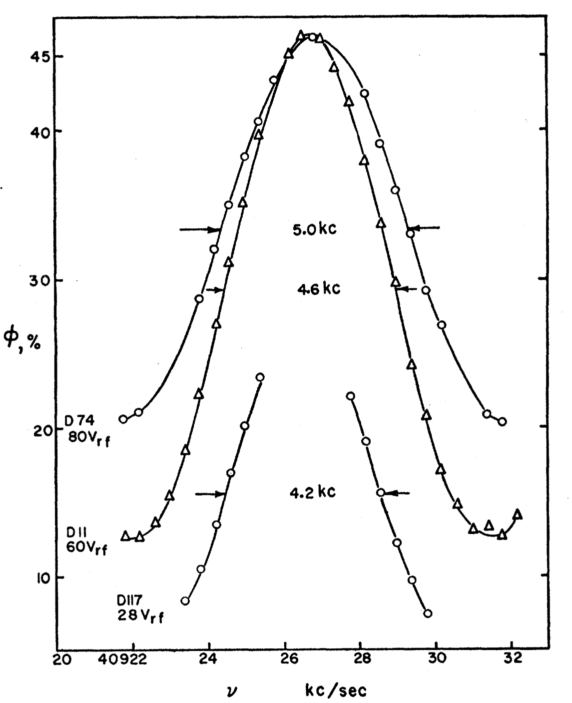

FIG. 2. Central peaks of the field-independent doublet (2,5) (3,6) at several typical rf amplitudes.

[9] N. F. Ramsey, Phys. Rev. 78, 695 (1950).

TABLE I. Results of all runs, deuterium.

Run No.	Page	V_{rf}	Phase[a]	$\Delta\nu'-40\,924$ kc/sec
1	D32	60	+	0.437
	33	60	+	0.487
	34	60	+	0.432
	35	60	+	0.454
	36	60	+	0.477
2	D40	60	−	0.459
	41	60	−	0.473
	42	60	−	0.462
	43	60	−	0.463
	44	60	−	0.463
3	D49	64	−	0.452
	50	64	−	0.460
	51	64	−	0.481
4	D57	60	−	0.474
	58	60	−	0.452
	59	60	−	0.453
	60	60	−	0.448
	61	60	−	0.449
5	D65	60	+	0.481
	66	60	+	0.436
	67	60	+	0.456
	68	60	+	0.466
	69	60	+	0.464
6	D73	60	+	0.445
	74	80	+	0.380
	75	80	+	0.411
	76	80	+	0.421
	77	60	+	0.450
7	D87	80	+	0.440
	88	80	+	0.424
	89	80	+	0.404
8	D93	80	+	0.425
	94	80	+	0.411
	95	60	+	0.469
	96	40	+	0.444
	97	40	+	0.455
	98	40	+	0.467
9	D103	40	+	0.459
	104	40	+	0.481
	105	60	+	0.434
	106	40	+	0.455
	107	40	+	0.443
10	D112	50	+	0.468
	113	50	+	0.462
	114	50	+	0.479
	115	40	+	0.439
	116	28	+	0.435
	117	28	+	0.439
	118	28	+	0.440
11	D123	28	+	0.440
	124	28	+	0.452
	125	28	+	0.477
	126	40	+	0.490
	127	50	+	0.452
	128	50	+	0.480
	129	50	+	0.424
12	D134	50	+	0.439
	135	40	+	0.438
	136	28	+	0.499
	137	28	+	0.460
	138	28	+	0.429
	139	50	+	0.428
	140	28	+	0.400

[a] The phase (+) or (−) indicates that the sign of the phase error is different for the two conditions.

(2,5) (3,6) was then measured at 12 preselected frequencies spaced approximately evenly about the two halfwidth points. For simplicity the points were taken in order of increasing or decreasing frequency. To avoid errors, caused by drift of excitation conditions during a traverse, in the determination of the line center to one part in a thousand of its width, traversals in opposite directions were taken in roughly equal numbers. No systematic effect depending on the direction of traverse was noted. The traverse of the (2,5) (3,6) line was followed by another traverse of the (3,5) line. A typical run consisted of seven determinations of $\nu_{3,5}$ interspersed with six determination of ν_π.

