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Multiple Scattering of 2.4-Mev Electrons in Nuclear Emulsion*

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The multiple scattering of 2.39±0.09 Mev electrons in Ilford G-5 emulsion has been measured for cell lengths of 10, 20, and 40 microns. The independent data of three observers were combined to obtain the best value of the mean scattering angles with and without experimental cutoff. The associated scattering factors for cell lengths between 10 and 40 microns are: $K_{c0}=24.9\pm1.2$ Mev degrees/(100 microns)$^{0.59}$ (for a cutoff at 4 times the mean deviation) and $K_0=25.1\pm0.8$ Mev degrees/(100 microns)$^{0.58}$. The corresponding values from the Molière theory are 23.5 and 25.7, respectively.

I. INTRODUCTION

THE multiple scattering of charged particles has been investigated from both the theoretical and experimental viewpoints by a number of authors.[1] For the most part, satisfactory agreement between theory and experiment has been reported. The exact theories of Molière,[2] Snyder and Scott,[3,4] Goudsmit and Saunderson,[5] and Lewis[6] are in essential agreement where their respective regions of validity overlap. The earlier and less rigorous theory of Williams[7,8] as adapted by Goldschmidt-Clermont[9] and Voyvodic and Pickup[10] agrees with the later theories[2–6] within a few percent. In recent years, experimental results have usually been compared with the Molière theory. This custom will be followed in the present work.

In the case of nuclear emulsion as a scattering material, the results of many investigations[11] covering a wide range of particle velocity and cell lengths show satisfactory agreement with theory to within 5–10%. The multiple-scattering technique has accordingly become a valuable tool in the analysis of events in nuclear emulsion.

Hisdal[12] has reported what appears to be a significant departure between experiment and theory. In this work the multiple scattering of 0.59-Mev electrons in Ilford G-5 emulsion was measured by a "center-of-gravity method." The measured mean scattering angle (without experimental cutoff) was smaller than the theoretical value by 61% or about 6 standard deviations. The author suggests that this discrepancy may be due to the failure of the theory for low-energy electrons or to the use of a small cell length (17.6 microns).

Multiple-scattering measurements of Husain[13] on 6.6-Mev electrons in nuclear emulsion are in agreement with theory for cell lengths as small as 38 microns. At higher electron energies the results of many investigations[10,14–19] are generally in agreement with theory. However, for 10-Mev electrons and cell lengths between 25 and 100 microns Bosley and Hughes[16] report scattering factors greater than the Molière theoretical values by about 6 standard deviations or more.

In the light of Hisdal's results the present work was undertaken to determine the validity of the theory for electrons in the region of 2 Mev for small cell lengths in nuclear emulsion.

While this work was in progress, a study of electron and positron multiple scattering in nuclear emulsion at energies of 1, 1.5, and 2.5 Mev was published by Heymann and Williams.[20] These authors measured chord angles directly with an ocular protractor using 50-micron chord lengths. For 2.5-Mev electrons the measured scattering factors were about 3 to 4 standard deviations greater than predicted by the Molière theory. Better agreement with theory could be obtained if inelastic collisions were taken into account. Their scattering factors at 1 and 1.5 Mev appear to be significantly lower than theory, which tend to confirm Hisdal's observations.

II. EXPERIMENTAL METHOD

A. Exposure of Emulsions

A number of 1-in.×3-in.×200-micron Ilford G-5 emulsions were exposed to a magnetically analyzed electron beam produced from an electron traveling-wave linear accelerator, a prototype of a proposed 3-Mev injector for the Berkeley synchrotron. An air-

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[*] This work was performed under the auspices of the U. S. Atomic Energy Commission.

[1] For a review of multiple-scattering theories and experiments see, e.g., references 10 and 32.

[2] G. Molière, Z. Naturforsch. **3a**, 78 (1948).

[3] H. S. Snyder and W. T. Scott, Phys. Rev. **76**, 220 (1949).

[4] W. T. Scott, Phys. Rev. **85**, 245 (1952).

[5] S. Goudsmit and J. L. Saunderson, Phys. Rev. **57**, 24 (1940); **58**, 36 (1940).

[6] H. W. Lewis, Phys. Rev. **78**, 526 (1950).

[7] E. J. Williams, Proc. Roy. Soc. (London) **A169**, 531 (1939).

