Scattering of Alpha Particles from Helium*

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Excitation curves and angular distributions for the scattering of α particles from helium have been obtained for bombarding energies of 3 to 6 Mev, using a differentially pumped gas scattering chamber and a singly charged α-particle beam from the Rice Institute Van de Graaff accelerator. A phase shift analysis of the data, yielding phase shifts accurate to about 2°, reveals that only S- and D-wave scattering occurs and shows that the well-known excited state at 2.9 Mev in Be^8 is a D state. The scattering data taken at the Department of Terrestrial Magnetism and the Universities of Illinois and Indiana are compared to the Rice Institute data by using the dispersion theory and the Landau K-function formalism developed for proton-proton scattering. The dispersion theory, applied to the Rice data, allows a rough estimate of the D-state level parameters, while the K function, applied to the Rice and D.T.M. data, is used to deduce a ground state width for Be^8 of 4.5±3 ev (c.m.). For excitation energies below 12 Mev, the scattering data are consistent with a description of the Be^8 nucleus as an interaction of two α-particles. Certain qualitative features of the potential of interaction of such a two-body configuration are discussed.

INTRODUCTION

IN recent years interest in the α-particle model of nuclei has revived because of the agreement between the experimentally determined level structure of C^{12} and O^{16} and the predictions of the model.[1,2] Be^8 is crucial to this model because it is the simplest of the α-particle nuclei, and because it may be studied directly by the scattering of α particles from helium. In principle, if the model applies, α-α scattering determines a potential of interaction between two α particles, and from this potential some of the properties of the other α-particle nuclei can be computed. One limitation of the study of Be^8 by α-α scattering is that only states of even spin and parity can be investigated. However, recent experiments of Moak and Wiseman[3] indicate that only three states exist in Be^8 below ~14-Mev excitation; these are the same states that are evidenced by α-α scattering.

The results of the early α-α experiments, which used natural α emitters as the source of energetic projectiles, were summarized by Wheeler[4] in 1941. In 1952, Cowie, Heydenburg, Temmer, and Little,[5] using the singly ionized α-particle beam of the 1-Mev and the 3-Mev Van de Graaff accelerators at the Carnegie Institution of Washington, Department of Terrestrial Magnetism, measured α-α angular distributions at energies between 0.15 and 3 Mev. Cyclotron data are available for the region of 12 to 23 Mev. The two most complete cyclotron experiments, done by Steigert and Sampson at the University of Indiana[6] and Nilson and Jentschke at the University of Illinois,[7] are in serious disagreement between the energies and 12 and 18 Mev. The Indiana data indicate an S state in Be^8 at about 7.5-Mev excitation; the Illinois data do not indicate such a state. Observations of the $Li^6(He^3,p)Be^8$ reaction, reported by Moak and Wiseman, do not show a state in Be^8 at 7.5 Mev; nor do neutron spectrometer studies of the $Li^7(d,n)Be^8$ reaction reported by Trail and Johnson.[3] The work reported in this paper covers α-particle bombarding energies of 2.96 to 5.9 Mev.

DESCRIPTION OF THE EQUIPMENT

In 1954, the provisions made for accelerating singly ionized α particles with the Rice Institute Van de Graaff accelerator allowed the extension of reliable α-α scattering data to 6 Mev. For this, and other experiments requiring gas targets, a large volume scattering chamber and a differentially pumped beam entrance tube were constructed.[8] The use of similar experimental equipment has been described elsewhere.[9]

As shown in Fig. 1, the scattering chamber was 30 inches in diameter and 13 inches deep. The large size permitted the installation of two scintillation detectors and provided a great amount of freedom in equipment design. The beam was brought into the chamber through a differentially pumped tube, re-entrant in the chamber wall, while collimating slits in the tube defined the beam to ±0.1°. The pressure of the scattering gas was measured with an oil manometer, backed by a diffusion pump, and read with a cathetometer calibrated to 0.05 mm. The specific gravity of the manometer oil (butyl

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† Now with General Electric Company, San Jose, California.
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[1] A. E. Glassgold and A. Galonsky, Bull. Am. Phys. Soc. Ser. II, 1, 181 (1956).
[2] D. M. Dennison, Phys. Rev. 96, 378 (1954).
[3] C. D. Moak and W. R. Wiseman, Phys. Rev. 101, 1326 (1956); C. C. Trail and C. H. Johnson, Phys. Rev. 98, 249 (1955).
[4] J. A. Wheeler, Phys. Rev. 59, 16 (1941).
[5] Cowie, Heydenburg, Temmer, and Little, Phys. Rev. 86, 593(A) (1952); G. M. Temmer and N. P. Heydenburg, Phys. Rev. 90, 340(A) (1953); N. P. Heydenburg and G. M. Temmer (private communication); Phys. Rev. 104, 123 (1956), preceding paper.
[6] F. E. Steigert and M. B. Sampson, Phys. Rev. 92, 660 (1953).
[7] R. Nilson and W. Jentschke (private communication).
[8] J. L. Russell and C. W. Reich, thesis, The Rice Institute, 1954 (unpublished).
[9] Tuve, Heydenburg, and Hafstad, Phys. Rev. 50, 806 (1936); Herb, Kerst, Parkinson, and Plain, Phys. Rev. 53, 998 (1939); Huntoon, Ellett, Bayley, and Van Allen, Phys. Rev. 58, 97 (1940); Worthington, McGruer, and Findley, Phys. Rev. 90, 899 (1953).

