Scattering of Protons from C^{12}†

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The elastic scattering of protons from carbon in the range of bombarding energies 1.5 to 5.5 Mev has been investigated, using protons from the Rice Institute Van de Graaff accelerator and a differentially pumped gas scattering chamber. Angular distributions and excitation curves at seven angles have allowed an explicit phase shift analysis of the data below 5 Mev. Only five nonzero phase shifts were necessary to fit the data to better than 10% at all energies and angles. The fourth and fifth excited states of N^{13} are identified at 4.808- and 5.37-Mev bombarding energy, and their respective assignments are $5/2^+$ and $3/2^+$, while their laboratory widths are 12 kev and 125 kev. The 4.43-Mev inelastic scattering γ radiation was studied by obtaining excitation curves and angular distributions which confirmed the $3/2^+$ assignment for the 5.37-Mev resonance. The large ratio of inelastic to elastic reduced widths for this level suggests that it and, possibly, the fourth excited state of N^{13} are due to an S-wave proton configuration about C^{12} in its 2^+ first excited state.

INTRODUCTION

CERTAIN features of the properties and structure of nuclei have led to the hypothesis that nucleons are arranged into shells in the nucleus.[1] Most theories which attempt to describe the nature of these shells are based on the independent particle model of the nucleus with strong spin-orbit forces. It is particularly interesting to apply this model to the closed-shell-plus-one nuclei, since it should be most applicable to such cases. One method of investigating the properties of the energy levels of such nuclei is to observe the elastic scattering of neutrons and protons from a closed shell nucleus. An application of the dispersion theory formalism[2,3] allows, in principle, a determination of the resonant energies, reduced widths, spins, and parities of any states observed. As a method of analyzing the elastic scattering data, this formalism is of use primarily when the separation of levels of the same spin and parity is large compared with their widths.

This condition is quite well fulfilled for the levels thus far known in N^{13}. Jackson *et al.*[4] have investigated the elastic scattering of protons by carbon from 0.4 to 4.4 Mev and have found the first three excited states of N^{13}. Martin *et al.*[5] have investigated this reaction from 2.2 to 7 Mev and have reported the existence of levels in N^{13} at proton energies of 3.2, 4.8, 5.37, and 5.9 Mev. Jackson *et al.* definitely ruled out the existence of a state at 3.2 Mev, but were unable to attain the energies necessary to observe the other three reported states. Moreover, in their phase-shift analysis of the elastic scattering data, they found it necessary to allow two of the phase shifts to depart from the values calculated from the dispersion theory. One explanation of this (but not the only one, as they pointed out) was the existence of states in N^{13} above the energy range of their experiment.

As would be expected on the basis of charge symmetry of nuclear forces, there is quite good agreement between the properties of the first three excited states of N^{13} and those of the corresponding states in C^{13}. Numerous studies of the properties of the fourth and higher excited states of C^{13}, as observed in the elastic scattering of neutrons by C^{12}, have been reported.[6] A study of the properties of the corresponding states in N^{13} would allow a further check of the charge symmetry hypothesis. Furthermore, from the energy difference of states with the same orbital quantum numbers, but with different total angular momenta, it is possible to obtain some idea of the magnitude of the spin orbit forces. For these reasons it was believed that an investigation of the elastic scattering of protons by carbon from 4 to 5.5 Mev would prove to be of considerable value in providing information pertinent to the current theories of nuclear structure and nuclear forces.

APPARATUS

In order for the results of a phase-shift analysis of elastic scattering data to be dependable, the absolute values of the differential cross section must be known to within a few percent. A gas target is ideally suited for such experiments, since the target thickness depends only on the dimensions of the detector slit system, the scattering angle, and the pressure and temperature of the gas. These quantities may be measured to a degree of precision higher than that necessary for an analysis of the scattering data.

In order to perform elastic scattering experiments employing gas targets, a large-volume scattering chamber and differential pumping system were designed and

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[1] M. G. Mayer and J. H. D. Jensen, *Elementary Theory of Nuclear Shell Structure* (John Wiley and Sons, Inc., New York, 1955).
[2] E. P. Wigner and L. Eisenbud, Phys. Rev. 72, 29 (1947).
[3] T. Teichmann and E. P. Wigner, Phys. Rev. 87, 123 (1952).
[4] H. L. Jackson and A. I. Galonsky, Phys. Rev. 89, 370 (1953); Jackson, Galonsky, Eppling, Hill, Goldberg, and Cameron, Phys. Rev. 89, 365 (1953).
[5] Martin, Schneider, and Sempert, Helv. Phys. Acta 26, 595 (1953).
[6] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. 27, 77 (1955).