[8] E. J. Williams, Phys. Rev. **58**, 292 (1940).

[9] Y. Goldschmidt-Clermont, Nuovo cimento **7**, 331 (1950).

[10] L. Voyvodic and E. Pickup, Phys. Rev. **85**, 91 (1952).

[11] For a summary of several experiments on the multiple scattering of charged particles in nuclear emulsion, see M. J. Berger, Phys. Rev. **88**, 59 (1952).

[12] E. Hisdal, Phil. Mag. **43**, 790 (1952); see also H. Överös, Phil. Mag. **45**, 158 (1954).

[13] A. Husain, Proc. Roy. Soc. (London) **A68**, 45 (1955).

[14] D. R. Corson, Phys. Rev. **84**, 605 (1951).

[15] W. Bosley and H. Muirhead, Phil. Mag. **43**, 63 (1952).

[16] W. Bosley and I. S. Hughes, Phil. Mag. **46**, 1281 (1955).

[17] Hanson, Lanzl, Lyman, and Scott, Phys. Rev. **84**, 634 (1951).

[18] L. V. Spencer and C. H. Blanchard, Phys. Rev. **93**, 114 (1954).

[19] H. A. Bethe, Phys. Rev. **89**, 1256 (1953).

[20] F. F. Heymann and W. F. Williams, Phil. Mag. **1**, 212 (1956).

free path was provided for the analyzed beam by means of a vacuum chamber that was a physical extension of the accelerator and part of its vacuum system. The analyzed beam traveled down an evacuated pipe 2 ft long with a $\frac{1}{4}$-in. slit system at the end and emerged into the air through a 1-mil aluminum exit foil. The emulsions were contained in a light-tight dural box adjacent to the exit foil, one face of which was a 1-mil aluminum foil. The analyzed beam passed through this foil at normal incidence and entered the emulsions making an angle of $\approx 10°$ with the emulsion surface.

The Harvard model cyclotron magnet with a pole diameter of $5\frac{1}{4}$ in. was used. The field was uniform to within one percent up to a radius of $2\frac{1}{4}$ in. Field measurements were made by the rotating coil method at the exposure current setting and a value of 978 gauss was obtained in the homogeneous region. The radius of curvature was 10 cm.

The emulsions were developed horizontally by a modified temperature cycle process using Amidol developer. The resulting tracks had a mean grain density of 35 grains/100 microns and a mean grain diameter of 0.7 micron. The track density was approximately 10^4 entering tracks per cm^2.

B. Scanning Procedure

All of the various multiple-scattering theories[2–8] except that of Goudsmit and Saunderson[5] are expressed, at least in their final form, in terms of the small-angle approximation ($\sin\alpha = \alpha$). To compare the present results with theory, cell lengths were chosen so that this approximation was valid. A convenient method of measuring the multiple scattering in this approximation is that proposed by Fowler.[21] In this method the coordinates, y_n, are measured relative to a base line whose direction closely approximates that of the projected track. The projected scattering angle is then defined as the difference between successive projected chord angles,

$$\alpha_n = \psi_{n+1} - \psi_n,$$

or, in terms of the first and second differences of the y-coordinates, d_n and D_n, respectively,

$$\alpha_n = \frac{(y_{n+2}-y_{n+1})-(y_{n+1}-y_n)}{t} = \frac{d_{n+1}-d_n}{t} = \frac{D_n}{t}, \qquad (1)$$

where t is the cell length. In the usual application of this method on tracks of high-energy particles, the approximation implicit in Eq. (1) is usually of no concern. In the present case, however, ψ_n can become appreciable and a proper allowance for this effect must be made. This problem was met here by limiting the α_n's accepted to those corresponding to ψ_n less than 10°. To prevent biasing the data no restriction was placed on ψ_{n+1}. It can be shown from the probability functions of Scott[22] that, although successive deflections are correlated, the width of the distribution of second differences is independent of this restriction. Measurements of the y coordinates were made for cell lengths of 10, 20, and 40 microns as determined by the stage coordinates under the following criteria:

1. For a track to be acceptable the projected and dip angles at the surface had to be within 5° of the beam direction. Only the central region of the beam was scanned.

2. Successive y-coordinate measurements were made along the track until one of the following occurred: (*a*) ψ_n became greater than 10° (no restriction being placed on ψ_{n+1}); (*b*) the dip angle became greater than 20°; (*c*) the track scattered out of the emulsion; (*d*) a total range of 400 microns was covered.