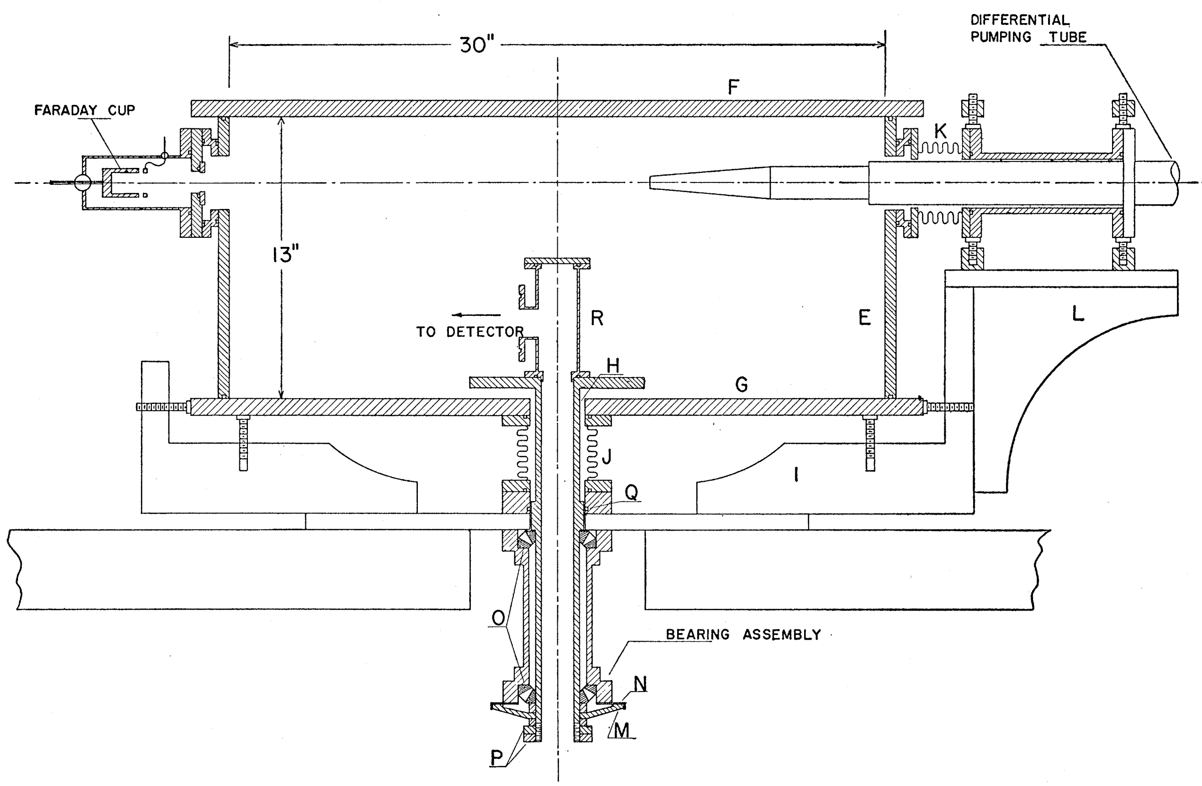


FIG. 1. The scattering chamber used for α-α scattering. The aluminum cylindrical chamber walls E and top and bottom plates F and G, are supported on adjusting screws by the rigid structure I, L, and are attached to the differential pumping tube and counter bearing assemblies by the sylphon bellows K and J. This technique prevents the deflection of the beam tube or counter axes by the large vacuum forces on the chamber. The rigid structure, I, L, resting on a table, is firmly attached to the differential pumping tube and the counter bearing housing and assures that right angle coincidence of these axes is maintained. One detector shaft, H, is shown supported in the taper bearings O that are loaded by nuts P. The counter angle may be read by means of the azimuth circle M and vernier N, while the necessary vacuum seal is made by an O-ring at Q. Not shown is the other detector bearing, which is inside those shown and coaxial with them, nor is there indicated the details of either scintillation counter, one of which is attached at R. The Faraday cup shown was replaced in some experiments with a re-entrant one.

sebacate) was measured to be 0.932. After initially evacuating the chamber, the helium pressure was satisfactorily regulated with a mechanical leak in series with a standard reducing valve and a high pressure cylinder of helium gas. The pressures employed in the experiments were about 0.4 cm Hg and the maximum beam energy loss to the target volume was 35±6 kev. The grade A helium used in these experiments had a purity somewhat better than 99.9%. The temperature was measured with a mercury thermometer in thermal contact with the lid of the chamber.