Figure 3 shows a typical set of data for the field-dependent line (3,5). The curves are separated by approximately half-hour intervals while the points are taken every 15 seconds. It is apparent that the data are subject to considerable fluctuation. We ascribe this to fluctuations in the magnetic field in the laboratory. These are easily observed as variations in galvanometer deflection when the frequency is adjusted to a point on the resonance curve where ϕ varies rapidly with ν. Fluctuations of about two milligauss were not uncommon. The magnetic field used to evaluate BH^2 was taken to be the mean of the fields found before and after observation of the doublet. The center of the (3,5) line was found from a symmetrical resonance curve sketched through the observed points.

The line (3,5) is single, and hence has a maximum intensity one-half that of the doublet (2,5) (3,6). Because of the fluctuations in the field the line was difficult to observe. Hence this line was observed only at an rf amplitude of 60 volts, where the line had a good intensity. It seems improbable that the frequency of the line (3,5) is sufficiently shifted with rf amplitude to have a significant effect on the correction term derived from it.

The field-independent line (2,5) (3,6) was observed at several values of rf voltage, as shown in Table I. Resonance curves at several typical values of rf voltage are shown in Fig. 2. We note that the strongest Ramsey[9] pattern occurs at 60 volts rf; hence the preponderance of data were taken at that amplitude. The panoramic of the (2,5) (3,6) line in Fig. 4 shows the Ramsey line shape. The operating conditions were not kept as stable during the run taken to show several subsidiary maxima as during the precision runs in which only the central maximum was observed.

REDUCTION OF DATA

For a single determination of ν_π, the ten line centers found by linear interpolation between the twelve observational points are averaged as in Eq. (5) of HRK. From this result is subtracted the quadratic correction corresponding to the average magnetic field for two traversals of the field-dependent line. This correction is of the order of 2000 cps. A sample reduction is shown in Table II. The resulting values of $\Delta\nu'$ (primed because still subject to correction for systematic effects) are shown in Table I and plotted in Fig. 5.

The presence of electrostatic fields in the region between the two rf quenchers would modify the value of $\Delta\nu(2S;\mathrm{D})$ through the dc Stark effect. A study in the case of hydrogen (HRK, Sec. 15) showed that stray electrostatic fields were so small that the Stark effect was negligible. Although the apparatus in the present work was the same in all relevant details as that used for hydrogen, a search for possible dc Stark effect was undertaken. Again no effect was discerned, as discussed under Corrections *C*. A search indicated that the phase shift (HRK, Sec. 6) was negligible (Corrections *F*).

Shifts of $\Delta\nu'$ with V_{rf} may arise from several causes. Ramsey[9] has discussed the transition process in a system in which the rf field has a constant amplitude within the

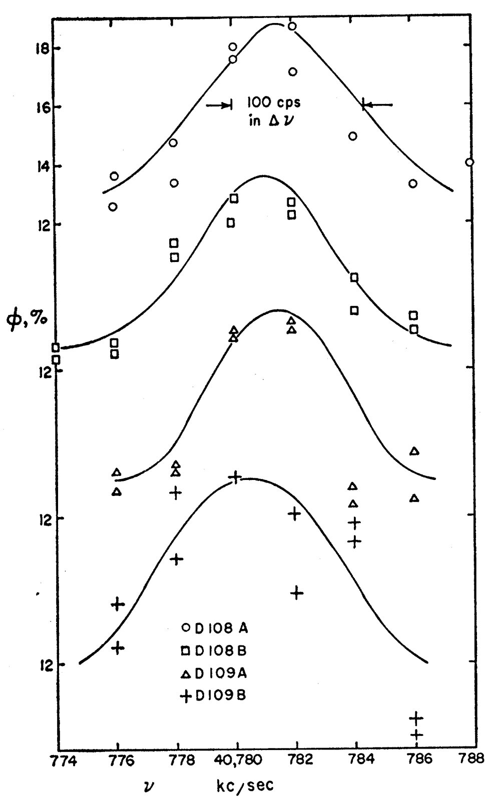

FIG. 3. Typical field-dependent resonances (3,5). To indicate the extent to which the experimental value of $\Delta\nu$ depends on the measured frequency of the line, the error in ν_{35} that produces an error of 100 cps in $\Delta\nu$ is indicated on the upper curve.