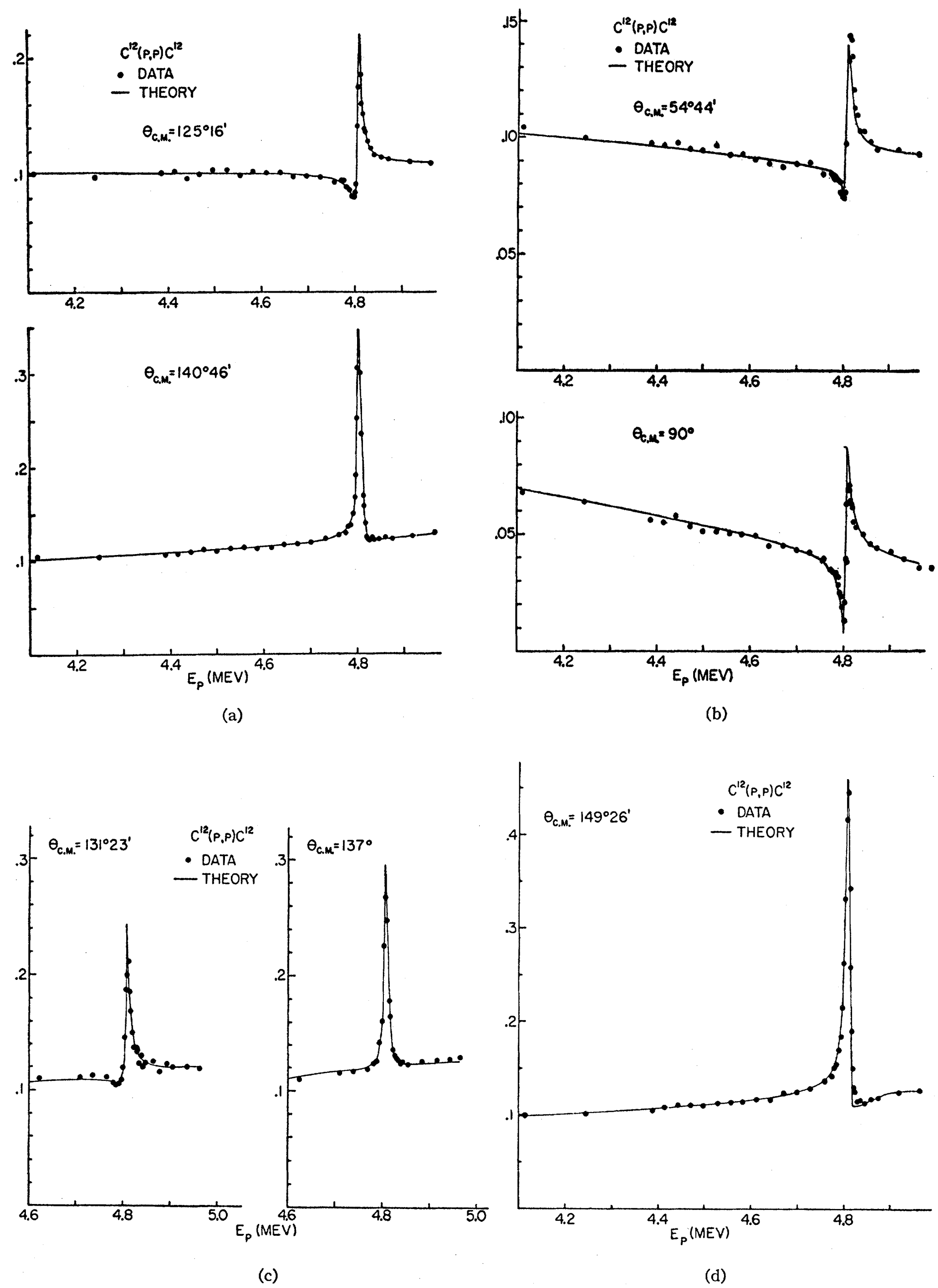


FIG. 1. The $C^{12}(p,p)C^{12}$ center-of-mass differential cross section (in barns/steradian) *vs* bombarding energy at the seven angles observed. The data points are solid circles, while the solid line indicates the fit of the theory to these data. The narrow anomaly at 4.808 Mev is the fourth excited state of N^{13}, and its assignment is $5/2^+$. The behavior of the off-resonance cross section is due to the presence of a $3/2^+$ resonance at 5.37 Mev.

built at The Rice Institute. Since this apparatus and the associated equipment have been described in the preceding paper,[7] no description will be given here. The target gas employed in the scattering experiments was methane at pressures of the order of $\frac{1}{2}$ cm Hg. At such pressures, the errors introduced by incomplete beam integration due to the small angle scattering of particles of the beam out of the Faraday cup were found to be negligible; the energy lost by the beam in traveling from the 90° analyzing magnet to the target volume was calculated to be 7 kev with an uncertainty of less than 3 kev. The scintillation crystals employed as detectors with the two photomultiplier counters were CsI(Tl) crystals of 12 mil thickness which allowed the pulse-height separation of protons scattered from hydrogen and carbon.

EXPERIMENTAL RESULTS

For convenience in the analysis, the differential cross section and the scattering angle are given in the center-of-mass system, while the proton energy is given in the

[7] Russell, Phillips, and Reich, Phys. Rev. **104**, 135 (1956), preceding paper.

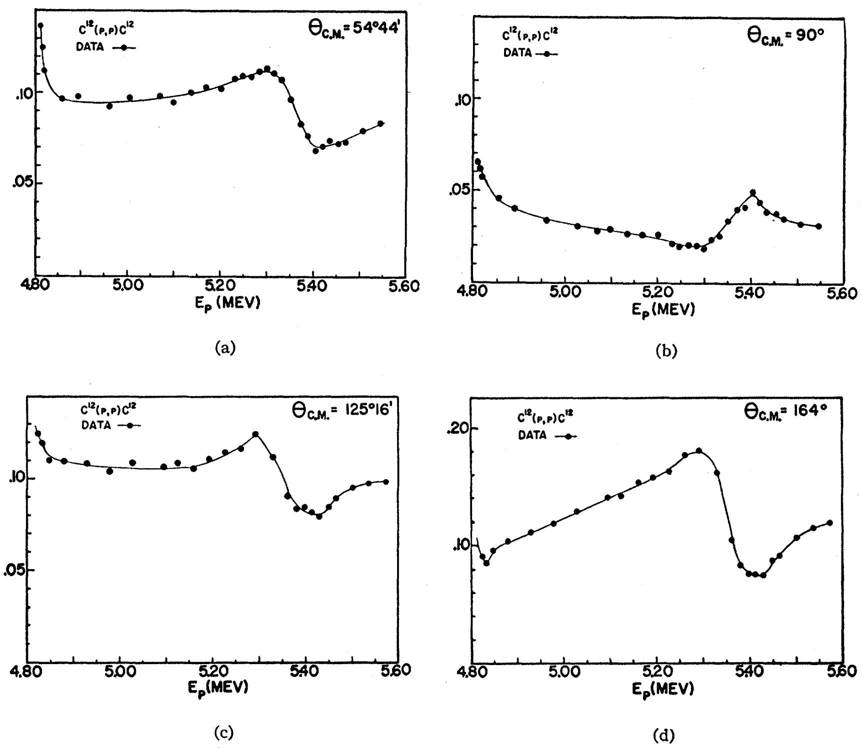


FIG. 2. The $C^{12}(p,p)C^{12}$ center-of-mass differential cross section (in barns/steradian) *vs* bombarding energy at four angles. The solid line is merely drawn through the data points and does not represent a theoretical curve. The shape of the anomaly near 5.4 Mev at the four angles is satisfactorily explained by a $3/2^+$ assignment to this level.

laboratory system. The differential cross section for the $C^{12}(p,p)C^{12}$ reaction from 4.1 to 5.0 Mev at seven angles of scattering is given in Fig. 1. Figure 2 shows the $C^{12}(p,p)C^{12}$ differential cross section from 4.8 to 5.6 Mev at four angles of scattering. The presence of anomalies at 4.8 and 5.4 Mev is the predominant feature of these excitation functions. Angular distributions, taken at proton energies of 4.613, 4.964, and 5.574 Mev, are shown in Fig. 3. For reasons to be discussed later, excitation functions for the $C^{12}(p,p)C^{12}$ reaction were taken at center-of-mass angles of 54° 44′ and 90° from 1.5 to 5.5 Mev. These curves are shown in Fig. 4.

The energy scale for these data was established by means of The Rice Institute annular magnet spectrometer.[8] The peak of the 4.8-Mev anomaly, when observed at 180°, was found to be (4.806±0.005) Mev. The position of the peak is a function of the scattering angle, and in establishing the energy scale, the assumption was made that the energy of the peak in the 149° 26.5′ cross section was 4.806 Mev. After a phase-shift analysis of the data in this energy region had been completed, it was found that this assumption introduced an error of about 0.2 kev into the energy scale. Since the energy lost by the beam in reaching the target volume could be calculated to better than 3 kev, the energy scale is believed good to ±8 kev at 4.8 Mev or better than 0.2%. Table I lists the uncertainties in the various experimentally measured quantities.

The methane gas used in the scattering experiments was supplied by the Phillips Petroleum Company of Bartlesville, Oklahoma, and had a claimed purity of better than 99%. No analysis was made to check the producer's claim, but on the assumption that most of the impurities were present in the form of light hydrocarbons the error introduced would be negligible. Natural carbon contains about 1% C^{13}. Since no values for the $C^{13}(p,p)C^{13}$ differential cross section have been quoted above about 1.6 Mev at the present time, the error introduced through this isotopic contaminant cannot be estimated. In addition to the uncertainties listed, there is the statistical uncertainty in the yield of the elastically scattered protons. In general, this statistical uncertainty was less than 2%, although in the region of the dip in the 90° cross section near 4.8 Mev this uncertainty was 4%. Comparisons with the University of Wisconsin data[4] were made at several angles and energies. The agreement between the two sets of data was always within the statistical accuracy of our data.

