PHYSICAL REVIEW VOLUME 104, NUMBER 5 DECEMBER 1, 1956

Effects of a Ring Current on Cosmic Radiation. Impact Zones*

ERNEST C. RAY
Department of Physics, State University of Iowa, Iowa City, Iowa
(Received August 2, 1956)

An investigation of the effects of a ring current on the "0900" (geomagnetic local time) impact zone of flare-associated cosmic-ray intensity increases has been carried out using 55 trajectories calculated on an automatic computer. The particles considered have rigidities of 2 Bv, 6 Bv, and 10 Bv, and arrive at the earth from the vertical direction. Reasonable ring currents shift the longitude of impact by as much as half an hour from that predicted by the dipole theory. The shift may be in either direction, depending on the exact ring parameters chosen. Experimental data so far published are not sufficiently accurate to test the theory. Because of the finite width of observed impact zones together with the rather small shift predicted here, the experimental test is inherently difficult.

INTRODUCTION AND SUMMARY

In a previous paper[1] the author reported the results of a study of the effect of a ring current such as that postulated by Störmer,[2] Chapman and Ferraro,[3] and Schmidt[4] on the latitude variation of cosmic radiation. The study was undertaken in an effort to develop cosmic radiation as a tool for exploring the nature of electromagnetic systems in the earth's vicinity. Toward the same end, the present work discusses the effects of such a ring current on the zones of impact on the earth of charged particles leaving the vicinity of the sun.

The impact zones for the case of a dipole magnetic field alone were first investigated by Störmer in connection with his auroral theory.[2] Firor[5] has applied the same considerations to cosmic radiation. Using individual trajectories previously measured in model experiments and calculated by several workers,[6] he constructed curves showing the latitude and longitude of arrival of solar particles at the earth as a function of their rigidity and of the time of the year. He then used these curves together with data from four occasions of large cosmic-ray intensity increases to conclude that the particles responsible for the increases had originated in the vicinity of the sun. Three of the cases were accompanied by large visual solar flares. The fourth was associated with the radio fadeout characteristic of a flare, although the flare itself was not seen.

The present discussion is based on the numerical integration of 55 trajectories.

The cases considered were particles arriving vertically with rigidities of 2 Bv, 6 Bv, and 10 Bv. As one might expect from the first paper,[1] the impacts occur at lower latitudes than they would if no ring were present. In general, the longitude effect is small. The difference in local time of impact between the pure dipole case and the case of dipole plus ring current amounts to as much as half an hour and may be in either direction, depending on the ring current parameters.

It is not possible, using present data, to either show or disprove the existence of these effects. More data on Firor's small-flare effect might resolve the question, although for two reasons it will be difficult to obtain the accuracy required. In the first place, very long counting periods will be necessary in order to obtain adequate statistics. In the second place, the lines of impact discussed here become rather broad impact bands when one takes account of trajectories arriving from nonvertical directions and of possible interplanetary electromagnetic systems which would have the effect of making the sun seem to be a finite-sized source.

CALCULATION AND TRAJECTORIES

In order to discover the effect of a ring current on the impact zones, it is necessary to find a number of individual particle trajectories in the combined magnetic field due to the earth's dipole and an assumed ring current. The similar problem in the field of the dipole alone is thoroughly discussed by Störmer.[2] The vector potential of a filamentary ring current is rather unwieldy, depending on complete elliptic integrals. In order to simplify the equations to a manageable set, a suggestion of Chapman[7] was adopted. The φ component of the vector potential due to the external current system is taken to be

$$A_\varphi=\begin{cases} M_r r\cos\lambda/a^3, & r\leq a \\ M_r\cos\lambda/r^2, & r\geq a. \end{cases}$$

Here λ is the geomagnetic latitude, M_r is a parameter playing the role of a dipole moment in fixing the strength of the magnetic field (it is measured in units of the earth's dipole moment), and r is the distance in Störmer units from the earth's dipole to the point of interest.

* Assisted by a joint research program of the Office of Naval Research and the U. S. Atomic Energy Commission.

[1] E. C. Ray, Phys. Rev. **101**, 1142 (1956); **102**, 1689 (1956). The function designating the dashed curve in Fig. 7 should be $j=0.29E^{-0.9}$.

