Neutron Polarization in (p,n) Reactions and Nuclear Optical Model

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The polarization of the neutron produced in a (p,n) reaction is calculated under the assumption of a direct mechanism for the reaction. The "cloudy crystal ball" potential modified by the spin-orbit coupling term is assumed to be the neutron-nucleus interaction. The model is applied to the reaction $Si^{29}(p,n)P^{29}$ for $E_p=6$ and 6.5 Mev. The maximum negative polarizations $\sim -75\%$ come in for the center-of-mass scattering angle $\theta \sim 75°$.

I. INTRODUCTION

THE theory of (p,n) reactions as proposed by Austern *et al.*[1] has been successul in the interpretation of the differential cross sections for (p,n) reactions measured off the pronounced resonances of the excitation function in the medium energy range. In this energy range, for most nuclei, the direct reaction mechanism should be the primary mechanism operative rather than compound nucleus formation.

It is well known that the neutron produced as a product in the (p,n) reaction is polarized. The polarization of neutrons from the $Li^7(p,n)Be^7$ reaction for a rather low energy on and near resonance was investigated[2,3] under the assumption of compound nucleus formation.

On the other hand, it seems interesting to calculate the polarization for the direct-reaction mechanism in a manner similar to that used for deuteron stripping reactions.

The "cloudy crystal ball" potential of Feshbach *et al.*,[4] modified by an additional spin-orbit term, has been successful in the description of the elastic scattering of polarized neutrons.[5] This model was applied by Cheston[6] to (d,p) reactions, for which a stripping mechanism was assumed at deuteron energies ~ 3 Mev. The same model is applied here.[7] Since the direct-reaction mechanism for the (p,n) reaction is used throughout the present paper, the extent of disagreement between the predictions and eventually forthcoming experiments will indicate the extent to which the direct mechanism is not suitable at the proton energies considered below, i.e., ~ 6 Mev.

II. CALCULATION

Following Austern *et al.*,[1] we assume that the neutron which is initially the most loosely bound (with a spin-zero core) and which is in a state of orbital angular momentum l_i and z-component m_i, is knocked out from the surface of the nucleus by the incoming proton which is then captured into a state of orbital angular momentum l_f and z-component m_f. In the free-wave (Born) approximation, the matrix element for the collision in the "outside" region ($r \geqslant r_0$) is:

$$M_B = \int d\sigma_n d\sigma_p \int d\xi d\mathbf{r}_n d\mathbf{r}_p \psi_f^*[\mu_f] \times \exp(-i\mathbf{k}_n\cdot\mathbf{r}_n)\chi_{\mu_n}^*(\sigma_n) V_{np}\psi_i[\mu_i] \times \exp(i\mathbf{k}_p\cdot\mathbf{r}_p)\chi_{\mu_p}(\sigma_p), \quad (r_n, r_p \geqslant r_0) \quad (1)$$

where

$$\psi_i[\mu_i] = \sum_{m_i\mu_n'} C_{l_i\frac{1}{2}}(j_i,\mu_i;\, m_i,\mu_n')\psi(l_i,m_i)\chi_{\mu_n'}(\sigma_n)\phi_0(\xi);$$

$$\psi_f[\mu_f] = \sum_{m_f,\mu_p'} C_{l_f\frac{1}{2}}(j_f,\mu_f;\, m_f,\mu_p')\psi(l_f,m_f)\chi_{\mu_p'}(\sigma_p)\phi_0(\xi);$$

where $\psi(l,m)$ = single-particle orbital, $\phi_0(\xi)$ = core wave function; $\chi_\mu(\sigma)$ = nucleon spin function; $C_{l\frac{1}{2}}$'s are the vector addition coefficients. Similarly, as given by Demeur[8] and Satchler,[9] we employ the zero range n-p interaction: $V_{np}=V_0\delta(\mathbf{r}_n-\mathbf{r}_p)$.

