Method of Analysis of Charged Pion Photoproduction

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It is explained why the usual analysis of the angular distribution of charged pion photoproduction with the help of a quadratic expression of $\cos\theta$ (θ being the meson emission angle) is physically meaningless. A substitute scheme is proposed in which the angular distribution in the center-of-mass system, multiplied by $(1-\beta\cos\theta)^2$ (β being the meson velocity), is analyzed by a quartic expression in $\cos\theta$. Such an analysis is given for several theories commonly in use.

It has been customary[1] to analyze the angular distribution of charged pion photoproduction in terms of a quadratic expression in $\cos\theta$, where θ is the meson emission angle:

$$d\sigma/d\Omega = A + B\cos\theta + C\cos^2\theta. \tag{1}$$

This expression is usually justified by the assertion that at low photon energies ($k_{\text{lab}} < 350$ Mev) it is reasonable to assume that only S and P waves will contribute substantially. If we accept this assumption, the highest power of $\cos\theta$ which will appear in the angular distribution is the square of the highest power in the first Legendre polynomials, that is, two.

This philosophy is supported by the theory in the case of pion-nucleon scattering and neutral-pion photoproduction. In the case of charged pion production, however, the theoretical predictions strongly discourage such analysis. All meson theories predicting charged pion photoproduction have in common the term which arises from the interaction of the meson current with the incident photon. This term is of the form

$$\frac{\boldsymbol{\varepsilon}\cdot\mathbf{q}\boldsymbol{\sigma}\cdot(\mathbf{k}-\mathbf{q})}{q_0k-\mathbf{q}\cdot\mathbf{k}}, \tag{2}$$

where $\mathbf{k}$ and $\boldsymbol{\varepsilon}$ are the photon momentum and polarization, respectively; and $\mathbf{q}$ and q_0 are the momentum and energy, respectively, of the meson. $\boldsymbol{\sigma}$ is the nucleon spin. (We take $\hbar=c=\mu=1$, where μ is the pion mass.) It is easy to see that this term contains all angular momenta. Even if the denominator did not contain $\mathbf{q}\cdot\mathbf{k}$, it would contain S, $P_{\frac{1}{2}}$, $P_{\frac{3}{2}}$, and $D_{\frac{3}{2}}$ terms in approximately equal strength, and the denominator mixes in all higher angular momentum states, again in comparable strength whenever $\beta\sim1$. Since at 200-Mev laboratory photon energy we already have $\beta=0.7$, it is clear that the appearance of the term given by Eq. (2) invalidates the argument leading to the form given by Eq. (1). For neutral production and pion-nucleon scattering, however, Eq. (1) can be used, since the photon does not interact directly with the neutral meson, and since pion-pion interaction can be neglected in pion-nucleon scattering. Thus the term of Eq. (2) appears neither in neutral pion production, nor in pion-nucleon scattering.

The fact that the analysis of charged-pion photoproduction in terms of Eq. (1) could be pursued at all up to now can be easily understood. Firstly, the present experimental errors in the angular distribution are considerable, especially when measurements from different laboratories are compared. Furthermore, satisfactory measurements have been carried out so far only in a limited angular range, from about 30° to 150°. Now, however, several measurements are in progress in the 0°–30° range[2,3] and a general increase of precision at all angles can be expected. In order to provide a meaningful interpretation of these measurements, and in order to facilitate the comparison with theory, this note suggests a new method of analysis of charged pion photoproduction data.

We will base our analysis on the assumption that the matrix element for charged photoproduction is of the form

$$\begin{aligned} M = {} & a\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon} + b\frac{\boldsymbol{\sigma}\cdot(\mathbf{k}-\mathbf{q})\boldsymbol{\varepsilon}\cdot\mathbf{q}}{q_0k-\mathbf{q}\cdot\mathbf{k}} \\ & + c[i\mathbf{q}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon}) - (\boldsymbol{\sigma}\cdot\mathbf{k}\mathbf{q}\cdot\boldsymbol{\varepsilon} - \boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\mathbf{q}\cdot\mathbf{k})] \\ & + d[2i\mathbf{q}\cdot(\mathbf{k}\times\boldsymbol{\varepsilon}) + (\boldsymbol{\sigma}\cdot\mathbf{k}\mathbf{q}\cdot\boldsymbol{\varepsilon} - \boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\mathbf{q}\cdot\mathbf{k})] \\ & + e[\boldsymbol{\sigma}\cdot\mathbf{k}\mathbf{q}\cdot\boldsymbol{\varepsilon} + \boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\mathbf{q}\cdot\mathbf{k}], \end{aligned} \tag{3}$$