[2] C. Störmer, *The Polar Aurora* (Clarendon Press, Oxford, 1955), p. 340.

[3] S. Chapman and V. C. A. Ferraro, Terrestrial Magnetism and Atm. Elec. **38**, 79 (1933).

[4] A. Schmidt, Z. Geophys. **1**, 1 (1924).

[5] F. Firor, Phys. Rev. **94**, 1017 (1954).

[6] C. Störmer, Astrophys. Norv. **1**, 1 (1934); K. Dwight, Phys. Rev. **78**, 40 (1950); A. Schluter, Z. Naturforsch. **6a**, 613 (1951); K. G. Malmfors, Arkiv Mat. Astron. Fysik **A32**, No. 8 (1945); E. Brunberg, J. Geophys. Research **58**, 272 (1953).

[7] S. Chapman, Nature **140**, 423 (1937).

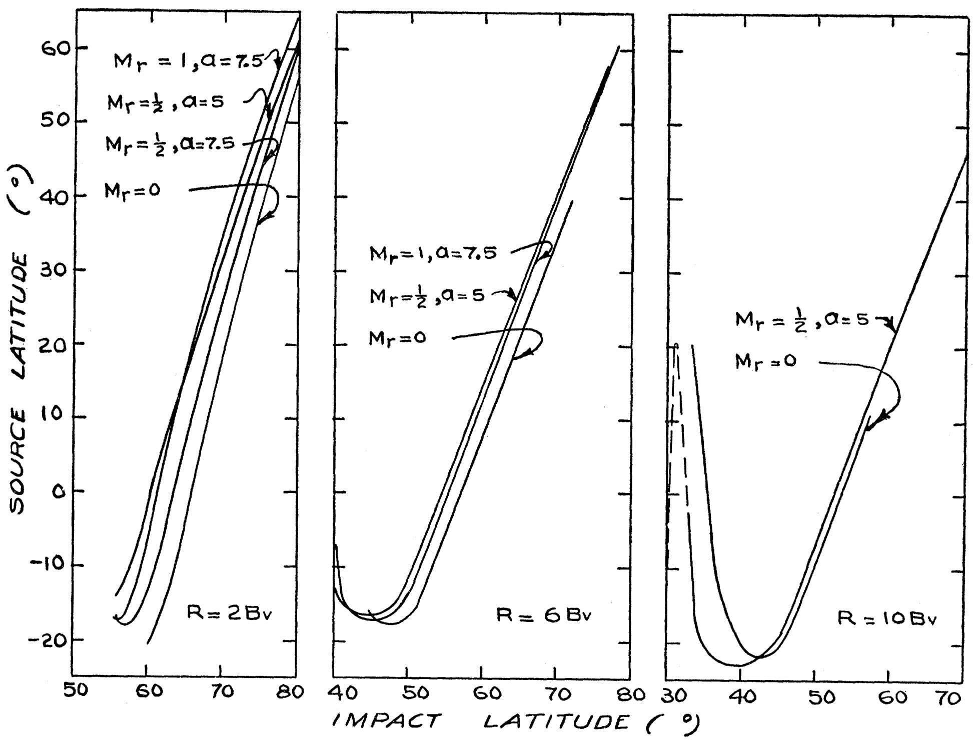

FIG. 1. Geomagnetic latitude of the source *versus* geomagnetic impact latitude.

The current flows from east to west parallel to the geomagnetic equator along the surface of a sphere of radius a (Störmer units) centered on the dipole. The current density is proportional to $\cos\lambda$. The r and λ components of the vector potential are zero.

It may well be that the exact locations of the impact zones depend significantly on the model (spherical current sheet, filamentary ring, etc.) adopted for the external current system. The model adopted here is as likely as any other to approximate an actual "ring" current the very existence of which is not established. On the other hand, the qualitative nature of the results as well as their general magnitudes ought not to be affected.