The target volume was defined by slit systems of ±1/2° resolution mounted on each detector. The two scintillation detectors were mounted on concentric, rotatable shafts. Angles of observation were determined by the alignment procedure to better than ±0.1°. The detector used for this experiment, a 3-mg/cm² CsI(Tl) scintillator,[10] made possible the detection of low-energy α particles in the presence of an intense γ ray and neutron flux. A five-point integral bias curve was taken for every data point with a multichannel analyzer. The total number of detected α particles was chosen to be the point of inflection of the integral bias curve, marking the onset of low-level detector noise. Generally, this number was within 2% of the value obtained by straight line extrapolation of the flat portion of the bias curve to zero pulse height.

The beam was collected in a Faraday cup, which in the early experiments was as shown in Fig. 1. In later experiments the cup was made re-entrant and placed only 5 inches from the center of the chamber. The Faraday cup vacuum was maintained by means of a diffusion pump and a liquid air trap, while the Faraday cup evacuated region was isolated from the target gas by means of an aluminum foil of about 1.2 mg/cm². Electrons, produced by the beam in passing through the foil, were prevented from reaching the Faraday cup by means of a permanent magnet and an electrostatic suppressing ring, held 200 volts below ground potential. The earlier α-α experiments employed only

[10] Schiffer, Kraus, and Phillips (unpublished). This technique need not be described here.

TABLE I. Estimated cross-section uncertainties for the α-α scattering, in percent.

Detection efficiency	2.0
Geometry	0.4
Target	
Purity	1.0
Pressure	0.3
Temperature	0.3
Current integration	
Faraday cup geometry	0.3
Switching uncertainty	0.5
Condenser calibration	1.0
Leakage	0.1
Charge state uncertainty	0.2
Energy (assuming the Mott relationship $2dE/E=d\sigma/\sigma$)	1.0
Total rms uncertainties	$\pm 3.0\%$

electrostatic suppression of secondary electrons in the Faraday cup region and were found to yield reproducible, but incorrect, absolute cross sections that necessitated normalization by α-argon scattering. This difficulty was eliminated by the addition of the permanent magnet. Current integration was accomplished by discharging a calibrated condenser to zero voltage, as measured by a quartz fiber electrometer. The charge state of the integrated α particles was computed from the data summarized in the review article by Allison and Warshaw[11] by using the known energy loss of the α particles in passing through the Faraday cup foil.

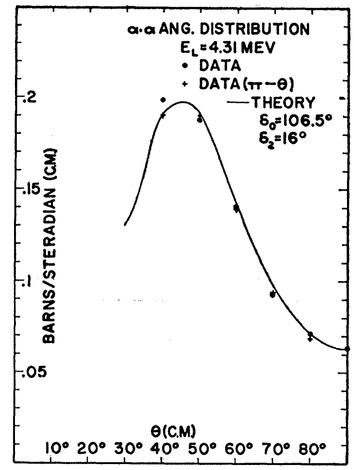
The energy of the incident α-particle beam was known to ±20 kev below 5.5 Mev and to ±50 kev above 5.5 Mev. Counting statistics varied widely because a fixed amount of charge (∼100 microcoulombs) was accumulated for each datum point. However, statistics were generally better than 1.5%.

Neglecting counting statistics, the root-mean-square error in the cross section, using the equipment as described above, was about ±3%. The estimated uncertainties are shown in Table I.

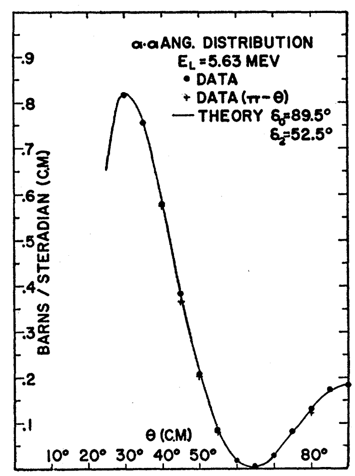
In December, 1955, the scattering of α particles from argon was measured with the equipment described above at several angles and energies between 2.5 and 4.5 Mev and, when corrected for the charge state of the α particle, was found to follow the Rutherford relationship, with maximum random deviations of 3%. These α-argon scattering experiments, then, may be considered to have demonstrated the proper functioning of the apparatus; or they may be considered to have confirmed the charge state of the α particle reported by Allison and Warshaw in the energy region 2.1 to 5 Mev to ±3%.

THE EXPERIMENT

The α-α experiments were done at the Rice Institute three times. First, four α-α excitation functions were taken from 3.8 to 5.0 Mev at laboratory angles of 15° 17.5′, 20°, 27° 22′, and 35° 3.5′. Angular distributions were taken at energies of 4.31 and 5.63 Mev. An α-argon scattering experiment over the same energy region indicated an error in current integration; although a $\csc^4(\theta/2)$ angular dependence was observed, the absolute value of the cross section was not the Rutherford value. These α-α data were therefore normalized with respect to the α-argon data to obtain absolute cross sections. Since the quantity of interest, theoretically, is the center-of-mass differential cross section, the data are tabulated in that form. The normalized angular distributions, of the first experiments, are shown in Fig. 2.



(a)



(b)

FIG. 2. α-α angular distributions at two bombarding energies. The absolute differential cross section in the c.m. system is plotted *versus* the c.m. scattering angle. The smooth curves are for the *S*- and *D*-wave phase shifts indicated.