The measurements were made independently by three observers using a Bausch and Lomb eyepiece scale. Observers 1 and 3 used Bausch and Lomb binocular microscopes and Observer 2 used a similar Leitz microscope. The eyepiece calibration factors under the magnification used (≈ 2000) were 1.062, 1.055, and 0.8642 microns/scale division, respectively. All three microscopes were equipped with Brower stages.[23] The error in range measurements for these stages is cyclic with an amplitude ≈ 5 microns and a period of 1 mm. Measurements of y_n were read to the nearest half-scale division and the particle was assumed to have traveled in a straight line between centers of successive grains.

III. ANALYSIS OF DATA

For singly charged particles the mean absolute projected scattering angle, $\langle|\alpha_0|\rangle$, for a cell length of t microns is given by

$$\langle|\alpha_0|\rangle = K_0/p\beta c(t/100)^{\frac{1}{2}}, \qquad (2)$$

where p = particle momentum (Mev/c); β = particle velocity in units of the velocity of light, c; and K_0 = scattering factor for chord angles (customarily expressed in units of Mev degrees for t in microns).

Theory predicts, and it has been experimentally confirmed,[24] that the distribution of scattering angles is nearly Gaussian. If a cut-off procedure is used to eliminate the single scattering tail a better approximation to a normal distribution results. In this case a new mean deviation $\langle|\alpha_{c0}|\rangle$ is obtained and is related to the corresponding scattering factor, K_{c0}, by an equation formally the same as Eq. (2). The cut-off angle is usually taken at four times the mean deviation. This cutoff criterion is used in the following analysis. K_{c0} and K_0 are characteristic of the scattering medium and

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[21] P. H. Fowler, Phil. Mag. 41, 169 (1950).

[22] W. T. Scott, Phys. Rev. 76, 212 (1949).

[23] Manufactured by W. Brower, 1 Vista del Moraga, Orinda, California.

[24] See, e.g., the sampling distributions of C. O'Ceallaigh and O. Rochat, Phil. Mag. 42, 1050 (1951).

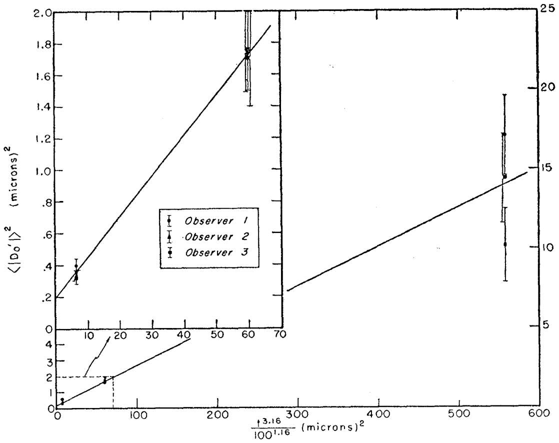

FIG. 1. Plot of $\langle|D_0'|\rangle^2$ as a function of $(t^{3.16}/100^{1.16})$. The straight line has been obtained from a least-square analysis of the data of each observer.

are slowly varying functions of particle "energy," $p\beta c$, and t.

Because of the presence of reading errors, grain noise, and stage noise, the observed width of the distribution of α_n will in general be greater than that due to multiple scattering alone. If the total noise, ϵ, is also assumed to be normally distributed and independent of $\langle|\alpha_{c0}|\rangle$ and $\langle|\alpha_0|\rangle$, the following relationships are valid:

$$\langle|\alpha_{0,c0}'|\rangle^2=\langle|\alpha_{0,c0}|\rangle^2+\langle|\epsilon|\rangle^2, \tag{3}$$

where $\langle|\alpha_{0,c0}'|\rangle$ and $\langle|\alpha_{0,c0}|\rangle$ are the observed and true mean scattering angles, respectively (with or without cutoff). In the small-angle approximation,

$$\langle|D_n|\rangle=\langle|\alpha_n|\rangle(\pi/180)t, \tag{4}$$

and the quantities in Eq. (3) can be written in terms of their corresponding mean coordinate second differences as follows

$$\langle|D_{0,c0}'|\rangle^2=\langle|D_{0,c0}|\rangle^2+\langle|\delta|\rangle^2. \tag{5}$$