where a, b, c, d, and e are, in general, complex scalars depending on q and k. The interpretation of the various terms in Eq. (3) is well known and need not be repeated. If we work in the center-of-mass system, the density of final states contains no angular factor, so that $|M|^2$ gives the entire angular dependence.

Calculating $|M|^2$ and averaging over $\boldsymbol{\varepsilon}$ and $\boldsymbol{\sigma}$, we

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[1] E.g., Watson, Keck, Tollestrup, and Walker, Phys. Rev. **101**, 1159 (1956).

[2] L. S. Osborne (private communication).

[3] A. S. Lazarus (private communication).

get the following expression:

$$|M|^2=|a|^2+\tfrac{1}{2}|b|^2\beta^2\frac{[1+\beta^2\gamma^2-2\beta\gamma\cos\vartheta](1-\cos^2\vartheta)}{(1-\beta\cos\vartheta)^2}$$
$$+\tfrac{1}{2}(|u|^2+|v|^2)\beta^2\gamma^2k^4(1-\cos^2\vartheta)+|w|^2\beta^2\gamma^2k^4\cos^2\vartheta$$
$$-\mathrm{Re}(a^*b)\beta^2\gamma\left(\frac{1-\cos^2\vartheta}{1-\beta\cos\vartheta}\right)-\mathrm{Re}(a^*w)2\beta\gamma k^2\cos\vartheta$$
$$+\mathrm{Re}(b^*v)k^2\beta^2\gamma\left(\frac{1-\beta\gamma\cos\vartheta}{1-\beta\cos\vartheta}\right)(1-\cos^2\vartheta)$$
$$+\mathrm{Re}(b^*w)\beta^3\gamma^2k^2\cos\vartheta\left(\frac{1-\cos^2\vartheta}{1-\beta\cos\vartheta}\right),\quad(4)$$

where

$$\beta=q/q_0,\quad \gamma=q_0/k,$$
$$u\equiv c+2d,\quad v=d+e-c,\quad w=d-e-c.\quad(5)$$

For a rough estimate we can use $\gamma=1$, but for a more accurate calculation,

$$q_0=k+0.4327[k+(k^2+45.226)^{\frac{1}{2}}]^{-1}.\quad(6)$$

All quantities are to be taken in the center-of-mass system.

Equation (4) as it stands is in an awkward form for analysis. However, multiplying through by $(1-\beta\cos\theta)$,[2] we obtain an expression which is a fourth-order polynomial in $\cos\theta$. We have, in fact,

$$(1-\beta\cos\theta)^2|M|^2$$
$$=A+B\cos\theta+C\cos^2\theta+D\cos^3\theta+E\cos^4\theta,\quad(7)$$

and the coefficients are given by

$$A=|a|^2+\tfrac{1}{2}\beta^2(1+\gamma^2\beta^2)|b|^2+\tfrac{1}{2}\beta^2\gamma^2k^4(|u|^2+|v|^2)$$
$$-\beta^2\gamma(\mathrm{Re}a^*b-k^2\,\mathrm{Re}b^*v),\quad(8)$$

$$B=\beta\{-2|a|^2-\beta^2\gamma|b|^2-\beta^2\gamma^2k^4(|u|^2+|v|^2)$$
$$[+\gamma[\beta^2\,\mathrm{Re}a^*b-2k^2\,\mathrm{Re}a^*w$$
$$-\beta^2(\gamma+1)k^2\,\mathrm{Re}b^*v+\beta^2\gamma k^2\,\mathrm{Re}b^*w]\},\quad(9)$$