Later, α-α excitation curves were taken at laboratory angles of 20° and 27° 22′ from 3 to 5.5 Mev, and angular distributions were taken at 2.96, 3.96, and 4.98 Mev. An α-argon experiment over the same energy region indicated that the error in charge integration remained, so that these data were also normalized to obtain absolute cross sections. In Fig. 3 are shown the normalized angular distributions taken in the second experiment.

[11] S. K. Allison and S. D. Warshaw, Revs. Modern Phys. 25, 779 (1953).

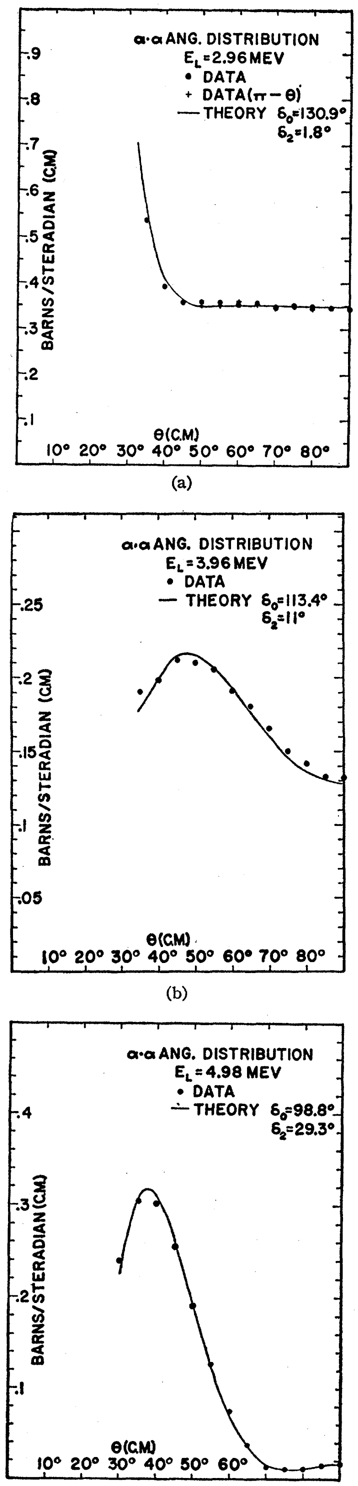


FIG. 3. α-α angular distributions at three bombarding energies. The absolute differential cross section in the c.m. system is plotted *versus* the c.m. scattering angle. The smooth curves are for the *S*- and *D*-wave phase shifts indicated.

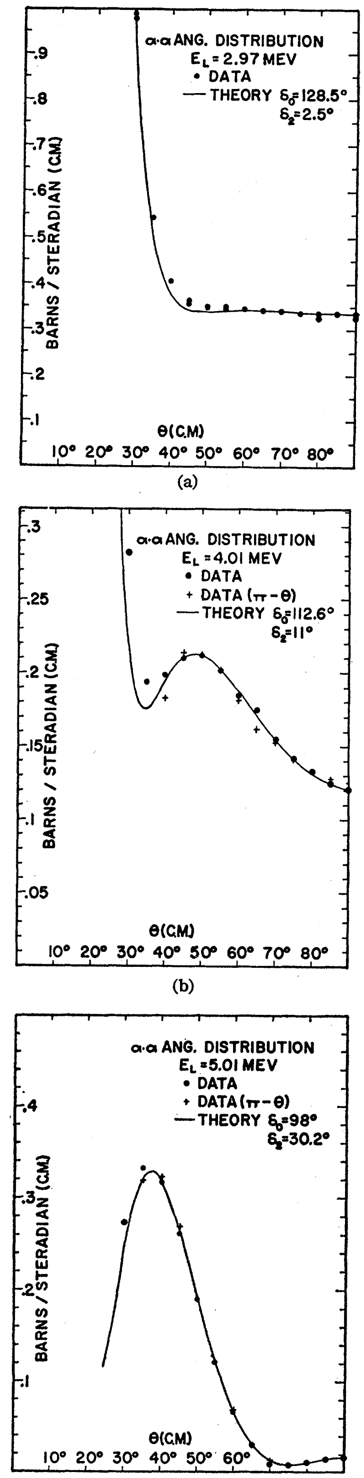


FIG. 4. α-α angular distributions at three bombarding energies. The absolute differential cross section in the c.m. system is plotted *versus* the c.m. scattering angle. The smooth curves are for the *S*- and *D*-wave phase shifts indicated.

With the equipment in its final form, as described in the previous section, the α-α experiment was repeated a third time. An α-argon experiment confirmed the Rutherford relationship to ±3%. Excitation functions for α-α scattering were measured at angles of 20° and 27° 22′ from 2.97 and 5.5 Mev. Figure 4 shows the final α-α angular distributions. All three sets of data, where comparable, were found to agree within their estimated accuracies of $\pm 3\%$. The earlier measurements that were normalized by α-argon scattering were apparently as accurate as the ones where no normalization was required.