On assuming that the outgoing neutron will scatter in a spin-orbit potential, $\exp(i\mathbf{k}_n\cdot\mathbf{r}_n)\chi_{\mu_n}(\sigma_n)$ in (1) must be replaced by:

$$\Psi_n[\mu_n] = \sum_{L,M_L}\sum_{J,M_J} a^*(L,M_L)C_{L\frac{1}{2}}(J,M_J;\, M_L,\mu_n) \times \psi(J,L,M_J) \quad (2)$$

in the notation of reference 6. The expression

$$\exp(i\mathbf{k}_p\cdot\mathbf{r}_p)\chi_{\mu_p}(\sigma_p)$$

is replaced in the notation of references 6 and 7 by:

$$\Psi_p[\mu_p] = \sum_{L_pM_p} b^*(L_p,M_p)\Phi(L_p,M_p)\chi_{\mu_p}(\sigma_p). \quad (3)$$

The incident proton wave is now assumed to be distorted by a nonspin-dependent potential. There are indications that, for the numerical examples considered below, the influence of an eventual spin-orbit coupling for the proton is rather small and results in smaller absolute values of the neutron polarization. The

[1] Austern, Butler, and McManus, Phys. Rev. **92**, 350 (1953).
[2] R. Adair, Phys. Rev. **96**, 709 (1954).
[3] A. Okazaki, Phys. Rev. **99**, 55 (1955).
[4] Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954).
[5] Adair, Darden, and Fields, Phys. Rev. **96**, 503 (1954).
[6] W. B. Cheston, Phys. Rev. **96**, 1590 (1954).
[7] Most of the general considerations presented herein were reported in a preliminary note: J. Sawicki, Nuovo cimento **2**, 1322 (1955).
[8] H. Demeur, J. phys. radium **16**, 73 (1955).
[9] G. R. Satchler, Proc. Phys. Soc. (London) **A68**, 1037 (1955).

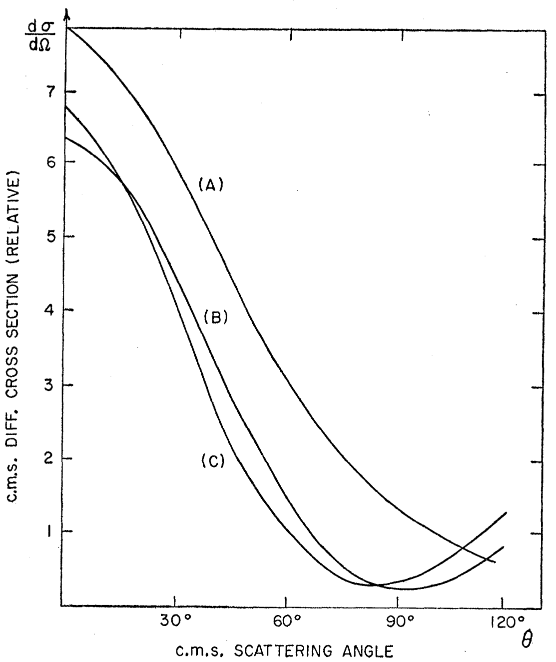

FIG. 1. Angular distribution of neutrons from $Si^{29}(p,n)P^{29}$ for $E_p=6$ Mev. (*A*) No proton and no neutron scattering; (*B*) no proton scattering; (*C*) scattering of neutrons and absorption of proton partial waves $L_p=0.1$.

Coulomb distortion of the incident proton wave is neglected. The rather small polarization due to Coulomb interactions in (d,p) reactions obtained by Grant[10] encourages to use the latter simplification for the proton energies considered here. The influence of the nuclear distortion of the incident proton on the neutron angular distribution and the polarization is shown below to be very small. (This was stated for only the angular distribution first in reference 1.)