Using this model of the ring current, one obtains the following equations of motion:

$$\begin{aligned}\frac{d^2R}{ds^2}&=R\frac{d\varphi}{ds}\left[\frac{4\gamma}{R^2}-\frac{3bR^2}{r^5}\frac{d\varphi}{ds}\right],\\ \frac{d^2z}{ds^2}&=-z\frac{3bR^2}{r^5}\frac{d\varphi}{ds},\\ \frac{d\varphi}{ds}&=\frac{2\gamma}{R^2}-\frac{b}{r^3}-D.\end{aligned}\tag{1}$$

R, φ, and z are the coordinates in Störmer units of a point on the trajectory[1]; $r^2=R^2+z^2$; γ is the same as Störmer's γ_1; s is the path length in Störmer units along the trajectory, and b and D are given by

$$b=1,\quad D=M_r/a^3 \quad \text{when} \quad r<a.$$
$$b=M_r+1,\quad D=0 \quad \text{when} \quad r>a.$$

Fifty-five trajectories were integrated numerically on an IBM 650 computer. The method used has been described by Milne.[8]

The numerical integrations were carried out from the earth's surface to $r\cong 5$. The solutions were continued to infinity by the following approximate analytic integrations. As first shown by Störmer[9] for the dipole case, the equations of motion can be put in the form

$$\frac{d^2x}{d\sigma^2}=\frac{1}{2}\frac{\partial P}{\partial x};\quad \frac{d^2\lambda}{d\sigma^2}=\frac{1}{2}\frac{\partial P}{\partial \lambda},\tag{2}$$

where λ is the geomagnetic latitude, $r^2d\sigma=2\gamma ds$, and $e^x=2\gamma r$. In the present case, for $r\geq a$,

$$P=(2\gamma)^{-4}e^{2x}-\sec^2\lambda+2(M_r+1)e^{-x}-(M_r+1)^2e^{-2x}\cos^2\lambda.$$

The first step in the approximation suggested by Störmer[9] consists in putting $\cos^2\lambda=\alpha^2$ in the last term in the expression for P. α^2 is taken as constant, and is some mean value of $\cos^2\lambda$. Equations (2) can then be integrated once, yielding

$$\begin{aligned}(dx/d\sigma)^2&=(2\gamma)^{-4}e^{2x}+2(M_r+1)e^{-x}-(M_r+1)^2\alpha^2e^{-2x}-K_1,\\ (d\lambda/d\sigma)^2&=K_1-\sec^2\lambda.\end{aligned}\tag{3}$$

The first of these equations cannot be integrated

[8] W. E. Milne, *Numerical Calculus* (Princeton University Press, Princeton, 1949), p. 143.

[9] C. Störmer, Avhandl. Norske Videnskaps-Akad. Oslo. I. Mat. Naturv. Kl. No. 2, 6 (1934).

analytically. The second yields

$$\sin\lambda = (1-K_1^{-1})^{\frac{1}{2}} \sin K_1^{\frac{1}{2}}(K_2-\sigma). \qquad (4)$$

Now we need a further approximation for the first of Eqs. (3). Upon changing back to the variable r in place of x and rearranging, one obtains

$$d\sigma = 2\gamma dr[(r^2-2\gamma^2K_1)^2+4\gamma(M_r+1)r - 4\gamma^4K_1^2-(M_r+1)^2a^2]^{-\frac{1}{2}}. \qquad (5)$$

The approximation consists in neglecting all except the first term in the square brackets, and then performing the integration from r_0 to ∞. One has, then,

$$\sigma = \sigma_0 + (2K_1)^{-\frac{1}{2}} \times\{\ln[r_0+\gamma(2K_1)^{\frac{1}{2}}]-\ln[r_0-\gamma(2K_1)^{\frac{1}{2}}]\}. \qquad (6)$$

Finally, a similar pair of approximations in $d\varphi/d\sigma$ yield

$$\varphi = \varphi_0 + \alpha^{-2}(\sigma-\sigma_0) - \tfrac{1}{4}(M_r+1)\gamma^{-2}K_1^{-1} \times\{\ln r_0^2 - \ln[r_0^2-2\gamma^2K_1]\}. \qquad (7)$$

Equations (4), (6), and (7) are then the continuation of the trajectory from r_0, σ_0 to infinity.

Störmer[9] discusses the effect of the approximation using α^2 for $\cos^2\lambda$. The effect of the approximation to (5) can be seen by expanding the radical using the binomial theorem. The first term is that term used in getting (6). The second term contributes less than 0.1% to the final value of σ.