In Fig. 5 are composite excitation functions where all the α-α excitation data have been plotted. The $27° 22'$ and $20°$ curves include overlapping data from all three experiments. $54° 44'$ center-of-mass angle ($27° 22'$ laboratory angle) corresponds to a zero of the second-order Legendre polynomial, $P_2(\cos\theta)$. It is seen, in Fig. 5, that the $27° 22'$ curve is a monotonic function of bombarding energy, while the other excitation functions show an anomalous variation near 6 Mev. These facts require a 2^+ assignment for the 2.9-Mev state in Be^8.

PHASE-SHIFT ANALYSIS

The formula for the center-of-mass differential cross section, as a function of the nuclear phase shifts, δ_L, is[12]

$$\sigma(\theta) = \Big| -(\eta/2k)\csc^2[\theta/2]\exp[i\eta\log_e\csc^2(\theta/2)] - (\eta/2k)\csc^2[(\pi-\theta)/2]\exp\{i\eta\log_e\csc^2[(\pi-\theta)/2]\} + (2/k)\sum_{L=0,2,4,\cdots}^{\infty}(2L+1)\sin\delta_L P_L(\cos\theta) \times\exp[i\alpha_L+\delta_L]\Big|^2,$$

where $\eta = 4e^2/\hbar v$, $\alpha_L = 2\sum_{s=1}^{L}\tan^{-1}(\eta/s)$, $\alpha_0=0$, and δ_L is the nuclear phase shift for the partial wave of order L; k is the wave number, v is the velocity of the incident α-particles, and θ is the center-of-mass scattering angle.

The analytical technique for deducing the phase shifts, δ_L, for each angular distribution of the experiments reported here, was as follows. It was assumed that only two phases, δ_0 and δ_2, were required to fit the data in the energy region of 3 to 6 Mev. Equation (2) was solved explicitly for δ_0 in terms of the experimental cross section at $54° 44'$(c.m.). Using the value of δ_0 obtained in this way, the experimental cross section at $40°$(c.m.) was solved for δ_2. These trial phase shifts, (δ_0,δ_2), as well as the trial values $(\delta_0+1°, \delta_2)$ and $(\delta_0, \delta_2+1°)$, were converted to a family of angular distributions which, when compared with the data, made possible an accurate estimate of better phase shifts. These new estimated phase shifts were then converted to an angular distribution, and were generally found to be an acceptable fit to the data. The ability to fit the data with only two phase shifts, δ_0 and δ_2, justified the original assumption. Since not all of the data were of the same dependability, it was decided not to make a least squares fit. The various final fits of the angular distributions are shown with the data in Figs. 2, 3, and 4.

The phase shifts, obtained as described above, were plotted *versus* energy, and are shown in Fig. 6. The branch of δ_0 near π, at low energies, was chosen to conform to the established fact that the ground state of Be^8 is an S configuration.[13] These phase shift curves were converted to excitation curves at laboratory angles

[12] N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Oxford University Press, New York, 1953).

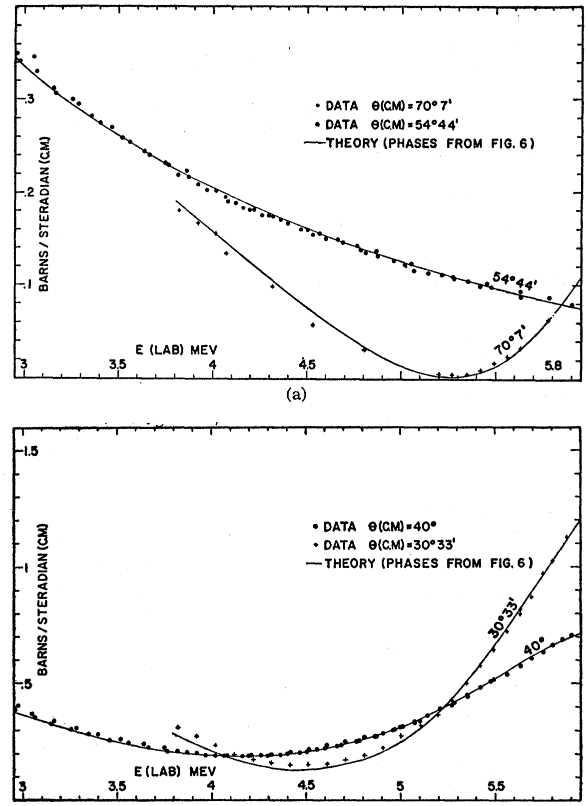


FIG. 5. Excitation curves for α-α scattering in the energy region 3 to 6 Mev, at four scattering angles. The differential cross section in the c.m. system is plotted *versus* the laboratory energy. The smooth curves are the cross sections calculated from the phase shifts shown in Fig. 6.

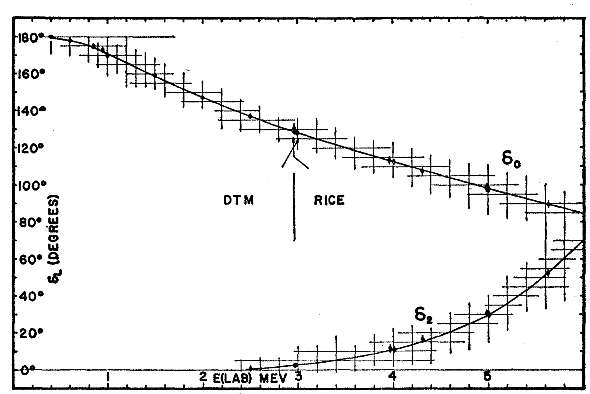


FIG. 6. S- and D-wave phase shifts for α-α scattering in the energy region 0.3 to 6 Mev. Included are the phase shifts of reference 5. Arbitrary smooth curves have been drawn through the data points.