TABLE II. Sample reduction of data, field-independent line (2,5) (3,6).

$\nu-40\,920$ kc/sec	$\phi(\%)$	$\nu_\pi-40\,920$ kc/sec	$\phi(\%)$	$\nu-40\,920$ kc/sec
5.800	36.59			
		7.127 2	35.28	8.600
5.400	32.99	7.102 4		
		7.094 9	30.81	9.000
5.000	28.84	7.107 7		
		7.118 6	27.15	9.400
4.600	24.69	7.115 5		
		7.110 8	22.89	9.800
4.200	20.65	7.116 1		
		7.106 7	19.03	10.200
3.800	17.18	7.105 4		
			15.52	10.600

$\bar{\nu}_\pi=40\,927.111$
$-\quad 2.658$

$\Delta\nu'=40\,924.453$ kc/sec

rf quenchers and is zero elsewhere. The frequency of the resonance peaks is then determined by the fields in the precession region if the length of the rf quenchers is negligible compared with that of the precession region. In the present case this condition is not closely met, and the finite average value of $(E_{\mathrm{rf}})^2$ in the quenchers will cause $\Delta\nu'$ to decrease as E_{rf} is increased through the coupling of the *S* and *P* states. In our case the oscillating field is not perfectly confined within the quenchers and the average value of $(E_{\mathrm{rf}})^2$ in the precession region also serves to decrease $\Delta\nu'$.

Magnetic mixing of all the α and β states occurs in the quenchers, since the line-width characteristic of an individual quencher is comparable with the line separation. A dependence of $\Delta\nu'$ on H_{rf} may thus occur.

The Bloch-Siegert[10] effect is negligible for the case here considered.

In traversing a quencher the metastable atom experiences a continuous range of rf amplitude rather than a rectangular pulse as postulated in Ramsey's treatment of separated oscillating fields. The effect of the continuous variation has not been analyzed; it is conceivable that the line frequency may depend on rf amplitude.

The combination of these effects causes the sign of the dependence of the line frequency on rf amplitude to be unknown. It is certain, however, that the best value of $\Delta\nu$ is obtained for vanishingly small values of the amplitude.

The dependence of the frequency of the doublet (2,5) (3,6) on rf amplitude was investigated. The data were taken in the sequence shown in Table I. After the runs 6, 7, 8, in which $\Delta\nu'$ was measured at 80 volts rf, it appeared that $\Delta\nu'$ at 80 volts rf was significantly lower than at 60 volts rf. The observed decrease in $\Delta\nu'$ appeared to be a real effect rather than a statistically fortuitous one, though the latter possibility is not excluded. However, the quantity of interest is the value of $\Delta\nu'$ extrapolated to zero rf amplitude. Accordingly,

[10] This is discussed in a convenient form by N. F. Ramsey, *Molecular Beams* (Clarendon Press, Oxford, 1956), p. 122.