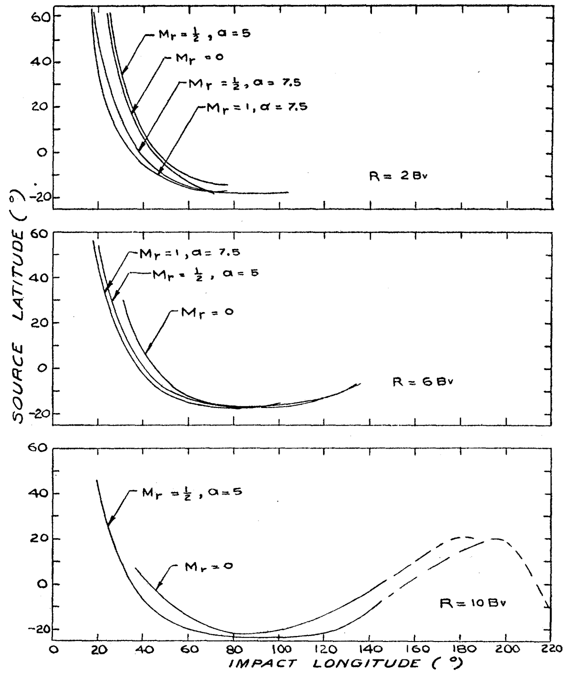

FIG. 2. Geomagnetic latitude of the source *versus* geomagnetic impact longitude. The longitude given is the dihedral angle between two planes, each of which contains the dipole. One plane contains the source, the other contains the impact point. Positive longitudes are measured from geomagnetic local noon toward the morning.

RESULTS OF CALCULATIONS

The results of the calculations are shown in Figs. 1 and 2. Figure 1 is a plot of the geomagnetic latitude of the source (here the sun) against the latitude of impact with the earth. Figure 2 is a plot of the latitude of the source against the geomagnetic longitude of the impact point. The various parts of each figure are for various values of particle rigidity. The individual curves are labeled with the ring current parameters M_r and a. M_r is in units of the earth's dipole moment, and, in these figures, a is in earth radii. Table I gives the calculated points from which these curves were drawn. The curves for $M_r=0$ are taken from Dwight's paper,[6] and from the work of Jory.†

In order to use these curves, one chooses a date, thus determining the geographic latitude of the sun. This is then converted to geomagnetic latitude, using the time of day. The result of this conversion is the source latitude in Figs. 1 and 2. One then uses the curves to find the geomagnetic local time and latitude of impact with the earth for a particle of selected rigidity.

These results refer mostly to the 0900 zone. Further calculations are in progress for the other zones and for some lower rigidities as well.

In order to facilitate the programing of the problem for the machine, one trajectory was first calculated most of the way by hand. The results of this calculation agree with the machine value to within the expected errors of the hand calculation. That is, the two calculations agree to within five parts in the fifth significant figure. The machine results were printed out after every other step along each trajectory. In addition to the coordinates of the particle, some of the functions calculated by the machine in carrying out a step were printed as well. Hand-calculated checks on these quantities show that the largest error produced by the machine came in the calculation of a square root to obtain r, and amounted usually to four or five parts in 10^6. This error on an occasion or two became as large as one part in 10^5. Study of the effects of this error shows that a more important error occurs in the extrapolation procedure. In using Eqs. (4), (6), and (7), one must first calculate K_1. This is done from Eq. (3). $d\lambda/d\sigma$ for this purpose is obtained by a numerical differentiation at the end of the printed trajectory. The error in the impact longitude resulting from the error in $d\lambda/d\sigma$ may amount to about 1° for the two or three points of highest impact latitude. For the other points it should be $\frac{1}{2}$° or less. The errors in source latitude are smaller.

COMPARISON WITH EXPERIMENT

Firor[5] has discussed four great flare-associated cosmic ray intensity increases in the light of impact zones as calculated for the earth's dipole field alone. It seems not

† F. S. Jory, Phys. Rev. **103**, 1068 (1956).

TABLE I. Impact zones as a function of source latitude for vertically arriving particles. Since the impact latitude was an initial condition for the numerical integration, it may be taken as exact. The asymptotic latitude and impact longitude are thought to be well within 1° of the true values in nearly all cases. For the two or three cases of highest latitude of impact, the impact longitude may be in error by as much as 1°. These comparatively large errors are due to the extrapolation procedure. Geomagnetic longitude is measured toward the west from the plane containing the dipole and passing through the sun.