$$C=\beta^2\{|a|^2-\tfrac{1}{2}(1+\beta^2\gamma^2)|b|^2-\tfrac{1}{2}\gamma^2k^4(1-\beta^2)(|u|^2+|v|^2)$$
$$+\gamma^2k^4|w|^2+\gamma[\mathrm{Re}a^*b+4k^2\,\mathrm{Re}a^*w$$
$$-\beta^2\gamma k^2\,\mathrm{Re}b^*w-k^2(1-\gamma\beta^2)\,\mathrm{Re}b^*v]\},\quad(10)$$

$$D=\beta^3\gamma\{|b|^2+\gamma k^4(|u|^2+|v|^2-2|w|^2)$$
$$-[\mathrm{Re}a^*b+2k^2\,\mathrm{Re}a^*w+\gamma k^2\,\mathrm{Re}b^*w$$
$$-(\gamma+1)k^2\,\mathrm{Re}b^*v]\},\quad(11)$$

$$E=\beta^4\gamma^2k^2\{-\tfrac{1}{2}k^2(|u|^2+|v|^2-2|w|^2)$$
$$+\mathrm{Re}b^*w-\mathrm{Re}b^*v\}.\quad(12)$$

The form given by Eq. (3) should be an excellent approximation for positive pion production. For negative production there is an additional term from the nucleon charge interaction, which contains S and D waves. This term, however, is of the order of (μ/M) and hence the analysis presented here should also be quite good for negative pion production. Formally, the method can also be applied to neutral production, although in that case nothing is gained compared to the usual quadratic analysis. In fact, in this latter case only three of the five coefficients in Eq. (7) are independent, since one can factor out $(1-\beta\cos\theta)^2$ on both sides. Nevertheless, if one wants to use isotopic spin formalism, and therefore maintain a symmetry among the charge states, one can also analyze neutral photoproduction in this manner.

Undoubtedly Eqs. (8)–(12) are more complicated than those arising from the quadratic analysis. They have the advantage, however, of promising a really good fit at all angles. As in the case of the quadratic analysis, these relations cannot be used to determine the theoretical coefficients $a,\cdots,e$ from the experimental values of $A,\cdots,E$. If, however, we know $a,\cdots,e$, we can calculate $A,\cdots,E$ and can compare them with experiments. The form given by Eq. (7), therefore, can serve as a common meeting place for all theories and all experiments.

Let us now summarize the practical scheme of analysis proposed above:

1. Tabulate experimental data on the angular distribution of charged pion production in the center-of-mass system.
2. Multiply this table by $(1-\beta\cos\theta)^2$.
3. Plot the result against $\cos\theta$.
4. Fit the resulting curve with the best fourth-order polynomial. In applying the least-squares fit principle, one should weight the deviations by the inverse of the experimental error multiplied by $(1-\beta\cos\theta)^2$, thus giving a weight to each point in the original angular distribution which is inversely proportional to its error.
5. The resulting curve is of the form given by Eq. (7). The coefficients $A,\cdots,E$ thus obtained can be compared with the coefficients predicted by the theory through Eqs. (8)–(12).

Since the absolute values given by the measurements and by the theories are more uncertain than the relative angular distributions, one can normalize the coefficients $A,\cdots,E$ by requiring unity for the integral over $\cos\theta$ (i.e., a total cross section of 2π), that is,

$$\int_0^\pi |M(\vartheta)|^2\sin\vartheta d\vartheta=1.\quad(13)$$

Finally we will calculate the coefficients $A,\cdots,E$ for positive-pion production from a few theories currently in use. These examples are given at $k_{\mathrm{c.m.}}=1.5$, that is, at $k_{\mathrm{lab}}=260$ Mev, where numerous data are available and where some new experiments[2,3] are planned.