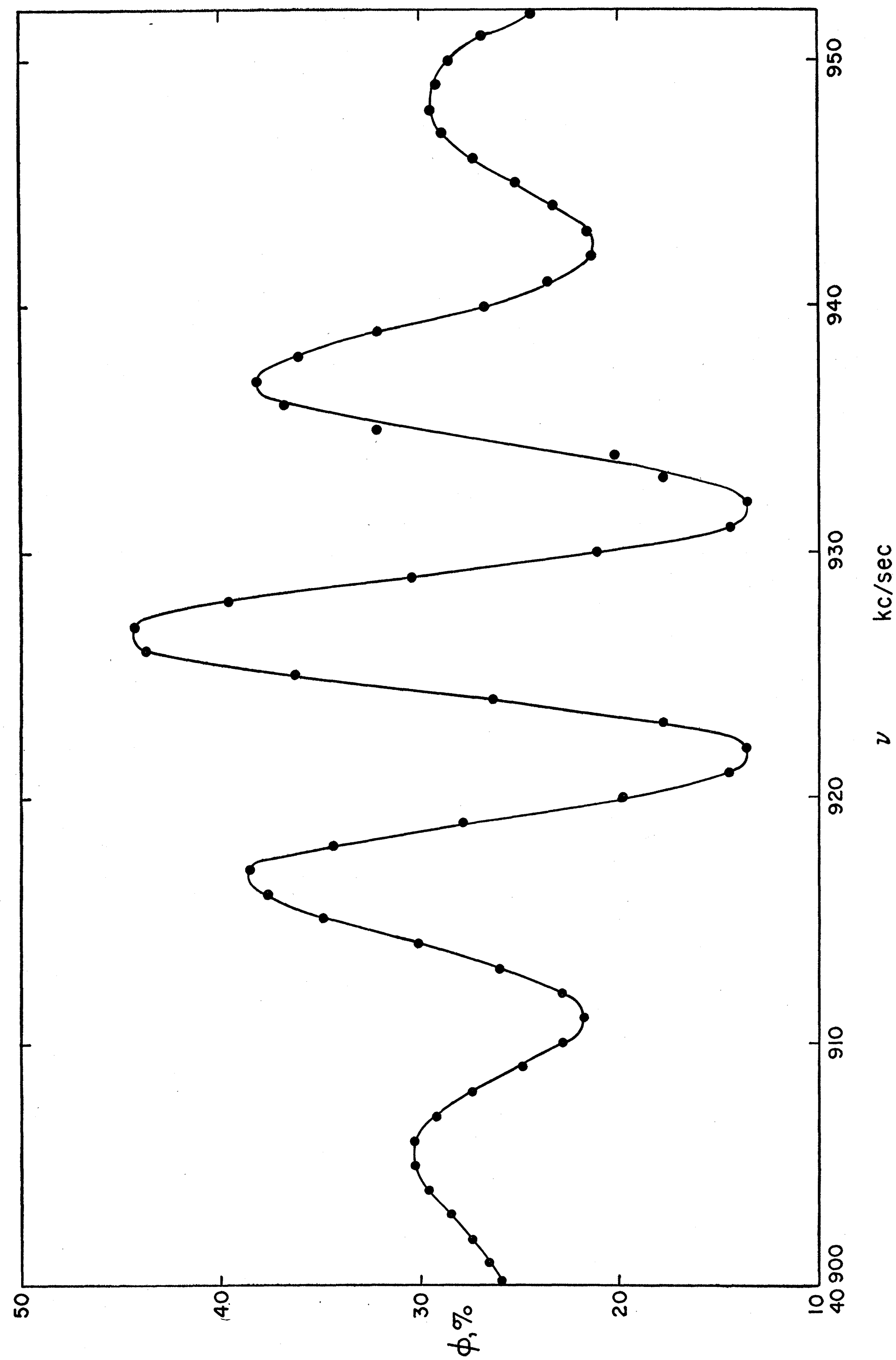

FIG. 4. Panoramic of the field-independent doublet (2,5) (3,6) showing several subsidiary maxima in the Ramsey pattern. Since the velocity distribution is considerably sharper than a Maxwellian distribution, the subsidiary maxima are not damped as rapidly as in Ramsey's theoretically calculated curve.

the remaining data (runs 9–12) were taken at a lower range of rf voltage, from 60 volts down to 28 volts, and the drop in $\Delta\nu'$ at 80 volts was not further investigated. At 28 volts the rf quenching was considerably less than at 60 volts. It did not seem to be worthwhile, in view of the rapidly decreasing precision with which the line center could be found as the rf voltage was decreased, to obtain data at still lower voltages.

Since we are unable to account quantitatively for the apparent drop in $\Delta\nu'$ at 80 volts rf, we exclude these data from further consideration. The remaining 55 values of $\Delta\nu'$ are treated by the method of least-squares under two assumptions: (a) $\Delta\nu'$ is the form $\Delta\nu'=\Delta\nu_0'+\gamma(V_{\rm rf})^2$ and (b) $\Delta\nu'$ does not depend on $V_{\rm rf}$. Assumption (a) leads to a vlaue of

$$\Delta\nu_0'=40\,924.447\pm0.004 \text{ kc/sec.}$$

The associated value of γ is $+0.0031\pm0.0015$ cps/volt^2. Assumption (b) gives a value of

$$\Delta\nu_0'=40\,924.456\pm0.002 \text{ kc/sec.}$$

The stated uncertainties are the probable errors.