Rigidity and ring parameters	Source latitude (degrees)	Impact latitude (degrees)	Impact longitude (degrees)
$R=2$ Bv	64.6	80	17
$M_r=1$	34.9	70	20
$a=7.5$	16.4	65	25
	3.7	62	31.6
	−5.3	60	39.3
	−9.9	59	46.3
	−13.7	58	53.1
	−16.2	57	60.2
	−17.3	56	70.8
	−16.9	55.5	77.4
$R=2$ Bv	60.0	80	18
$M_r=\frac{1}{2}$	26.0	70	26
$a=7.5$	6.6	65	34
	−5.7	62	44.0
	−13.2	60	54.7
	−16.0	59	62.1
	−18.0	58	71.1
	−17.7	57	83.0
	−17.4	56	95.6
	−17.5	55.5	104.5
$R=2$ Bv	61.8	80	25
$M_r=\frac{1}{2}$	31.9	70	30
$a=5$	15.5	65	36
	4.9	62	42.2
	−1.6	60	49.4
	−5.9	59	53.6
	−9.3	58	59.0
	−12.0	57	64.9
	−13.7	56	73.7
	−14.1	55.5	78.6
$R=6$ Bv	53.1	75	18
$M_r=1$	12.5	60	30
$a=7.5$	−1.4	55	39
	−13.5	50	55.1
	−16.9	44	92.6
	−15.6	42	110.4
	−12.7	41	122.2
	−6.6	40	136.5
$R=6$ Bv	54	75	20
$M_r=\frac{1}{2}$	14.2	60	32
$a=5$	0.5	55	40
	−11.7	50	54.9
	−16.8	44	89.2
	−15.8	42	106.2
	−14.6	41	116.9
	−10.6	40	128.4
$R=10$ Bv	46.4	70	19
$M_r=\frac{1}{2}$	7.8	55	30
$a=5$	−6.2	50	40
	−18.6	45	56.4
	−23.5	40	84.3
	−22.1	36	115.7
	−7.1	33	144.9
	11.6	31.5	174.8
	−9.7	30	218.8

to be possible to use these data to determine accurately the local time dependence of the impact zones. None of the data gives any clear evidence of increases in intensity as the point of observation enters such a zone. The experimental evidence for the reality of these zones consists in the significantly higher increases in intensity observed at stations which were located in an impact zone at the time a flare began compared with the intensity increases observed at stations located only in the background zone. (See Firor's paper[5] for a discussion of the background zone.) If sufficient data were collected along these lines, the impact zones would eventually be determined as accurately as one might wish, but large flares are so rare that this is not a practical program.

A second potential source of information lies in data from balloon flights. One should be able to find important information about the kinds of particles which the sun is capable of emitting, and eventually determine the location of the impact zones. The latter is probably a very difficult program unless smaller flares make their effects felt at high altitude. This may well be the case, however. Koshiba and Schein[10] report an observed increase in heavy particle intensity between 0900 and 1100 on June 24, 1955 at high altitude. They interpret this as the effect of the 0900 impact zone on solar produced heavy nuclei.

Probably the best potential source of information for the present purpose is the small-flare effect as analyzed by Firor.[5] He plotted the Climax neutron intensity *versus* time of day averaged over three-month periods, and found that the impact zones were clearly visible in the difference between the intensity on days of high flare index and those of low. Since he grouped the counts in one-hour periods in order to get adequate statistics, his data cannot reveal the presence or absence of the shift of impact time of half an hour or less suggested by the present calculations. A more extended set of data might detect a shift this small, although the comparison will likely be difficult.

It is particularly to be noticed that the ring current does nothing to explain the high-latitude impacts not accounted for by Firor's theory. If the mechanism which produces these impacts operates in any significant way at lower latitudes, it may render meaningless or confusing the comparison with experiment of the present theory.

ACKNOWLEDGMENTS

The author is indebted to Professor James A. Van Allen for several helpful discussions during the course of the work, as well as to Mr. John G. Pettit of Collins Radio Company who programmed the problem for an IBM 650 and carried out the machine calculations.

[10] M. Koshiba and M. Schein, Bull. Am. Phys. Soc. Ser. II, 1, 229 (1956).