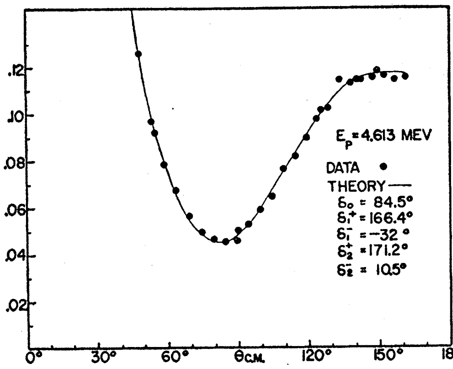
ANALYSIS OF THE $C^{12}(p,p)C^{12}$ DATA BELOW 5 MEV

Introduction and General Expressions

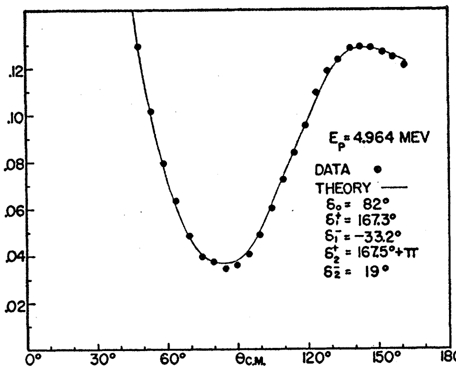
If elastic scattering is the only process energetically possible, the partial wave expansion of the center of mass differential cross section for the elastic scattering of protons by spin-zero nuclei is

$$\sigma(\theta)=(1/k^2)(|f_c|^2+|f_i|^2), \qquad (1)$$

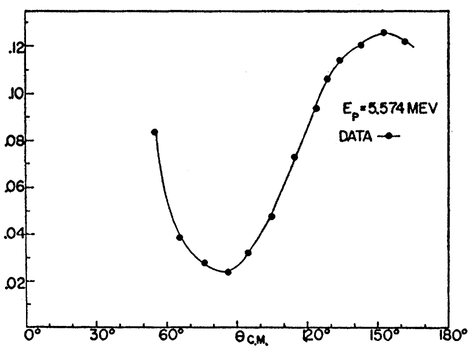
[8] Gossett, Phillips, and Eisinger, Phys. Rev. **98**, 724 (1955).



(a)



(b)



(c)

FIG. 3. The $C^{12}(p,p)C^{12}$ center-of-mass differential cross section (in barns/steradian) *vs* center-of-mass angle. The solid lines on the two lower energy angular distributions represent theoretical fits using the five phase shifts shown, while the solid line on the highest energy angular distribution merely connects the data points.

where

$$f_c = -(\eta/2)\csc^2(\theta/2)\exp[i\eta\log_e\csc^2(\theta/2)] + \sum_{l=0}^{\infty}\exp(i\alpha_l)P_l(\cos\theta)[(l+1)\exp(i\delta_l^+)\sin\delta_l^+ + l\exp(i\delta_l^-)\sin\delta_l^-],$$

$$f_i = \sin\theta\sum_{l=1}^{\infty}\exp(i\alpha_l)P_l'(\cos\theta)[\exp(i\delta_l^+)\sin\delta_l^+ - \exp(i\delta_l^-)\sin\delta_l^-].$$

Here, $k=\lambda\!\!\!^{-}{}^{-1}=\mu v/\hbar$, μ being the reduced mass of the system, and v the relative velocity,

$$\eta = ZZ'e^2/\hbar v, \quad \alpha_l = 2\sum_{s=1}^{l}\arctan(\eta/s),$$

TABLE I. Estimated cross-section uncertainties for p-C^{12} scattering.

Quantity	Uncertainty
Geometry	0.5%
Angular uncertainty	0.1°
Current integration	3%
Detection efficiency	2%
Gas purity	1%
Root-mean-square uncertainty	3.8%

with $\alpha_0=0$, θ is the center-of-mass scattering angle, $P_l(\cos\theta)$ is the lth order Legendre polynomial, $P_l'(\cos\theta) = dP_l(\cos\theta)/d(\cos\theta)$, and $\delta_l^{\pm}$ is the non-Coulomb phase shift of the partial wave of orbital angular momentum l and total angular momentum $j=l\pm\frac{1}{2}$.

The expression f_c, the coherent scattering amplitude, represents those protons whose spins do not change direction in the scattering process, while the expression f_i, the incoherent scattering amplitude, represents those protons whose spins have been reversed during scattering. This expression has several mathematical properties which made its application to a specific problem somewhat simpler than might be expected. Since these properties have been discussed in detail elsewhere,[9] they will be mentioned only briefly here.

Each term in f_c and f_i is a complex number, and thus may be treated as a "vector" in the complex plane. For convenience, these terms will be called partial wave vectors, and spectroscopic notation will be used in referring to them. Thus, the partial wave vector involving $\delta_1^+(l=1, j=1+\frac{1}{2}=\frac{3}{2})$ will be called the $P_{3/2}$ vector. f_c and f_i may be obtained by adding these various vectors graphically, and the resulting cross section may be calculated from (1).

As a resonance of a given l and j is traversed, the corresponding phase shift, $\delta_l^{\pm}$, changes by approxi-

[9] R. A. Laubenstein and M. J. W. Laubenstein, Phys. Rev. **84**, 18 (1951).

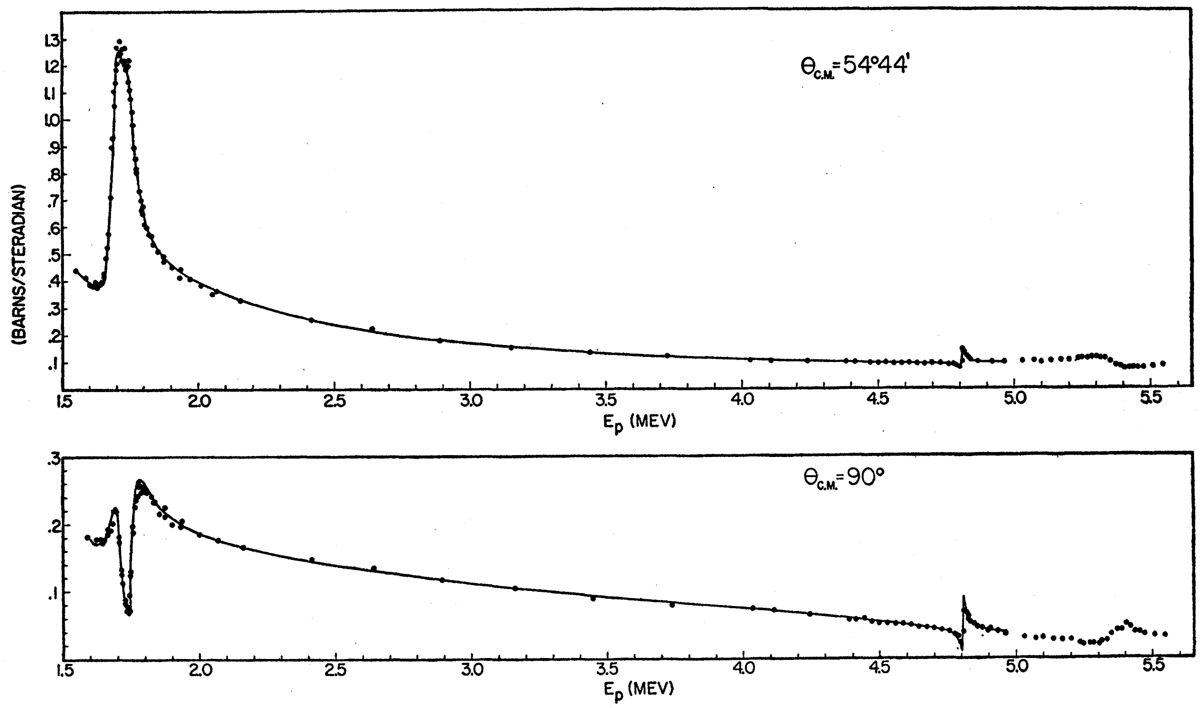


FIG. 4. The $C^{12}(p,p)C^{12}$ center-of-mass differential cross section *vs* bombarding energy at two forward angles. The solid line indicates the theoretical fit to the data.

mately 180°, and the tip of the corresponding partial wave vector moves counterclockwise along the circumference of a circle. This circle has a diameter of $(j+\frac{1}{2})P_l(\cos\theta)$ and is tangent, at the point corresponding to $\delta_l^j=0$, to a line making an angle α_l with the real axis. This property enables one to obtain some idea of the spin and parity of a resonance. If cross-section data are available at an angle close to 180°, where f_i is small, then

$$k[\sigma(\theta)_{\max}]^{\frac{1}{2}}-k[\sigma(\theta)_{\min}]^{\frac{1}{2}}\simeq|f_c|_{\max}-|f_c|_{\min}=(j+\tfrac{1}{2})P_l(\cos\theta).$$

Thus, the maximum and minimum cross sections at a backward angle give some indication of the spin and parity of a resonance.