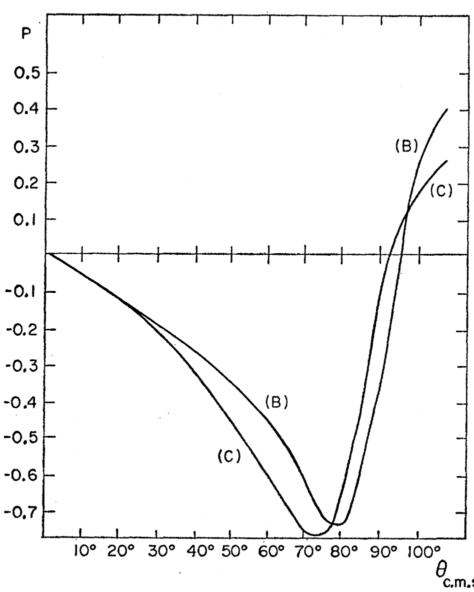

FIG. 2. Polarization P of neutrons from $Si^{29}(p,n)P^{29}$ for $E_p=6$ Mev. (*B*) No proton scattering; (*C*) absorption of proton partial waves $L_p=0.1$.

[10] I. P. Grant, Proc. Phys. Soc. (London) **A68**, 244 (1955).

If $l_i=0$, the polarization is due to the spin-orbit coupling of the neutron scattering potential only (see below). On the other hand, if $l_i\neq 0$, the polarization effect enters, even if one introduces a central interaction. It seems, therefore, interesting to investigate the $l_i=0$ case for the case of **LS** coupling. After all, the $l_i=l_f=0$ case considered below is the simplest one.

The reaction matrix elements for this case are easily obtained in a manner similar to that used in reference 7. With arbitrary normalization, they may be written in the form:

$$
\begin{aligned}
M(\mu_i\rightarrow\mu_n=+\tfrac{1}{2})
&=\sum_{L,M_L}\sum_J\{a(L,M_L)b^*(L,M_L)C_{L\frac{1}{2}}{}^2(J,\,M_L+\tfrac{1}{2};\,M_L,\tfrac{1}{2})\\
&\quad\times C_{0\frac{1}{2}}(\tfrac{1}{2},\mu_i;\,\mu_i-\tfrac{1}{2},\,\tfrac{1}{2})+a(L,M_L)b^*(L,\,M_L+1)\\
&\quad\times C_{L\frac{1}{2}}(J,M_L+\tfrac{1}{2};M_L,\tfrac{1}{2})C_{L\frac{1}{2}}(J,M_L+\tfrac{1}{2};M_L+1,\,-\tfrac{1}{2})\\
&\quad\times C_{0\frac{1}{2}}(\tfrac{1}{2},\,\mu_i;\,\mu_i+\tfrac{1}{2};\,-\tfrac{1}{2})\}R_{LJ}, \qquad (4a)
\end{aligned}
$$

$$
\begin{aligned}
M(\mu_i\rightarrow\mu_n=-\tfrac{1}{2})
&=\sum_{LM_L}\sum_J\{a(L,M_L)b^*(L,\,M_L-1)\\
&\quad\times C_{L\frac{1}{2}}(J,\,M_L-\tfrac{1}{2};\,M_L,\,-\tfrac{1}{2})\\
&\quad\times C_{L\frac{1}{2}}(J,\,M_L-\tfrac{1}{2};\,M_L-1,\,\tfrac{1}{2})\\
&\quad\times C_{0\frac{1}{2}}(\tfrac{1}{2},\,\mu_i;\,\mu_i-\tfrac{1}{2},\,\tfrac{1}{2})+a(L,M_L)b^*(L,M_L)\\
&\quad\times C_{L\frac{1}{2}}{}^2(J,\,M_L-\tfrac{1}{2};\,M_L,\,-\tfrac{1}{2})\\
&\quad\times C_{0\frac{1}{2}}(\tfrac{1}{2},\,\mu_i;\,\mu_i+\tfrac{1}{2},\,-\tfrac{1}{2})\}R_{LJ}, \qquad (4b)
\end{aligned}
$$

where

$$
\begin{aligned}
R_{LJ}=\int_{r0}^{\infty}&drr^2k_0(t_ir)f_0(t_fr)[j_L(k_nr)\\
&-\beta(L,J)h_L{}^{(1)}(k_nr)][j_L(k_pr)-\beta(L)h_L{}^{(1)}(k_pr)]; \qquad (5)
\end{aligned}
$$