We doubt that the slope γ has a physical significance. For example, if the lowest value of $\Delta\nu'$ at 28 volts is removed from the body of data, the slope is reduced to one-half of the stated value and has the same probable error. We also note that γ is positive rather than negative as would be expected if the rf stark effect gave rise to a significant change in $\Delta\nu'$.

Since we cannot choose between (a) and (b), we take $\Delta\nu_0'$ to be the mean of the extremes of (a) and (b) with an uncertainty large enough to cover both. The resulting value of $\Delta\nu_0'$ is

$$\Delta\nu_0'=40\,924.451\pm0.008 \text{ kc/sec.}$$

It appears to us that the quoted uncertainty includes all reasonable interpretations of the data. The mean value of $\Delta\nu'$ at each rf amplitude (except 64 volts) considered in the reduction of the data lies within the range quoted for $\Delta\nu_0'$. The discrepancy at 64 volts, where only three observations were made, lies within the sum of the uncertainties for $\Delta\nu_0'$ and for $\Delta\nu'$ at 64 volts.

CORRECTIONS AND UNCERTAINTIES

(*A*) *Statistical uncertainty*.—The spread in the data can be accounted for largely by the fluctuations of the magnetic field. Since they appeared to be random, their effect was reduced by the acquisition of a statistically significant body of data.

(*B*) *rf Stark effect*.—The rf Stark effect was discussed in Appendix A of HRK. Since the experimental evidence is not clear, we assign an uncertainty large enough to cover the maximum effect consistent with the data. The systematic observation of the line center at low rf amplitudes has reduced the possibility that a significant error in the final result has been introduced by the ambiguity in our extrapolation to zero rf amplitude.

(*C*) *dc Stark effect*.—No systematic differences in the value of $\Delta\nu'$ were found with the electrostatic shield (HRK, Sec. 10) present hot, cold, or absent.

(*D*) *Inhomogeneity of the magnetic field*.—Inhomogeneity of the magnetostatic field gives rise to an error which is rather large and uncertain, compared with that in hydrogen. The error occurs because a measurement of the field-dependent line yields the average value of H in the precession region. The quadratic correction, however, requires a knowledge of $\langle H^2\rangle$, which in all reductions to this point we have assumed to be equal to $\langle H\rangle^2$. An exploration of the field with a flipcoil has indicated that the difference $\langle H^2\rangle-\langle H\rangle^2$ yields a correction of -10 ± 5 cps.

(*E*) *Frequency standard*.—The laboratory standard was reset to radio station WWV several times during a run. The error in an individual determination of $\Delta\nu'$ is thus random in sign and magnitude and is already included in the statistical uncertainty.

(*F*) *Phase shift*.—The phase shift error is discussed in Sec. 14 of HRK. The value of $\Delta\nu'$ was measured at 60 volts rf for both phase settings. The mean values differ by 3.3 ± 6 cps. An uncertainty of ±3 cps must, however, be added to our result to include a possible uncompensated phase error, since a greater volume of data was taken with one phase than with the other. The phase error is here less than in HRK presumably because the circuit losses are smaller at lower frequency.

(*G*) *Overlap error*.—The effect of the two adjacent σ lines on the central π line cancels out because they are both equally separated from the π line and have equal