In this experiment $\langle|\delta|\rangle$, K_{c0}, and K_0 were determined by using the method of Menon *et al.*[25] and Corson.[14] This method makes use of Eq. (5) where the dependence of $\langle|D_{c0}|\rangle$ and $\langle|D_0|\rangle$ on cell length is explicitly given from theory. The further assumption is made that $\langle|\delta|\rangle$ is independent of cell length.[26]

In the region, 10 microns $\leq t \leq 40$ microns, the theoretical (Molière) dependence of K_{c0} and K_0 on t can be represented by a power law:

$$K_{c0}(p\beta c,t)=k_{c0}(t/100)^{0.09}, \tag{6a}$$

$$K_0(p\beta c,t)=k_0(t/100)^{0.08}, \tag{6b}$$

where k_{c0} and k_0 are constants for monoenergetic particles. K_{c0} values were obtained by numerically integrating the Molière distribution function. The usual $\sqrt{\frac{2}{3}}$ factor relating the chord method with the tangent method has been used. Using Eqs. (2)–(6b), we have

$$\langle|D_{c0}'|\rangle^2=\left[\frac{k_{c0}}{p\beta c}\frac{\pi}{180}\right]^2\frac{t^{3.18}}{100^{1.18}}+\langle|\delta|\rangle^2, \tag{7a}$$

$$\langle|D_0'|\rangle^2=\left[\frac{k_0}{p\beta c}\frac{\pi}{180}\right]^2\frac{t^{3.16}}{100^{1.16}}+\langle|\delta|\rangle^2, \tag{7b}$$

where k_{c0} and k_0 are in units of Mev degrees for t in microns. Thus, a plot of $\langle|D_{c0}'|\rangle^2$ and $\langle|D_0|\rangle^2$ against $t^{3.18}/100^{1.18}$ and $t^{3.16}/100^{1.16}$, respectively, should be consistent with a straight line through the experimental points.

The experimental values $\langle|D_0'|\rangle^2$, as obtained by the three observers, for cell lengths of 10, 20, and 40 microns, are plotted against $t^{3.16}/100^{1.16}$ in Fig. 1. A similar plot of $\langle|D_{c0}'|\rangle^2$ *vs* $t^{3.18}/100^{1.18}$ is qualitatively the same as that in Fig. 1 and is not shown. Both sets of data are consistent with a straight line. In calculating $\langle|D_{c0}'|\rangle$ and $\langle|D_0'|\rangle$, zero values of D_n have been given a weight of $a/4$,[27] where a $(=\frac{1}{2}$ scale division) is the smallest unit of measurement of the y displacement. The errors are standard deviations and have been calculated on the bases of both external and internal consistency.[28] The assigned error is the larger of the two. The two types of errors never differed by more than 24% and were always consistent within statistical accuracy.

The best straight line through the experimental points for each of the three observers was obtained by the method of least squares. The results of this analysis appear in Table I. Figure 1 includes the curve corresponding to the appropriate weighted mean-square

TABLE I. Least-squares analysis of the multiple-scattering measurements.[a]

Ob-server	$(\text{slope})^{\frac{1}{2}}$ (radians)	$\langle\lvert\delta\rvert\rangle$ (microns)	Weighted root-mean-square slope (radians)	
1	0.164±0.011	0.341±0.099	0.155±0.006	Cut-off data
2	0.142±0.032	0.447±0.043		
3	0.158±0.024	0.335±0.055		
1	0.165±0.011	0.458±0.064	0.156±0.002	Non-cut-off data
2	0.145±0.011	0.444±0.045		
3	0.161±0.011	0.377±0.061		

[a] Errors are standard deviations.

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[25] Menon, O'Ceallaigh, and Rochat, Phil. Mag. 42, 932 (1951).

[26] The general validity of these assumptions regarding ϵ or δ is discussed by Menon *et al.*[25] and by Goldschmidt-Clermont.[9] Menon *et al.* have observed an increase in $\langle|\delta|\rangle$ with increasing cell lengths due to the contribution of the stage noise component. The stages used in this experiment were checked for this effect against a Bausch and Lomb straight line. In all cases the stage noise was found to be ≈0.2 micron. No significant dependence on cell lengths between 10 and 40 microns was found over the region of stage coordinates used in this experiment.