$f_0(t_fr)$ and $k_0(t_ir)=h_0{}^{(1)}(it_ir)$ are the bound-state radial wave functions; $\beta(L,J)=(1-\bar{\eta}_{LJ})/2$, $\beta(L)=(1-\bar{\eta}_L)/2$, where $\bar{\eta}_{LJ}(\bar{\eta}_L)$ are the average reflection factors defined as, e.g., reference 4 [if the neutron (proton) potential equals zero, then $\bar{\eta}_{LJ}\equiv 1(\bar{\eta}_L\equiv 1)$ and $\beta(L,J)\equiv 0$ $(\beta(L)\equiv 0)$, i.e., we have no neutron (proton) scattering]. All other symbols have their usual meaning.

If we now define the axis of quantization by the vector $\mathbf{k}_p\times\mathbf{k}_n$, the off-diagonal terms in (4a)–(4b) vanish. Then the components of the polarization vector

TABLE I. Neutron polarization P (in %) in $Si^{29}(p,n)P^{29}$ for $E_p=6.5$ Mev for: (B) no proton scattering and (C) $\beta(0)=\beta(1)=\frac{1}{2}$ (absorption of proton partial waves $L_p=0.1$).

C.M.S. angle $\theta°$	0	10	20	30	40	50	60	70	80	90	120
(B) P	0	−7.6	−15.7	−25.8	−37.9	−52.1	−69.5	−80.8	−30.9	61.12	69.5
(C) P	0	−7.3	−15.2	−26.7	−41.5	−57.9	−76.8	−39.7	65.2	98.2	55.6

in the reaction plane are: $P_x\equiv P_y\equiv 0$ and the polarization P is:

$$P=P_z=\{|M_{00}(+\tfrac{1}{2})|^2-|M_{00}(-\tfrac{1}{2})|^2\}/\{|M_{00}(+\tfrac{1}{2})|^2+|M_{00}(-\tfrac{1}{2})|^2\}, \quad (6)$$

where:

$$M_{00}(+\tfrac{1}{2})=\sum_L\sum_{M_L}Y_{LM_L}{}^*\left(\frac{\pi}{2},0\right)Y_{LM_L}\left(\frac{\pi}{2},\theta\right)\times\left\{\left(\frac{L+1+M_L}{2L+1}\right)R_L{}^+ +\left(\frac{L-M_L}{2L+1}\right)R_L{}^-\right\}, \quad (7a)$$

$$M_{00}(-\tfrac{1}{2})=\sum_L\sum_{M_L}Y_{LM_L}{}^*\left(\frac{\pi}{2},0\right)Y_{LM_L}\left(\frac{\pi}{2},\theta\right)\times\left\{\left(\frac{L+1-M_L}{2L+1}\right)R_L{}^+ +\left(\frac{L+M_L}{2L+1}\right)R_L{}^-\right\}, \quad (7b)$$

where $R_L{}^+=R_{L,\,J=L+\frac{1}{2}}$, $R_L{}^-=R_{L,\,J=L-\frac{1}{2}}$; $Y_{LM_L}(\vartheta,\varphi)$ are spherical harmonics as defined by Blatt and Weisskopf,[11] θ being the center-of-mass (c.m.) scattering angle. If the **LS** coupling equals zero, then $R_L{}^+=R_L{}^-=R_L$ and $P\equiv 0$. It is easily seen that $P(\theta=0°)=P(\theta=180°)=0$.