[13] P. B. Treacy, Proc. Phys. Soc. (London) **A68**, 204 (1955).

of 15° 17.5′, 20°, 27° 22′, and 35° 3.5′, and are shown with the data at those angles in Fig. 5.

While fitting the angular distributions, it was found that the two data points at angles smaller than 20° in the laboratory were inconsistent with the rest of the angular distribution; i.e., a theoretical curve passing through about twenty data points at angles larger than 20° would not pass through the two most forward data points. The deviation for these two angles was in the direction to be explained by the scattering into the detector of particles whose origin was other than the target volume. The discrepancy was about 10% in the worst cases. For these reasons, the phase shifts were chosen to fit the data at angles greater than 20°. Above 5 Mev the effect disappeared, and a satisfactory fit to the data at all angles was obtained. The estimated error for S- and D-wave phase shifts is $\pm 1.5°$ from 3 to 5.5 Mev and $\pm 2°$ from 5.5 to 5.9 Mev.

The three sets of data on α-α scattering, obtained at the Department of Terrestrial Magnetism, the Rice Institute, and the University of Illinois, are apparently consistent in that phase shifts appear to be reasonably continuous. Figure 7 shows all three sets of data. However, the uninvestigated region, between 6- and 12-Mev bombarding energy, allows the possibility that the higher energy S- and D-wave phase shifts are misplaced by an integer times π.

A comparison of the low-energy phase shifts to the Indiana data does not reveal as much continuity; indeed, it would appear necessary to postulate a D state in the region 3 to 6 Mev in Be^8 in order to join the highest Rice value to the lowest Indiana value.

DISCUSSION OF THE LOW-ENERGY PHASE SHIFTS

The first excited state of Be^8, as indicated by the phase shift analysis of the scattering data shown in Fig. 7, is a D state. Approximate level parameters for this state may be obtained by fitting the experimental phase shifts in the region of the resonance with the one

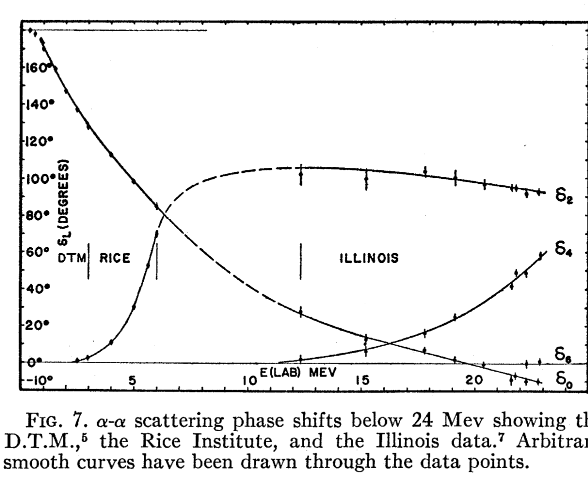


FIG. 7. α-α scattering phase shifts below 24 Mev showing the D.T.M.,[5] the Rice Institute, and the Illinois data.[7] Arbitrary smooth curves have been drawn through the data points.

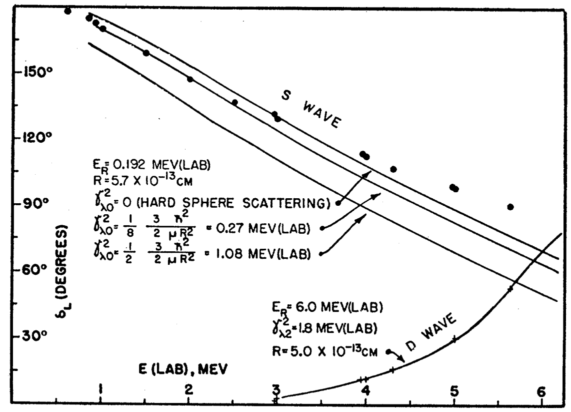


FIG. 8. An analysis of the 2.9-Mev D state in Be^8 using the dispersion theory. The experimental D-wave phase-shift data points are compared to a reasonable fit using the dispersion theory and the parameters indicated. Also shown is a similar comparison of the S-wave data to the prediction of the dispersion theory.

level dispersion formula of Wigner and Eisenbud.[14] To obtain a fit, the value of the resonance energy E_R was assumed to be 6.0 Mev, and two data points then determined the hard sphere radius and the width γ_λ^2. The comparison of the theory with the data is shown in Fig. 8. The level parameters obtained in this manner are shown in Table II along with those deduced by Nilson *et al.* for the G state. These widths are near the Wigner limit,[15] and are therefore consistent with a two-body description of Be^8.