[27] W. H. Barkas and D. M. Young, University of California Radiation Laboratory Report UCRL-2579, 1954 (unpublished).

[28] R. T. Birge, Phys. Rev. **40**, 207 (1932).

TABLE II. Comparison of the observed scattering distribution with the Molière theory.

Cell length (microns)	Ω_b	Percent deflections greater than $4\langle\lvert\alpha_{c0}\rvert\rangle$ Experimental ($\pm$ standard deviation)	Theoretical
10	32	2.4 ± 0.7	2.4
20	64	$1.7_{-0.6}^{+0.9}$	2.3
40	128	$0.8_{-0.4}^{+0.8}$	2.1

slope and intercept.[29] The weighted root-mean-square slopes of Table I correspond to the best determinations of the mean scattering angles with an without cutoff for 100-micron cells, uncorrected for systematic errors.

IV. SYSTEMATIC AND RANDOM ERRORS

The systematic errors due to (1) the approximation $\tan\alpha_n=\alpha_n$, (2) the apparent change in cell length and angle due to the inclination of the incident plane with the plane of measurement ($\approx 10°$), (3) the variation of ψ_n between $\pm 10°$, (4) emulsion distortion, and (5) tracks escaping from the surface (flat chamber effect)[2] have been considered. With the exception of (5), all of these effects cause an apparent increase in the mean scattering angle. For the small angles measured here, the correction due to (1) is less than one percent and is neglected. The correction factor for (2) is $\cos^{\frac{3}{2}} 10°$. Since this is less than one percent it is also neglected. The correction factor due to (3) is given to a sufficient approximation by

$$(1-\langle\psi_n^2\rangle)\Bigg/\left(1+\frac{\langle\lvert D_{0,c0}\rvert\rangle}{t}\langle\lvert\psi_n\rvert\rangle\right).$$

From an inspection of the ψ_n distribution, $\langle\psi_n^2\rangle^{\frac{1}{2}}$ is nearly 7° for all cell lengths and observers. The appropriate correction for both K_{c0} and K is calculated to be 2%. Emulsion distortion is believed to be negligible for the following reasons: (a) the algebraic mean values of the distributions of D_n with and without cutoff were consistent with zero; (b) the mean deviations of the third difference distributions are consistent with the expected values of $(\frac{3}{2})^{\frac{1}{2}}\langle\lvert D_{0,c0}'\rvert\rangle$[30]; and (c) there is no tendency for the points of Fig. 1 to exhibit a pronounced positive curvature.[31] The correction due to (5) would be difficult to calculate directly. Since escaping tracks may be correlated with large deflections, the observed distributions could be biased. If this correlation were present, an increasing amount of the single scattering tail should be missing with increasing cell lengths. A test of this effect can be made by an examination of the observed second difference distributions in the single-scattering region. These were tested by calculating the relative number of deflections greater than $4\langle\lvert\alpha_{c0}\rvert\rangle$ for each cell length. This was done for each observer using the $\langle\lvert\delta\rvert\rangle$'s from Table I and Eqs. (4) and (5). From the combined data, the experimental ratios of deflections greater than $4\langle\lvert\alpha_{c0}\rvert\rangle$ for each cell length have been obtained and are given in Table II along with the expected ratios from the Molière theory. Also included in Table II is the Molière parameter Ω_b, a measure of the average number of collisions. These values of Ω_b were obtained from the published curve of Gottstein *et al.*[32] The region of validity of the Molière theory corresponds to values of Ω_b greater than ≈ 20. The agreement between the experimental and theoretical ratios of Table II is seen to be satisfactory. One concludes that the outscattering effects are probably small relative to the statistical errors.

Another check of outscattering effects was made by plotting K_0 as a function of cell length. The $\langle\lvert\delta\rvert\rangle$'s of Table I, obtained from fitting a straight line to the cut-off mean deviations, are independent of the non-cut-off mean deviations, $\langle\lvert D_0'\rvert\rangle$ and $\langle\lvert D_0\rvert\rangle$. Thus, three independent determinations of K_0 can be made. Using Eqs. (2), (4), and (5), the appropriate $\langle\lvert\delta\rvert\rangle$'s of Table I, and the $p\beta c$ given in section V, K_0 has been calculated from the weighted mean $\langle\lvert D_0'\rvert\rangle$ at each cell length. These values of K_0 are compared with the theoretical curve of K_0 in Fig. 2. The curve corresponding to K_{c0} is also shown for comparison. From Fig. 2 it appears that the effects of outscattering are small relative to the statistical errors.