An analysis of the scattering data consists of extracting the phase shifts from these data. Once the phase shifts are known, it is desirable to relate them to parameters characteristic of the resonance levels involved. On the basis of the single-level approximation,

$$\delta_l^{\pm}=-\tan^{-1}(F_l/G_l)_{r=a}+\tan^{-1}[(\tfrac{1}{2}\Gamma_\lambda)/(E_\lambda+\Delta_\lambda-E)]=\phi_l+\beta_l^{\pm}. \quad (2)$$

F_l and G_l are the regular and irregular Coulomb wave functions, respectively, as defined by Bloch *et al.*[10]; ϕ_l and $\beta_l^{\pm}$ are the "hard sphere" and resonant phase shifts. The relation between the experimental width, Γ_λ, and the reduced width, γ_λ^2, is

$$\Gamma_\lambda=2k\gamma_\lambda^2/A_l^2, \quad \text{where} \quad A_l^2=F_l^2+G_l^2.$$

The resonant energy, E_R, is defined as that energy for which $E_\lambda+\Delta_\lambda-E=0$, where

$$\Delta_\lambda=-(\gamma_\lambda^2/a)(d\ln A_l/d\ln\rho+l)_{\rho=ka}.$$

In order to obtain the reduced width γ_λ^2, and the characteristic energy, E_λ, of a level, it is necessary to choose a value for the interaction radius a. The value used for this analysis was $a=4.77\times10^{-13}$ cm, which was the value used by the Wisconsin group[4] and was chosen in order that the results of the two analyses might be directly comparable.

Analysis of the 4.8-Mev Anomaly

Several considerations allow one to eliminate certain l and j assignments for a given resonance. If the resonance shows a *pronounced* dip when observed at 90°, then it cannot, in general, have odd parity. This follows from Eq. (1), since at 90° the terms involving odd l occur only in f_i. In general, f_i is somewhat smaller than f_c off-resonance; its effect on resonance, then, will be principally to increase the cross section, or at most to produce a slight dip. A preliminary experiment, using methane of unknown purity, contained data at an angle of 161.6° (c.m.). From the maximum and minimum cross sections of the resonance, when observed at this angle, it was decided that the resonance

[10] Bloch, Hill, Broyles, Bouricius, Freeman, and Breit, Revs. Modern Phys. 23, 147 (1951).

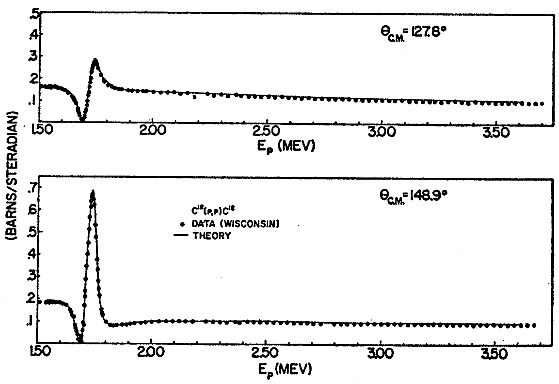


FIG. 5. The $C^{12}(p,p)C^{12}$ center-of-mass differential cross section *vs* bombarding energy. The points represent the Wisconsin data and the solid lines represent the theoretical fit obtained by using the phase shifts of the present analysis.

could be $D_{3/2}$, $D_{5/2}$, or possibly $G_{7/2}$. It is also possible to place an upper limit on the l values which form the state. The Wigner limit[3] on reduced widths requires that $\gamma_\lambda^2 \leq \frac{3}{2}(\hbar^2/\mu a)$. For an a of 4.77×10^{-13} cm, $\gamma_\lambda^2 \leq 14 \times10^{-13}$ Mev cm. At 4.8 Mev, a state with an observed width of 12 kev and an l of 5 requires $\gamma_\lambda^2=\Gamma_\lambda A_5^2/2k =31\times10^{-13}$ Mev-cm, which is more than a factor of two too large. Thus, the resonance at 4.8 Mev has even parity and is formed by protons having an $l\leq 4$.

In order to fit the data below 5 Mev, it was decided to fit the angular distribution at 4.613 Mev first. Consequently, phase shifts were calculated from Eq. (2), using the Wisconsin level parameters, and these phase shifts were then modified in the manner that they reported necessary.[4] Since the resulting fit was quite bad (e.g., ~80% high at 54° 44′), it was decided to extract the phase shifts explicitly from the data. The method adopted for this assumed two things: (1) the resonance was formed by D-wave protons, and (2) all phase shifts were zero for partial waves having $l>2$. The following reasoning was then used: (1) at 54° 44′ and 125° 16′, $P_2(\cos\theta)=0$; At these angles, f_c is essentially constant across the resonance. (2) at 90°, $P_1(\cos\theta)=0$. At this angle, f_i is essentially constant across the resonance. (3) Using Eq. (1), one may obtain a relation of the form

$$[k^2\sigma(\theta)_{\max}-|f_c|^2]^{\frac{1}{2}}-[k^2\sigma(\theta)_{\min}-|f_c|^2]^{\frac{1}{2}} =|f_i|_{\max}-|f_i|_{\min}=\sin\theta P_2'(\cos\theta)$$

at 54° 44′ and at 125° 16′. These two expressions may be used to calculate $|f_c|$ at the two angles. (4) Similarly, the (constant) value of $|f_i|$ at 90° may be determined from the 90° cross-section data. At this angle, $f_i=\sin(\delta_1^+-\delta_1^-)$, since by assumption $\delta_l^\pm=0$ for $l>2$. (5) The values of $|f_c|$ and $|f_i|$, determined from (3) and (4), are values at 4.8 Mev. It was assumed that they had these same values at 4.613 Mev. (6) The expressions given in (3) and (4) provide three relations between the three unknowns: δ_0, δ_1^+, and δ_1^-. These relations were solved (graphically) to determine possible values of δ_0, δ_1^+, and δ_1^-. One set of possible values is $\delta_0=82°$, $\delta_1^+=162.5°$, and $\delta_1^-=-32°$. The striking thing about this set of phases is that δ_0 and δ_1^- agree quite well with the values calculated from Eq. (2), using the Wisconsin level parameters. δ_1^+, however, is about 18° higher than the calculated value. (7) From these values of δ_0, δ_1^+, and δ_1^-, the phase shifts δ_2^+ and δ_2^- were extracted explicitly from the cross-section data at 54° 44′ and 125° 16′. The resulting phase shifts fit the angular distribution at 4.613 Mev to within 10% at all angles. It was possible to improve the fit by calculating increments ($\Delta\delta_l^\pm$) to the corresponding phase shifts in the following manner. It was assumed that the experimental cross section at a given angle differed from the calculated one by a correction term which was linear in the $\Delta\delta_l^\pm$. That is, $\sigma(\theta)_{\exp}=\sigma(\theta)_{\text{calc}} +\sum_l[\partial\sigma(\theta)/\partial\delta_l^\pm]\Delta\delta_l^\pm$. Such equations may be formed at five angles, and the five $\Delta\delta_l^\pm$ may be determined. It was found that adding $\frac{1}{3}$ of the calculated increments to the corresponding phase shifts gave quite acceptable results. The resulting phase shifts, along with the angular distribution they predict, are included in Fig. 3. Once these phase shifts were obtained, it was not difficult to obtain a fit to the angular distribution at 4.964 Mev. Furthermore, an examination of the partial wave vectors at 4.613 Mev showed that the only partial wave vector which could reproduce the observed resonance shape at all the angles observed was the $D_{5/2}$ vector. This established the spin and parity of the 4.8-Mev resonance as $5/2^+$.