The width of the ground state of Be^8 appears to be too narrow to be directly measured by an α-α scattering experiment.[16] However, the Landau K-function formalism, applied by Jackson and Blatt to proton-proton scattering,[17] may be applied to low-energy α-α scattering as a technique for extrapolating the S-wave phase shift to zero energy. This formalism considers a function K defined by the relation

$$K \equiv \pi \cot\delta_0/(e^{2\pi\eta}-1)+h(\eta),$$

where

$$h(\eta)=\mathrm{Re}[\Gamma'(-i\eta)/\Gamma(-i\eta)]-\log\eta,$$

and assumes that K may be expanded as a power series in the energy, as $K=D(-1/a+\frac{1}{2}r_0k^2-Pr_0{}^3k^4+\cdots)$. This expansion is equivalent to assuming that the de Broglie wavelength of the colliding particles is long, compared with the range of the potential of interaction of the particles. The region of applicability to proton-proton scattering is to about 20 Mev. Since an α particle has twice the momentum of a proton of the

[14] E. P. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1947).
[15] T. Teichmann and E. P. Wigner, Phys. Rev. **87**, 123 (1952).
[16] P. B. Treacy, reference 13, placed a lower limit of 0.15 ev for the width, while unsuccessful attempts to observe the resonance by N. P. Heydenburg and G. M. Temmer (private communication); Phys. Rev. **104**, 123 (1956), preceding paper, place an upper limit of about 3.5 ev on the width.
[17] J. D. Jackson and J. M. Blatt, Revs. Modern Phys. **22**, 77 (1950).

same energy, and since the range of the α-α potential of interaction is about four times the range of the proton-proton interaction, the expected energy region of applicability to α-α scattering is for bombarding energies below about 3 Mev. Since a value of δ_0 of zero or $n\pi$ causes the function K to diverge, a consideration of possible poles of K is necessary to justify employing a power series in the energy to describe K. The S-wave phase shift apparently has no zeros or values of $n\pi$ in the energy region above the ground state and below about 20 Mev. However, it is zero, or $n\pi$, near 20 Mev and may also vanish somewhere below the ground state. A singularity of K at 20 Mev would not materially effect the low-energy region. Since there is no evidence that the S-wave phase shift is ever negative below the resonance, it has been assumed that K is a proper function in that region. If δ_0 were negative below the ground state, it appears that this would imply a potential whose outer part is repulsive.[18] Since arguments to be given later indicate that the potential is probably attractive at large distances, it seems likely that K is not singular at low energies and that the application of the K-function formalism is reasonable.

The range expansion was applied to the Rice Institute and to the Department of Terrestrial Magnetism data, and the fit is shown in Fig. 9. It was found that only terms up to k^4 were required to fit the data below 3 Mev. In Fig. 9 are also shown the values of the "zero energy scattering length," "effective range," and shape parameter P—all obtained by the fitting. The fact that P is not zero shows that there is more potential shape information to be derived from α-α scattering than from low-energy p-p scattering. The energy expansion begins to deviate from the experimental K function in the vicinity of 4 to 5 Mev.

It is clear from the definition of K that when $\delta_0=\pi/2 \pm n\pi$ (where n is an integer), then $K=h(\eta)$. The energy expansion of K, then, must necessarily pass through

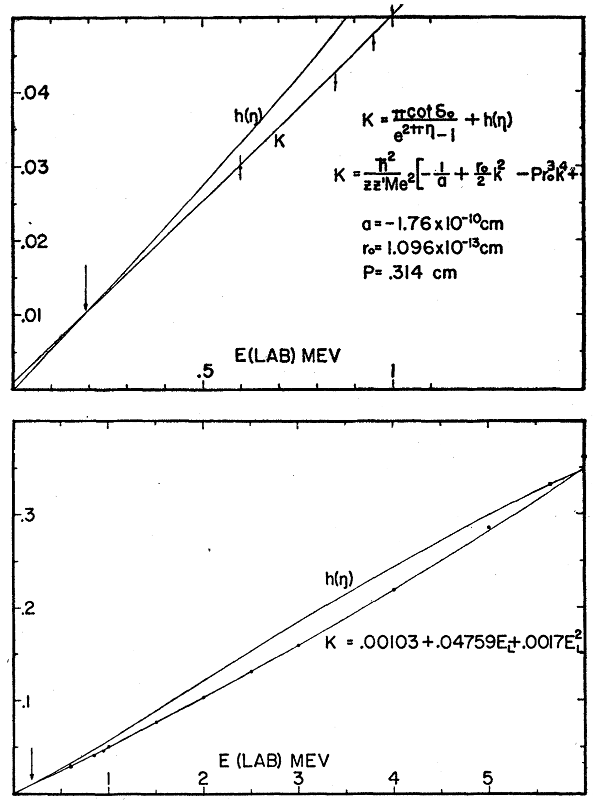


FIG. 9. Application of the Landau K function to the determination of the width of the S-wave ground state width of Be^8. The function $H(\eta)$ and $K(E)$ are shown in the lower part of the figure and plotted *versus* energy. An analytical fit to the data is given at the top of the figure. It also shows the intersection of K and h at the ground state of Be^8, along with the derived values of the "zero energy scattering length" a, "effective range" r_0, and shape parameter P.