From this discussion we conclude that the only correction greater than 1% to be applied to the mean scattering angles of Table I is a 2% correction arising from (2).

The random errors arise from statistics and momentum definition. The standard deviation of the mean

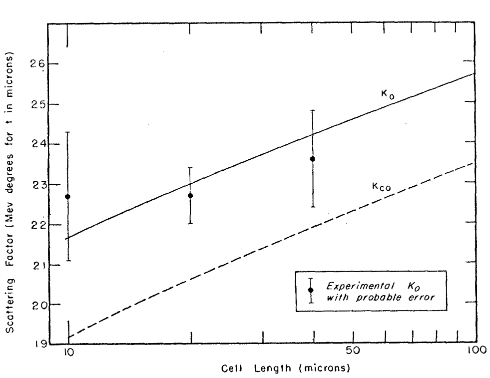

FIG. 2. Experimental values of the scattering factor without cutoff (K_0) plotted as a function of cell length and compared with Molière theory.

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[29] The intercepts are characteristic of the noise level for each of the three sets of data and need not be in agreement. Since they are in fairly good agreement here, they are combined in order to plot an average curve.

[30] This can be shown from the probability functions of Scott (reference 22).

[31] K. Gottstein and J. H. Mulvey, Phil. Mag. 42, 1089 (1951).

[32] Gottstein, Menon, Mulvey, O'Ceallaigh, and Rochat, Phil. Mag. 42, 708 (1951).

TABLE III. Experimental and theoretical scattering factors for t between 10 and 40 microns.

K_{c0} Mev deg/(100 microns)$^{0.59}$		K_0 Mev deg/(100 microns)$^{0.58}$	
Experimental (± standard deviation)	Theoretical (Molière)	Experimental (± standard deviation)	Theoretical (Molière)
24.9±1.2	23.5	25.1±0.8	25.7

deviation for a sample of n events is[12]

$$[(\pi-2)/2n]^{\frac{1}{2}}\langle|D_{0,c0}'|\rangle.$$

The momentum uncertainty corresponding to the slit width of $\frac{1}{4}$ in. is 3%. These errors have been added root-mean-square-wise in the calculations of the internal errors.

V. DETERMINATION OF K_0 AND K_{c0}

Knowledge of the corrected mean scattering angles of Table I permits the calculation of K_0 and K_{c0} if $p\beta c$ is known. The $H\rho$ of the analyzed beam, 9.78×10^3 gauss cm, corresponds to a kinetic energy of 2.46 Mev. The distribution in range intervals over which the measurements were taken is approximately symmetric with a mean at 200 microns. The energy loss in the two 1-mil Al foils and in 100 (=200/2) microns of emulsion are 0.02 Mev and 0.05 Mev, respectively. Therefore, the mean kinetic energy appropriate to these measurements is 2.39±0.09 Mev. The associated $p\beta c$ is 2.85±0.09 Mev. From this value and the corrected weighted root-mean-square slopes of Table I, k_0 and k_{c0} are readily calculated. These values of k_0 and k_{c0} are numerically equal to K_0 and K_{c0} when referred to 100-micron cell lengths. K_0 and K_{c0} are given accordingly in Table III along with the expected values from the Molière theory. The agreement between the experimental and theoretical scattering factors is satisfactory. The effect of inelastic electronic scattering can be taken into account by increasing the theoretical values of Table III by 2–3%.[10,33,34] If such a correction is made, good agreement would still exist between the experimental and theoretical scattering factors.

VI. CONCLUSIONS

From Table III it is evident that the results obtained here for 2.39±0.09 Mev electrons and cell lengths of 10–40 microns are in agreement with the Molière theory. The observed dependence of K_0 with cell length (Fig. 2) in the region 10–40 microns is consistent with that predicted by the Molière theory. The experimental ratios of events greater than $4\langle|\alpha_{c0}|\rangle$ (Table II) are consistent with those obtained from the Molière distribution function.

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[33] L. A. Kulchitsky, and G. D. Latyshev, Phys. Rev. **61**, 254 (1942).

[34] J. Fano, Phys. Rev. **93**, 117 (1954).