The phase shifts in the region between the two angular distributions were extracted from the data, and from the observed energy variation of δ_2^+, the parameters γ_λ^2 and E_λ for this resonance were obtained. A two-level formula for the phase shift was used, since another $D_{5/2}$ resonance is known to exist in N^{13}.[4] The

TABLE II. Parameters of the first five excited states in N^{13}. $a=1.45(\sqrt[3]{12}+\sqrt[3]{1})\times10^{-13}$ cm$=4.77\times10^{-13}$ cm.

Level:	$S_{1/2}$[b]	$P_{3/2}$[b]	$D_{5/2}$[b]	$D_{5/2}$	$D_{3/2}$[d]
E_R (Mev)[a]	0.461	1.698	1.748	4.808	5.37
E (Mev)[a]	−1.076	1.704	1.809	4.816	
Γ (kev)[a]	34	60	66	12	125
$\gamma_\lambda^2\times10^{-13}$ (Mev cm)[a]	8.22[c]	0.477[c]	3.169[c]	0.048[c]	0.18,[c] 2.9[e]
E_R (Mev)	2.369	3.511	3.558	6.380	6.90
E (Mev)	0.951	3.516	3.612	6.387	
Γ (kev)	31	55	61	11	115
$\gamma_\lambda^2\times10^{-13}$ (Mev cm)	7.58	0.440	2.92	0.044	0.17,[c] 2.7[e]
$\gamma_\lambda^2/(3\hbar^2/2\mu a)$	0.54	0.031	0.21	0.0031	0.012,[c] 0.2[e]

[a] Energies in the laboratory system. All other energies are in the center-of-mass system with the ground state of N^{13} taken to be zero.
[b] Taken from H. L. Jackson and A. I. Galonsky, Phys. Rev. **89**, 365 (1953).
[c] Refers to elastic scattering reduced width.
[d] The parameters for the $D_{3/2}$ state are obtained from the estimates described in the text and are not of comparable accuracy to those for the other states.
[e] Refers to inelastic scattering reduced width.

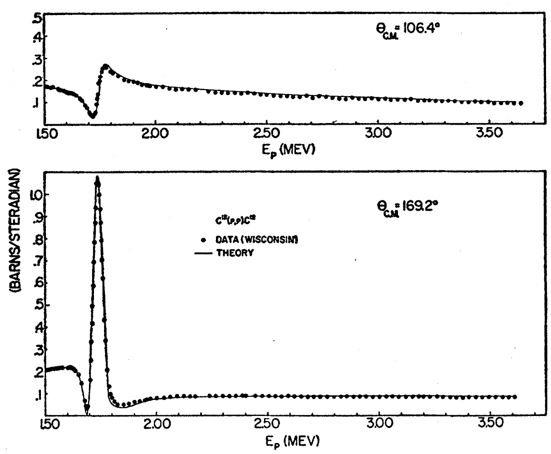


FIG. 6. The $C^{12}(p,p)C^{12}$ center-of-mass differential cross section *vs* bombarding energy. The points represent the Wisconsin data and the solid lines represent the theoretical fit obtained by using the phase shifts of the present analysis.

resonant portion of this expression is

$$\tan\beta_2{}^+=\frac{k/A_2{}^2}{-\frac{1}{a}\left[\frac{d(\ln A_2)}{d(\ln\rho)}+2\right]+\left[\frac{(\gamma_\lambda{}^2)_1}{E_{\lambda 1}-E}+\frac{(\gamma_\lambda{}^2)_2}{E_{\lambda 2}-E}\right]^{-1}},$$

where the subscripts 1 and 2 refer to the lower and upper levels, respectively. The parameters thus obtained for the 4.8-Mev resonance, along with those obtained by Jackson *et al.*[4] for the first three excited states, are given in Table II.

In calculating the $D_{3/2}$ phase shift from the parameters given, the hard-sphere contribution was assumed to vary linearly with energy from a value of $-3.6°$ at 4.613 Mev to $-4.4°$ at 4.964 Mev. The calculated hard-sphere values vary linearly with energy from a value of $-8.7°$ at 4.613 Mev to $-10.2°$ at 4.964 Mev.

COMPARISON WITH A PREVIOUS ANALYSIS

The Wisconsin group found it necessary to increase δ_0 by an amount which varied linearly with energy.[4] With such a correction, δ_0 would be expected to be about 93° at 4.613 Mev. Yet the extracted value, 84.5°, differs by less than 3° from the value calculated from (2), using the Wisconsin level parameters.[4] Furthermore, $\delta_1{}^+$ at 4.613 Mev is about 21° higher than the calculated value, and no anomalous behavior of $\delta_1{}^+$ was reported necessary to fit the data at 3.6 Mev, which was the highest energy covered by the Wisconsin analysis. No reasonable energy variation of $\delta_1{}^+$ could make the values extracted in the region of 4.8 Mev consistent with those quoted in the region of 3.6 Mev. Some analysis of the region between 3.6 Mev and 4.6 Mev was thus indicated.

Since the Wisconsin data consist of excitation curves at angles greater than 90°, excitation curves at center-of-mass angles of 54° 44′ and 90° were taken at this laboratory. These are shown in Fig. 4. Furthermore, through the cooperation of Professor H. T. Richards of the University of Wisconsin, a copy of the Wisconsin data and phase shifts was obtained. Upon applying the phase shifts to the 54° 44′ data, it was found that the agreement between the calculated cross section and the measured one was quite good in the region of the 1.7-Mev anomaly. At 2 Mev, however, there was a disagreement of about 15%, and at 3.6 Mev the calculated coherent scattering cross section alone was more than 50% higher than the measured cross section. During the course of these calculations, the source of the discrepancy became apparent. At the backward angles the $P_{3/2}$ vector was approximately perpendicular to the resultant f_c vector, while at the forward angles the two were approximately parallel. Thus, if the $P_{3/2}$ phase shifts were incorrectly chosen, the main effect at the backward angles would be to change the phase of f_c, while at the forward angles the main effect would be to change its magnitude. Thus, an error in the $P_{3/2}$ phase shift would be difficult to detect at backward angles, but quite noticeable at forward angles.

The data above 1.5 Mev were thus reanalyzed in the light of this information. The fits to the data at 54° 44′ and 90° are shown in Fig. 4. The fits to the Wisconsin data from 1.5 to 3.6 Mev are shown in Figs. 5 and 6.

BEHAVIOR OF THE PHASE SHIFTS FROM 1.5 TO 5 MEV

The variation with energy of the extracted phase shifts is shown in Fig. 7. The S-wave phase shift agrees with the values calculated using the Wisconsin level parameters up to about 4.6 Mev, where the difference in the two is approximately 3°. Similarly, the $P_{1/2}$ phase shift agrees very well with the hard-sphere scattering value up to 4.6 Mev. Above this energy, the deviations from the calculated values become larger, being $+5.6°$ for δ_0 and $+2.3°$ for $\delta_1{}^-$ at 4.964 Mev. The deviation

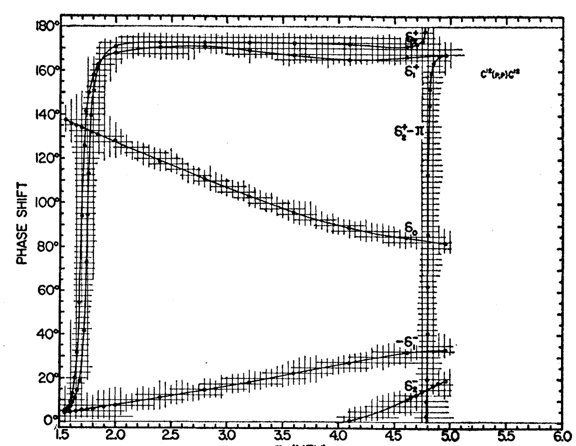


FIG. 7. The phase shifts for the $C^{12}(p,p)C^{12}$ reaction extracted explicitly from the cross-section data.

of δ_2^+ from the calculated values, using the two-level formula, varies linearly with energy from 0° at 2.4 Mev to +5.8° at 5 Mev. The variation of δ_2^- above 4 Mev indicates the presence of a $D_{3/2}$ resonance in the region above 5 Mev. The variation of δ_1^+ is quite different from the calculated values. Furthermore, since it increases with energy above 3.5 Mev, the indications are that this behavior is due to the presence of a $P_{3/2}$ level in the energy region above that of the present experiment. Since there is no difference between the extracted and calculated values of δ_1^+ below 1.8 Mev and only +2° at 2 Mev, it is believed that this difference is due to causes other than the Wisconsin choice of parameters for the $P_{3/2}$ level.