$h(\eta)$ at 94.5±1.4 kev center-of-mass energy, where $\delta_0=\pi/2$.[19] This fact permits a determination of the ground state width. The upper part of Fig. 9 is an enlarged view showing the intersection of K and $h(\eta)$. The width of the ground state was computed by solving the expansion of K to determine the energy change required to change the S-wave phase shift from 45° to 135° across the ground state. The width, then, in the c.m. system is given by the expression $\Gamma=\pi[(e^{2\pi\eta}-1)(dh/dE-dK/dE)]^{-1}$. The width obtained was 4.5±3 electron volts, center-of-mass energy. The error was assigned by assuming a ±1.4 kev (c.m.) uncertainty for the ground state energy and a ±10% uncertainty in the slope of K. This width is about 15% of the Wigner limit (assuming a radius of 5.7×10^{-13} cm), and although small is consistent with a two-body description of Be^8. Furthermore, the width derived in this way, overlaps the established limits.[16] The parameters of the ground state are also shown in Table II.

TABLE II. The excited states of Be^8, shown by α-α scattering below 12-Mev excitation energy. The properties of the G state are those of Nilson and Jentschke, while the properties of the ground state and first excited state are deduced (see text) from the low-energy S- and D-wave phase shifts measured at D.T.M. and Rice.

Excitation energy (Mev)	Spin ($\hbar$)	$\Gamma_{\mathrm{c.m.}}$	γ^2(Mev, c.m.)	R(cm $\times10^{-13}$)	$\gamma^2/3\hbar^2/2\mu R^2$)[a]
0	0	4.5±3 ev	0.15	5.7	0.15
2.9	2	2.0 Mev	0.9	5.0	0.7
11.6	4	6.7 Mev	1.5	4.5	0.95

[a] The Wigner limit was calculated in each case for the radius shown.

[18] A function $\Delta_L(k,r)$ may be defined by $\partial\Delta(k,r)/\partial r=-(1/k) \times V(r)A^2(k,r)\sin^2[\Delta_L(k,r)+\phi_L(k,r)]$, where $V(r)$ is the nuclear potential defined by $V(r)=V_{\mathrm{total}}-V_{\mathrm{Coulomb}}$. $A_L^2=F_L^2+G_L^2$, where F_L and G_L are the regular and irregular Coulomb wave functions, and $\phi_L=\tan^{-1}(F_L/G_L)$. One can show that

$$\delta_L(k)=\int_0^\infty \frac{\partial\Delta(k,r)dr}{\partial r}=-\frac{1}{k}\int_0^\infty V(r)A_L^2(k,r)\sin^2(\Delta+\phi)dr.$$

[19] Jones, Donahue, McEllistrem, Douglas, and Richards, Phys. Rev. **91**, 879 (1953).

QUALITATIVE FEATURES OF THE α-α INTERACTION POTENTIAL

If a two-body description of Be^8, in terms of a single central potential for all partial waves, is a sufficient description of the ground state and the first two excited states, then certain qualitative arguments, based on the scattering data, may be made about the α-α potential of interaction. First, because the ground state is slightly unbound, the net attractive volume of the potential well (defined as the radial distance over which the potential is negative times the square root of the average depth of the potential in this region) must be of the order of $\frac{1}{3}\times10^{-12}$ (Mev)$^{\frac{1}{2}}$-cm. Second, if the S-wave phase shift is negative above 20 Mev, as is indicated by the Illinois scattering data, there must be some spatial region in which the potential of interaction is more repulsive (positive) than the Coulomb potential.[18] Third, because the higher order partial waves, δ_2 and δ_4, first begin to deviate from zero in the positive direction, the outermost effect of the potential must be attractive (negative). Fourth, the D- and G-wave phase shifts begin to deviate from zero at energies which correspond to a maximum range of the potential of about 5×10^{-13} cm, as determined by the hard sphere sizes required in the previously discussed dispersion theory calculations. These considerations present a qualitative picture of the α-α interaction potential as an attractive potential trough, of the order of 2×10^{-13} cm wide and a few Mev deep, located at a radius of about 5×10^{-13} cm and a potential core which must be more repulsive than the Coulomb potential. This qualitative information about the potential shape stimulates an interest in the problems of the explicit construction of the potential from the phase-shift data. Such calculations are being attempted.

Of course, nothing may be inferred about the nature of the potential for distance less than about 2×10^{-13} cm because of the large uncertainty of the energies at these distances and the finite binding energies of the individual nucleons inside the α particles. Also, the finite extent in energy of the phase shift information limits the definition of the potential for the smaller radii.

It is interesting to note that the arguments given above, when applied to the data of Steigert and Sampson, would indicate a soft core, or attractive, potential in order to obtain the 7.5-Mev S state that they report. Thus it appears that additional experimental α-α scattering work needs to be done in the energy region above 6-Mev bombarding energy so that this serious disagreement may be resolved.

A consideration of the α-α scattering data, then, appears to allow an interpretation of the Be^8 nucleus, for excitation energies less than 12 Mev, as a two-body interaction between α particles, and further, some qualitative information about the potential of interaction may be obtained.

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