We shall give the coefficients for four theories:

I. Born approximation including Dirac moments, that is, first-order covariant perturbation theory. Here

$$a=1,\quad b=\frac{1}{1-0.0743k},\quad v=w=iu=-\frac{0.0743}{k}. \tag{14}$$

II. Cut-off theory in its initial form.[4] Here

$$a=b=1,\quad u=1.16h_{33},\quad v=0.76h_{33},\quad w=0.40h_{33}, \tag{15}$$

where

$$h_{33}=(e^{i\delta_{33}}\sin\delta_{33})/q^3. \tag{16}$$

III. Low's theory.[5,6] Here

$$\begin{aligned} &a=b=1,\\ &u=\frac{0.350}{k(1+0.0743k)}+\frac{0.156}{q_0}\left(\frac{0.75k}{f^2q^3}e^{i\delta_{33}}\sin\delta_{33}-1\right),\\ &v=w=-\frac{0.0654}{k(1+0.0743k)}\\ &\qquad+\frac{0.0778}{q_0}\left(\frac{0.75k}{f^2q^3}e^{i\delta_{33}}\sin\delta_{33}-1\right). \end{aligned} \tag{17}$$

IV. Theory derived from the general dispersion relations.[7] In this case

$$\begin{aligned} &a=\frac{1}{1+0.149q_0}-0.065q_0+\frac{iF_S}{3}(2\delta_{S1}+\delta_{S3}),\\ &b=\frac{1}{1+0.149q_0},\\ &u=-\frac{0.175}{f^2}h^{(+-)}+0.222F_Me^{i\delta_{33}}\sin\delta_{33},\\ &v=\frac{0.175}{f^2}h^{(--)}-\frac{0.065}{q_0}+\frac{i(F_Q+\frac{1}{3}F_M)}{3}e^{i\delta_{33}}\sin\delta_{33},\\ &w=\frac{0.175}{f^3}h^{(--)}-\frac{0.065}{q_0}-\frac{i(F_Q-\frac{1}{3}F_M)}{3}e^{i\delta_{33}}\sin\delta_{33}, \end{aligned} \tag{18}$$

[4] G. F. Chew, Phys. Rev. **96**, 1669 (1954).
[5] G. F. Chew and F. E. Low, Phys. Rev. **101**, 1579 (1956).
[6] M. J. Moravcsik, Phys. Rev. (to be published).
[7] G. F. Chew *et al.* (to be published).

TABLE I. Predictions of the coefficients $A,\cdots,E$ for the angular distribution of positive pion photoproduction from various theories, which are explained in the text. The coefficients are normalized to give a total cross section of 2π. $k_{\rm lab}=260$ Mev.

Coefficient	I	II	III	IV
A	0.5099	0.5532	0.5626	0.5795
B	−0.8241	−1.0345	−1.0302	−1.1128
C	0.2872	0.4366	0.3861	0.4576
D	0.0655	0.1261	0.1813	0.1581
E	0	−0.0643	−0.0830	−0.0762

where

$$\begin{aligned} &F_S=1-\frac{1}{2}\left[1+\frac{1-\beta^2}{2\beta}\log\left(\frac{1-\beta}{1+\beta}\right)\right],\\ &F_M=\frac{3}{4q^2}\left[1+\frac{1-\beta^2}{2\beta}\log\left(\frac{1-\beta}{1+\beta}\right)\right],\\ &F_Q=\frac{1}{{q_0}^2}\left\{1-\frac{3}{4\beta^2}\left[1+\frac{1-\beta^2}{2\beta}\log\left(\frac{1-\beta}{1+\beta}\right)\right]\right\}, \end{aligned} \tag{19}$$

and

$$\begin{aligned} &h^{(+-)}=\tfrac{1}{3}(h_{11}-h_{31}+2h_{13}-2h_{33})\\ &h^{(--)}=\tfrac{1}{3}(h_{11}-h_{13}-h_{31}+h_{33}) \end{aligned} \tag{20}$$

with

$$h_{ij}=(e^{i\delta_{ij}}\sin\delta_{ij})/q^3. \tag{21}$$

The resulting coefficients, normalized to give unit total cross section, are given in Table I. The phase shifts used in the numerical evaluation were

$$\delta_{33}=24°,\ \delta_{11}=-2°,\ \delta_{31}=\delta_{13}=-2°,\ \delta_{S1}=9°,\ \delta_{S3}=-8°,$$

and the coupling constant was taken to be $f^2=0.072$.

Finally, we might add that, if the experiments ever reach the precision of being able to detect very small corrections to Eq. (3) [such as terms of the order $(\mu/M)^2$], the foregoing analysis can be extended easily to include higher powers of $\cos\theta$. At the present time, however, the scheme described above suffices and will continue to do so for some time to come.

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