III. NUMERICAL RESULTS AND DISCUSSION

For the numerical computations the $Si^{29}(p,n)P^{29}$ reaction (ground state, $l_i=l_f=0$, $Q=-5.503$ Mev) was employed, the incident proton energies being 6 and 6.5 Mev. The final state neutron scattering potential was taken to be:

$$V_n \text{ (in Mev)}=-40(1+i0.05)-2\mathbf{L}\cdot\mathbf{S}, \quad r\leqslant R_0=1.45A^{1/3}\times 10^{-13}\text{ cm}$$
$$=0, \quad r\geqslant R_0. \quad (8)$$

The "Butler radius" r_0 was arbitrarily chosen to be the "nuclear radius" R_0. To take into account the initial-state proton scattering, the waves with $L_p=0$ and 1 are assumed to be absorbed by the nucleus $[\beta(0)=\beta(1)=\frac{1}{2}]$, all other partial waves L_p being unaffected (similarly to the procedure in reference 1). This should overestimate the effects of proton nuclear scattering.

The Coulomb effect of the final-state (bound) proton was taken into account in a similar fashion, as was done by Yoccoz.[12] For $r>r_0$: $f_0(t_fr)=W_{-n\frac{1}{2}}(2t_fr)/r$, where $W_{-n\frac{1}{2}}(2t_fr)$ is the Whittaker function and $n=Ze^2M_p{}^*/(\hbar^2t_f)$, all the remaining symbols having their usual meaning. In the important region of r, $f_0(qr)$ is fairly well approximated by some "equivalent" $Ah_0{}^{(1)}(it_f'r)$. The integrals R_{LJ} are performed by numerical methods.

Three cases are discussed: (*a*) $\beta(L,J)\equiv 0$, $\beta(L)\equiv 0$ (no neutron and no proton scattering); (*b*) $\beta(L,J)$ corresponding to (8), $\beta(L)\equiv 0$ (no proton scattering); (*c*) $\beta(L,J)$ corresponding to (8), $\beta(0)=\beta(1)=\frac{1}{2}$, all other $\beta(L)\equiv 0$.

The angular distributions for $E_p=6$ Mev are presented in Fig. 1. The only important effects of the scattering terms are: the zeros of the angular distributions disappear, the first minimum for (*b*) and (*c*) is displaced with respect to (*a*) towards smaller angles, and the first minimum for (*c*) is very slightly shifted with respect to (*b*) towards smaller angles. The curves for $E_p=6.5$ Mev have been found very similar in shape but steeper.

The neutron polarizations for $E_p=6$ Mev are presented in Fig. 2. The main features of the curves are: (1) P is negative up to $\theta\sim 90°$; (2) the first maximum of $|P|$ is rather large ($\sim$75%) and comes in near $\theta\sim 75°$; (3) for $\theta\gtrsim 80°$, P decreases very rapidly and changes sign, and then increases in positive values up to the second maximum of $|P|$.

Cheston's P for the $C^{12}(d,p)C^{13}$ reaction and for $E_d=3.29$ Mev[6] is also negative for smaller angles (contrary to the values of P obtained by Horowitz and Messiah[13] and Newns[14]). This feature of P, discussed in terms of (8), is due to the **LS** coupling model used.

The neutron polarizations for $E_p=6.5$ Mev are presented in Table I. All the main features of P for $E_p=6$ Mev remain preserved save that the rather high positive values of P appear at large angles. This is mainly due to the fact that the differential cross

TABLE II. P (in %) in $Si^{29}(p,n)P^{29}$ for $E_p=6$ Mev.

C.M.S. angle $\theta°$	20	40	60	80	100
P (no Coulomb effect)	−11.1	−25.0	−45.7	−71.5	24.3
P (Coulomb effect included)	− 7.0	−15.2	−26.5	−39.3	−63.0

[11] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952).

[12] J. Yoccoz, Proc. Phys. Soc. (London) **A67**, 813 (1954).

[13] J. Horowitz and A. M. L. Messiah, J. phys. radium **14**, 695 (1953).

[14] H. C. Newns, Proc. Phys. Soc. (London) **A66**, 477 (1953).

sections decrease much more rapidly for large angles than for small angles with an increase of E_p.