$C^{12}(p,p)C^{12}$ REACTION ABOVE 5 MEV

The predominant feature of the excitation functions from 5 to 5.6 Mev is the presence of a rather broad anomaly in the region of 5.4-Mev proton energy. The parameters deduced for this state are also shown in Table II. The analysis of the data in this region is more complicated than that in the lower energy region for a number of reasons. The major complication arises because of the occurrence of inelastic scattering to the first excited state of C^{12}. This becomes energetically possible at a proton energy of 4.80 Mev, but does not show up appreciably below 5 Mev. Because of this additional decay mechanism, the partial wave expansion as used in the preceding analysis is no longer valid, since it is derived on the assumption that elastic scattering is by far the most probable process. Furthermore, as will be shown later, an excitation function for the 4.43-Mev γ radiation resulting from the inelastic scattering process indicates a relatively large cross section for this process and also a rather complicated level structure above 5.2 Mev. This excitation function consists of two fairly broad overlapping resonances below 6 Mev, superimposed on a fairly rapidly increasing off-resonance yield.[5] An additional factor is an enhanced probability of scattering for partial waves having $l>2$. It is apparently not necessary to consider their effect in the energy region previously discussed, but in the higher energy regions their effects are perhaps enhanced.

In order to attack the problem, it is necessary to modify the partial wave expansion to take into account the presence of reactions other than elastic scattering. This may be done most readily in terms of the scattering matrix. The scattering matrix involves, among other quantities, the various partial widths of a level.[11] If one drops the requirement that the elastic scattering width be equal to the total width [in which case Eq. (1) follows] and allows it to be some fraction, a_p, of the total width, this has the effect of replacing the quantities $\exp(i\delta_l^+)\sin\delta_l^+$ in (1) with the more involved expressions $(1-a_{pl}^\pm)\exp(i\phi l)\sin\phi_l+a_{pl}^\pm\exp(i\delta_l^\pm)\sin\delta_l^\pm$, where the $\delta_l^\pm$ and ϕ_l are defined by (2). In this case a knowledge of the maximum and minimum cross sections of a resonance observed at an angle near 180° determines the quantity $a_{pl}^\pm(j+\frac{1}{2})P_l(\cos\theta)$ rather than $(j+\frac{1}{2})P_l(\cos\theta)$. Since the $a_{pl}^\pm$ enter into the expression as unknowns, this quantity is not of much use in selecting a possible spin and parity assignment for the level. However, some significant information concerning the level may be obtained even without an explicit phase-shift analysis of the data. From the known phase shifts at 4.964 Mev, it is possible to show that only one resonating phase shift will yield the observed shape of the 5.4-Mev anomaly at all angles. This phase shift is δ_2^-, the $D_{3/2}$ phase shift, as might be expected in view of its energy variation below 5 Mev. Due to the rather large width of the anomaly at 5.4 Mev, some effect on the phase shifts in the energy region, slightly below 5 Mev, is to be expected. The only two phase shifts which show marked effects below 5 Mev are δ_1^+ and δ_2^-. The possibility of the 5.37-Mev resonance being a $P_{3/2}$ state can be eliminated since a $P_{3/2}$ resonance should show a rise before a dip in the 90° cross section, but this is not observed.

Some information concerning the reduced widths for elastic and inelastic scattering of the level may be obtained. Since the state is a D state, the resonance effects at 54° 44′ and 125° 16′ are contained only in the expression f_i. If it is assumed that f_c, at these angles, remain essentially constant at the values they have at 4.964 Mev, it is possible to calculate the diameter the $D_{3/2}$ circle would have at these angles if a_{p2}^- were constant across the resonance. This may be done, as described previously, by solving the expression

$$[k^2\sigma(\theta)_{\max}-|f_c|^2]^{\frac{1}{2}}-[k^2\sigma(\theta)_{\min}-|f_c|^2]^{\frac{1}{2}}=3a_{p2}^-\sin\theta\cos\theta.$$

From this, the value of a_{p2}^- is calculated to be 0.48 at 54° 44′ and 0.39 at 125° 16′. It should be emphasized that these estimates are rather rough. The assumption that f_c changes but little from their values at 4.964 Mev is questionable on the basis of the energy variation of the various phase shifts. Furthermore, the inelastic scattering width is a rapidly increasing function of the energy due to the increasing penetrability of the inelastically scattered proton, and thus a_{p2}^- is not strictly constant across the resonance. Some information can be obtained from this value of a_{p2}^-, inaccurate though it may be. For an a_{p2}^- of 0.45, the ratio of the elastic scattering width to the inelastic scattering width is 0.82. From the relation between the observed width and the reduced width given previously, it follows that $(\gamma_\lambda^2)_{\text{elastic}}/(\gamma_\lambda^2)_{\text{inelastic}}=0.061$. Thus, the reduced width for inelastic scattering for this state is about 16 times as large as the reduced width for elastic scattering. From a study of γ rays produced by inelastic scattering

[11] J. M. Blatt and L. C. Biedenharn, Revs. Modern Phys. 24, 258 (1952).

the width of the resonance was deduced to be approximately 125 kev. Then, at 5.37 Mev (the peak of the γ-ray excitation curve) the two partial widths satisfy the relation: $\Gamma_{\text{elastic}}+\Gamma_{\text{inelastic}}=125$ kev. It then follows that $(\gamma_\lambda^2)_{\text{elastic}}\cong 0.18\times 10^{-13}$ Mev cm, and $(\gamma_\lambda^2)_{\text{inelastic}}\cong 2.9\times 10^{-13}$ Mev cm. These values are considered to be rough estimates only, but even so, they allow one to draw some reasonable inferences concerning the nature of this state.

GAMMA RADIATION FROM THE PROTON BOMBARDMENT OF CARBON

A. $C^{12}(p,\gamma)$ Reaction

The $C^{12}(p,\gamma)$ reaction has been studied in the region of the first three excited states of N^{13} by several investigators.[12–14] These observations were performed either by observing the positrons from the decay of the N^{13} formed in the capture process or by observing the γ radiation resulting from the positron annihilation. This method is not practical in the region of the 4.8-Mev anomaly, however. The threshold of the $C^{13}(p,n)$ reaction is about 3.2 Mev. Natural carbon contains about 1% C^{13}; and if the $C^{13}(p,n)$ reaction has a cross section of 100 mb in the region of 4.8 Mev, the resultant N^{13} positron activity could easily obscure that due to the $C^{12}(p,\gamma)$ process. Thus, it was decided to search for the capture γ radiation directly. The target used was a 0.36-mg/cm² carbon foil, and the detector was a NaI(Tl) crystal 1 inch in diameter by 1 inch thick mounted on a DuMont 6292 photomultiplier tube, placed at 90° with respect to the beam direction. The beam was allowed to pass through the target and stop in a 10-mil thickness of tantalum 24 inches from the target position. The pulses from the photomultiplier tube were amplified and fed into an Atomic Instrument Company 20-channel pulse analyzer, model 520. No γ radiation above background was observed in any of the channels corresponding to possible γ-ray energies. From these results, it was concluded that the differential cross section at 90° for the $C^{12}(p,\gamma)$ reaction at 4.8 Mev is less than 12 μb/steradian if the transition to the ground state involves any of the excited states of N^{13} and is less than 1 μb/steradian for the direct ground-state transition.