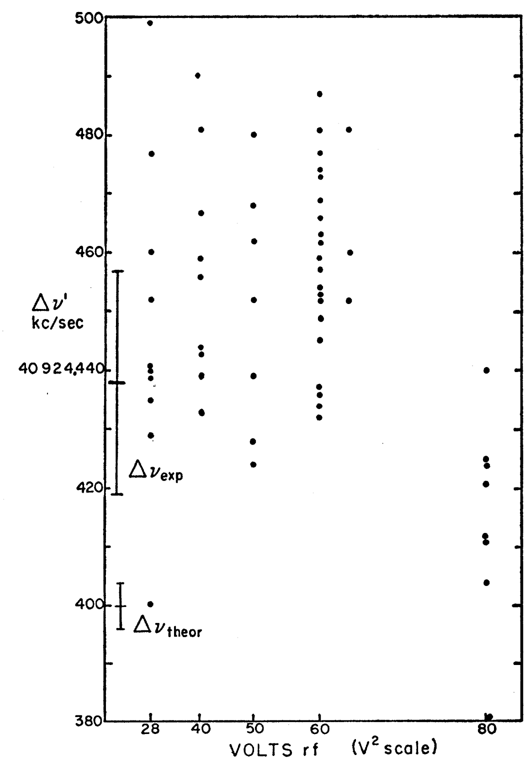

FIG. 5. Experimental values of $\Delta\nu(2S;{\rm D})$ as a function of rf amplitude. The upper of the two results plotted at $V_{\rm rf}=0$ is the experimental result after application of the corrections for systematic effects. The value of $\Delta\nu_{\rm theor}$ has been calculated from $\Delta\nu(1S;{\rm D})$ and $R_{\rm theor}$.

transition probabilities. The effect of the π line (1,6) has given no observable asymmetry as measured by the depth of the two minima immediately adjacent to the central maximum of the (2,5) (3,6) line. We estimate, however, a correction of -3 ± 3 cps.

(*H*) *Doublet error*.—The correction to $\Delta\nu$ for the unequal transition probabilities of the two doublet components (2,5) and (3,6) is $+1\pm1$ cps. The correction is positive because the amplitude used was less than that required to give a maximum transition probability.

(*I*) *Asymmetry*.—The values of $\Delta\nu'$ obtained by averaging over the upper and lower halves of the field-independent line do not differ by a statistically significant amount.

(*J*) *Variation of rf voltage*.—The rf voltage is set at the same value, as observed on a GR-1800-A voltmeter, for each point. The uncontrolled fluctuations are random and average out in a large body of data. Since the rf quencher presents an inductive reactance to the transmission line, the current will vary as the frequency is varied. As the total variation in frequency is one part in ten thousand in traversing the line, neglect of the variation of current with frequency leads to negligible error.

We take the sum of the uncertainties in the corrections rather than the root-square sum and obtain a total correction of -12 ± 12 cps.

RESULTS AND DISCUSSION

We arrive at a final result,

$$\Delta\nu(2S;\mathrm{D})=40\,924.439\pm0.020\text{ kc/sec},$$

where we have taken the sum of all the stated uncertainties. When this result is divided by the ground-state separation,[2]

$$\Delta\nu(1S;\mathrm{D})=327\,384.302\pm0.030\text{ kc/sec},$$

we obtain

$$R_{\mathrm{exp}}(\mathrm{D})=\tfrac{1}{8}(1.000\,034\,2\pm0.000\,000\,6).$$

The result is to be compared with the previously reported value for hydrogen (HRK),

$$R_{\mathrm{exp}}(\mathrm{H})=\tfrac{1}{8}(1.000\,034\,6\pm0.000\,000\,3).$$

In both cases, the stated uncertainty is considerably greater than its root-square value derived from all known sources of error.

Since

$$R_{\mathrm{theor}}=\tfrac{1}{8}(1.000\,033\,3),$$

the discrepancy for deuterium,

$$\Delta R(\mathrm{D})=R_{\mathrm{exp}}-R_{\mathrm{theor}}=\tfrac{1}{8}(9\pm6)\times10^{-7},$$

while the discrepancy for hydrogen is (HRK)

$$\Delta R(\mathrm{H})=\tfrac{1}{8}(13\pm3)\times10^{-7}.$$

In view of the method of assigning uncertainties this discrepancy is a real effect, though less precisely determined for D than for H. There is thus no evidence for a differential hyperfine structure anomaly for H and D between the 1*S* and 2*S* states.

Tentative estimates by Mittleman of higher order quantum-electrodynamic terms indicate that the discrepancy is the same for hydrogen and deuterium to the order of accuracy attained in this experiment.[5] Calculations by Dr. M. Mittleman of these effects are to be published shortly.

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