Note added in proof.—Coulomb effect of the incident proton wave was taken into account in the computations for $E_p=6$ Mev and case (b) i.e., no proton nuclear scattering. The angular distribution is considerably flattened, its deep minimum being shifted towards larger angles. The corresponding maximum of negative polarization is also shifted in this direction P (in %) in $Si^{29}(p,n)P^{29}$ is given in Table II.

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Neutrons from the Proton Bombardment of P^{31}†

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The reaction $P^{31}(p,n)S^{31}$ has been studied at $E_p=17.2$ Mev. The energy spectrum of the neutrons was determined by means of proton recoil measurements in nuclear emulsions. The mass excess, $M-A$, of S^{31} was calculated to be -10.04 ± 0.20 Mev. Excited states of S^{31} have been located at 1.15 ± 0.15, 2.28 ± 0.20, 3.35 ± 0.20, 4.51 ± 0.15, 5.94 ± 0.30, and 6.41 ± 0.20 Mev.

THE level structure of the $A=31$ isobars is poorly known. In P^{31}, the excited states at 1.26- and 2.23-Mev excitation are well verified,[1–3] and levels have also been reported at 0.4, 0.9 and 3.4 Mev.[4] In the mirror nucleus, S^{31}, the mass of the ground state is known to within 200 kev from β-decay work,[4] but no excited states had been observed. The reason for this lack of information about S^{31} is that all reactions leading to it are either neutron-emitting reactions or very endoergic, or both, with the exception of the reaction $S^{32}(He^3,\alpha)S^{31}$, which has never been studied. It was decided to investigate S^{31} by means of the reaction $P^{31}(p,n)S^{31}$. The difficulty in accurately measuring neutron energies coupled with the beam spread of cyclotron protons, necessary because of the endoergic character of the $P^{31}(p,n)S^{31}$ reaction, precluded the possibility of obtaining very accurate information on the states of S^{31}. However, because of the difficulty in reaching this nucleus, it was felt than any information on S^{31} would be of value.

A target of P^{31} was prepared by dissolving red phosphorus in absolute ethyl alcohol, and painting it onto a thin polystyrene film. The thickness of the target corresponded to an energy loss of 100 kev for 17.5-Mev protons. Unfortunately, the target was reversed prior to the exposure, so that the proton

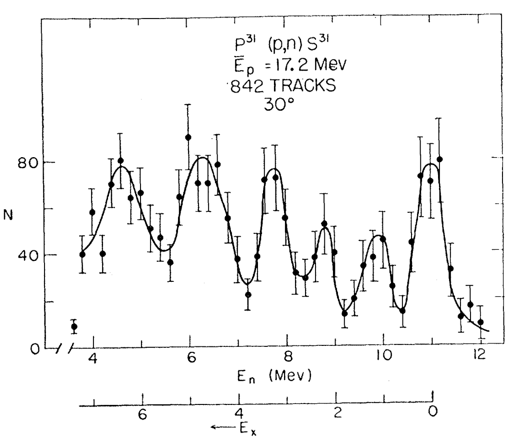

FIG. 1. Data at 30°. N is the corrected number of neutrons per 200-kev interval; E_n is the neutron energy; E_x is the excitation energy in S^{31}.

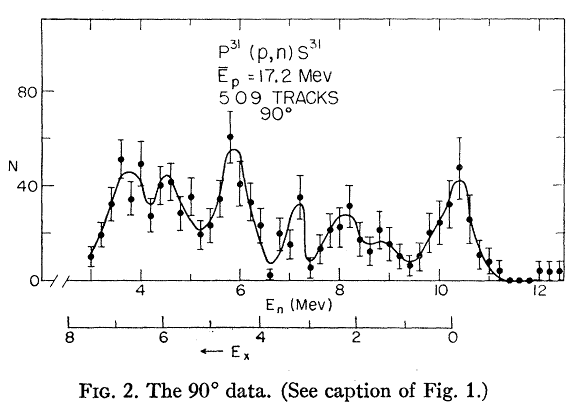

FIG. 2. The 90° data. (See caption of Fig. 1.)

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