B. $C^{12}(p,p'\gamma)$ Reaction

The $C^{12}(p,p'\gamma)$ reaction has been studied above 5 Mev.[5,15] Martin *et al.*[5] reported the existence of two strong 4.43-Mev γ-ray resonances at proton energies of 5.37- and 5.9-Mev bombarding energy. In conjunction with the $C^{12}(p,\gamma)$ work previously described, an excitation function for the $C^{12}(p,p'\gamma)$ reaction was taken at 90° up to a bombarding energy of 5.7 Mev. In addition, angular distributions of the 4.43-Mev γ radiation were taken at proton energies of 5.188, 5.297, and 5.425 Mev. This excitation function and the angular distributions are shown in Fig. 8.

The target chamber used in taking the angular distributions was a thin-walled brass cylinder 8 inches tall and 2 inches in diameter, the inside of which was lined with a 5-mil thickness of tantalum. The target was a 0.15-mg/cm² carbon foil mounted on a 10-mil gold backing, which effectively stopped the beam at the target. The detector used in this experiment was the same detector used in the (p,γ) work. It was mounted on a turntable, with the face of the crystal 13 inches from the target.

An absolute cross section was obtained in the following manner. After the completion of the angular distributions, the target assembly was removed from the chamber and a calibrated Po-Be source was inserted in its place. This enabled the product of the detector efficiency and solid angle to be determined. From the known target thickness and current integrator calibration, it was possible to calculate the absolute cross section. The excitation function at these three energies was then normalized to these three values and the cross-section scale thus established. The rms uncertainty in the cross section is 25%, excluding statistical uncertainties in the yield data. These statistical uncertainties in the angular distribution data are of the order of 3% at 5.188 Mev and less than 2% at 5.297 and 5.425 Mev. The peak cross section for the $C^{12}(p,p'\gamma)C^{12}$ reaction at 5.37-Mev bombarding energy was established to be (15.8±4.0) mb/steradian at 90°. This is to be compared with that of Maeder *et al.*,[15] who obtain a peak cross section of (6.8±3.0) mb/steradian at 105°.

If the angular distribution function, $W(\theta)$, is expressed as a series of Legendre polynomials $(A_0P_0+A_2P_2+A_4P_4)$, and the ratio A_4/A_0 is plotted against the ratio A_2/A_0, the graph in Fig. 9 results. On such a plot an angular distribution of the form considered here is represented by a single point. Some theoretical angular distributions for various pure cases are also shown on the graph. It should be noted that the three experimental angular distributions fall on a straight line. Furthermore, this straight line passes through the point $(2, \frac{3}{2}, 0, \frac{3}{2})$, which corresponds to the angular distribution to be expected from a pure $\frac{3}{2}^+$ state in N^{13}, for an inelastically scattered proton emerging as an S wave. The energy at which the line connecting the three data points passes through this point is near the resonance energy of the 5.37-Mev state. This fact tends to confirm the assignment of $D_{3/2}$, made previously from a consideration of the elastic scattering data. The three angular distributions may be fitted by assuming interference between a $D_{3/2}$ and a $D_{5/2}$ state in N^{13}. The theoretical angular distribution is made up of an arbitrary mixture of a $D_{3/2}$ state, a $D_{5/2}$ state, and an interference term between the two. If the experi-

[12] Fowler, Lauritsen, and Lauritsen, Revs. Modern Phys. **20**, 236 (1948).
[13] D. M. Van Patter, Phys. Rev. **76**, 1264 (1949).
[14] J. D. Seagrave, Phys. Rev. **84**, 1219 (1951).
[15] Maeder, Martin, Müller, and Schneider, Helv. Phys. Acta **27**, 166 (1954).

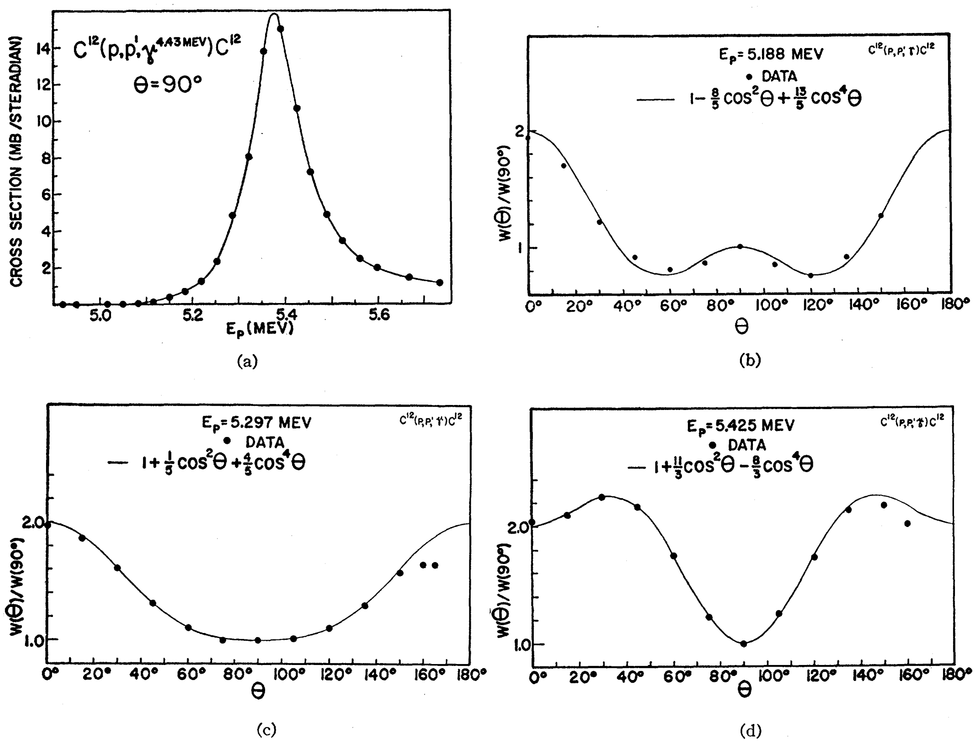


FIG. 8. An excitation curve at 90° for the 4.43-Mev γ radiation and three angular distributions normalized to unity at 90°. The variation with energy of the coefficients in the angular distributions indicates interference between levels in N^{13}. The form of the three distributions is consistent with the assumption that the interference is between a $D_{3/2}$ and a $D_{5/2}$ level in N^{13}. The vanishing γ-ray yield below 5 Mev justifies the form of the partial-wave expansion used in the analysis of the elastic scattering data below this energy.

mental angular distributions are written in the form: $W(\theta)/W(90°)=1+N_2\cos^2\theta+N_4\cos^4\theta$, the corresponding theoretical expression computed, and coefficients of the corresponding powers of $\cos\theta$ equated, it is found that the three relations thus obtained for the various mixtures of interfering states have no common solution unless $1-N_2=N_4$. This condition holds for each of the angular distributions investigated. Since this relationship holds, however, the three relations are not independent, and it is not possible to obtain unique values for the mixtures of the states.

Since the straight line through the data points tends, with increasing energy, toward the point representing the angular distribution from a pure $D_{5/2}$ state, one might be led to conclude that the $D_{5/2}$ state with which the $D_{3/2}$ state is interfering lies above the energy region investigated. Further experimental evidence, however, would be necessary before this conclusion can be definitely established; in fact, the 4.806-Mev resonance may provide the interference.

DISCUSSION

An energy level diagram summarizing the known states in C^{13} and N^{13} up to 8 Mev is given in Fig. 10. The spins and parities of the first three excited states in C^{13} are those obtained by Shire *et al.*[16] and Stanley[17] from an analysis of the angular distributions and (p,γ) angular correlations of the γ radiation from the $B^{10}(\alpha,p,\gamma)C^{13}$ reaction. The possible spin and parity assignments of the level at 6.87 Mev in C^{13} result from an analysis[18] of the angular distribution of the neutrons elastically scattered by C^{12} in the region of this resonance. In the range of neutron energies from 2.6 to 4.15 Mev, a phase-shift analysis[19] of the angular distributions of the neutrons elastically scattered by C^{12} indicates a $D_{3/2}$ resonance at a neutron energy of 2.95 Mev. The spins and parities of the first three levels in N^{13} are those obtained by Jackson and Galonsky[4] from a phase-shift analysis of the elastic scattering of protons by C^{12}. The corresponding quantities for the fourth and fifth levels are those of the present analysis. Evidence for the state at 7.4 Mev in N^{13} comes from the resonance at 5.9-Mev proton energy in the excitation function of the 4.43-Mev γ radiation which results from the inelastic scattering of the protons to the first excited state of C^{12}.[5]

The similarity of the spins and parities of the corresponding levels is quite striking. Furthermore, recent experimental evidence[20] from the $C^{12}(d,p)$ and $C^{12}(d,n)$ reactions is consistent with the assumption that the reduced widths of the first three corresponding levels in C^{13} and N^{13} are nearly the same. Some comparison of the reduced widths of the fourth excited states can be made, since the fourth excited state of C^{13} is presumably a D state. From the observation that the laboratory

[16] Shire, Wormald, Lindsay-Jones, Lunden, and Stanley, Phil. Mag. 44, 1197 (1953).
[17] A. G. Stanley, Phil. Mag. 45, 430 (1954).
[18] R. Ricamo, Nuovo cimento 10, 1607 (1953).
[19] P. Huber and R. Budde, Helv. Phys. Acta 27, 512 (1954).
[20] Beneson, Jones, and McEllistrem, Phys. Rev. 101, 308 (1956).

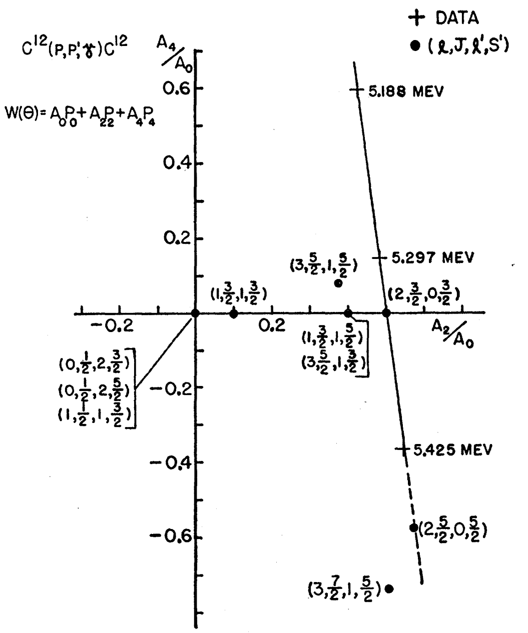


FIG. 9. A_4/A_0 plotted *vs* A_2/A_0, where $\sigma(\theta)=A_0+A_2P_2(\cos\theta)+A_4P_4(\cos\theta)$ is the 4.43-Mev γ-ray cross section. The notation used in describing the various points is this: (l,J,l',s') where l is the relative orbital angular momentum of the input channel, J is the spin of the compound state, l' is the orbital angular momentum of the outgoing proton, and s' is the channel spin of the outgoing proton and excited C^{12} nucleus.

width of this state is ≤ 10 kev,[21] the reduced width of this level is $\leq 0.095\times 50^{-13}$ Mev cm, which is only a factor of two larger than the corresponding quantity for the 6.38-Mev level in N^{13}. Since the neutron beam used in that experiment had an energy spread of about 10 kev, it is possible that the width of the level is less than 10 kev, in which case the agreement between the two values is perhaps even better. This similarity in the properties of corresponding levels in the mirror nuclei C^{13} and N^{13} lends support to the hypothesis of the charge symmetry of the nuclear forces.

Other significant information may be obtained from the reduced width of a level. The ratio of a reduced width of a level to the Wigner limit is regarded as a measure of the degree to which that level may be considered as a single particle state. This ratio for the first five excited states in N^{13} is included in Table II. This ratio would tend to indicate that the fourth excited state of N^{13} cannot be regarded as a single-particle state at all. In conjunction with the work on the 5.37-Mev anomaly, however, one may make some interesting conjectures concerning the nature of the fourth and fifth excited states in N^{13}. From the estimates of the reduced widths for the two processes occurring at the 5.37-Mev anomaly, the ratio of the

[21] Bockelman, Miller, Adair, and Barschall, Phys. Rev. 84, 69 (1951).

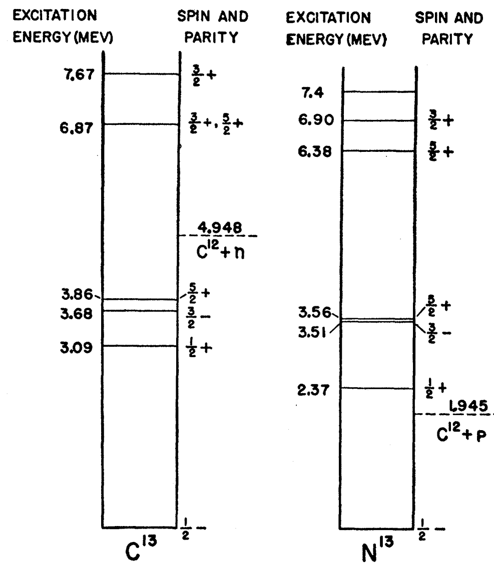


FIG. 10. The known energy levels in the mirror nuclei C^{13} and N^{13} up to 8-Mev excitation.

reduced width for inelastic scattering to the Wigner limit is about 0.2.

It thus appears that this state prefers to exist, not as a state consisting of a D-wave proton outside a core of C^{12} in its ground state, but rather as a state consisting of an S-wave proton about a central core of C^{12} excited to its 2^+, first excited state. It then may be conjectured that a similar process could account for the observed narrowness of the 4.8-Mev state. This state could also be thought of as consisting primarily of an S-wave proton about a central core of C^{12} in its 2^+, first excited state. Because of the vanishingly small barrier penetration of the S-wave proton, no inelastic scattering can occur from this state. Above about 5 Mev, however, such a reduced width might be expected to influence the inelastic scattering markedly. On this basis, it is possible that the $D_{5/2}$ state at 4.808-Mev bombarding energy provides the interference noted in the 4.43-Mev γ-ray measurements. Furthermore, if this model is valid, the fourth and fifth excited states of N^{13} might be considered as members of a single level which has been split by the action of the spin-orbit (or possibly spin-spin) forces so that their reduced widths for both processes would be expected to be about the same. Table II shows that the elastic widths are approximately the same. Finally, the energy difference of these two levels provides a measure of the strength of the splitting mechanism for the configuration. Since the splitting of this D-state doublet is probably only about one-tenth of that of the other D-state doublet (of which the third excited state is a member), the ratio of inelastic to elastic reduced widths of about sixteen seems very reasonable for the fifth excited state.