We have studied ω_e'/ω_p as a function of $1/H^2$ for magnetic fields ranging from 750 to 1700 gauss. For each run, from three to thirteen points were taken in this interval. Deviations from a least-squares straight line fit to the data of each run were less than three parts per million for over half of all points taken. These deviations were primarily due to random errors in tuning the electron microwave cavity at the individual points. We have found, by analysis of all data without rejection, that any systematic deviations from a straight line are less than one part in one million.

The free electrons were produced by photoelectric emission from a film of a few molecular layers of potassium deposited upon the inner surface of a highly evacuated spherical bulb of Pyrex $\sim\frac{1}{2}$ cm in diameter. The resonance was observed by the use of typical microwave techniques. The electron line widths in these measurements varied from one part in 2000 to one part in 35 000. The electrostatic fields, and hence the frequency shifts and line widths, were a function of the intensity and distribution of the light over the surface of the bulb; the lighting conditions were varied from run to run.

Figure 1 summarizes all the data taken. Several different electron bulbs, light sources, and cavities were used. The lines represent least-squares fits to the data of each run.

The average of extrapolated intercepts for all these runs, without rejection of any data, is $\omega_e/\omega_p=657.462\pm0.006$. The limit of error includes 95% of the runs, and is believed to represent a maximum error.[3]

A relativistic correction necessitated by the finite velocities of the electrons is taken to be 0.001 ± 0.001, where the error is again to be regarded as a maximum. Addition of this correction yields

$$\mu_0/\mu_{p(\mathrm{oil})}=657.463\pm0.007 \tag{3}$$

for a spherical sample of mineral oil, where no magnetic corrections have been applied. This result is to be compared with that of Gardner and Purcell:[1]

$$\mu_0/\mu_{p(\mathrm{oil})}=657.475\pm0.008. \tag{4}$$

Applying a diamagnetic correction factor[4–6] of $(2.94\pm0.10)\times10^{-5}$ to the field at the proton, we obtain for the final corrected value of the magnetic moment of the free proton in units of the Bohr magneton:

$$\mu_p/\mu_0=(657.444\pm0.007)^{-1} = (1.521042\pm0.000016)\times10^{-3}. \tag{5}$$

The present result (3), uncorrected for the spherical sample of mineral oil, when combined with the data[7–9] available for the magnetic moment of the free electron,

$$\mu_e/\mu_{p(\mathrm{oil})}=658.2293\pm0.0010, \tag{6}$$

also referred to a spherical sample of mineral oil, yields for the magnetic moment of the free electron in Bohr magnetons:

$$\mu_e/\mu_0=1.001165\pm0.000011 =1+(\alpha/2\pi)+(0.7\pm2.0)(\alpha^2/\pi^2). \tag{7}$$

This is to be compared with the current theoretical estimate[10,11]:

$$\mu_e/\mu_0=1.0011454 =1+(\alpha/2\pi)-2.973(\alpha^2/\pi^2). \tag{8}$$

A detailed report on this experiment is in preparation.

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Angular Distribution of Nuclear Reaction Products

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In an earlier paper,[1] the author applied the continuum theory of nuclear reactions[2,3] to predict the angular distribution of γ rays following inelastic neutron scattering. Unfortunately the formulas presented there contain an error and a numerical misprint. We wish now to give the corrected formulas, and to generalize them within the S-matrix formalism to include interference between two or more compound nucleus levels. Applications of the incorrect formula to two recent experiments have been published[4,5]; we find that correction of the errors leads to considerably better agreement between experiment and theory.

Let a target nucleus of spin J_0 capture particles with total angular momentum j_1 to form a compound nucleus with spin J_1. This re-emits particles with total angular momentum j_2, leaving an excited nucleus with spin J_2. Consider now the angular distribution (relative to the incident beam) of radiation with total angular momentum j_3, from the decay of J_2 to the final nucleus J_3. We denote the corresponding orbital angular momentum for the particles by l. When the "particles" are photons, j is the multipole order, and $(-)^l$ must be regarded as indicating the parity (i.e., magnetic or electric character) of the γ ray.

For incident particles of spin s_1, the differential cross section is

$$\frac{d\sigma(\theta)}{d\omega}=\frac{\lambda\!\!\!^{-2}}{4(2J_0+1)(2s_1+1)}\sum_\nu A_\nu P_\nu(\cos\theta),$$

where A_ν contains a factor for each stage of the process. Summing over j_1, j_1', l_1, l_1', j_2, j_3, j_3', l_3, l_3', J_1, and J_1', we have

$$A_\nu=\sum B_\nu(j_1l_1j_1'l_1'J_0;J_1J_1')B_\nu(j_3l_3j_3'l_3'J_3;J_2J_2)\times I_\nu(j_2J_1J_1'J_2J_2)S(J_1;j_2l_2;j_1l_1)\times S^*(J_1';j_2l_2;j_1'l_1')\gamma(j_3l_3)\gamma^*(j_3'l_3').$$

The S's are elements of the scattering matrix for the transitions indicated, and so the total cross section is

$$\sigma=\frac{\pi\lambda\!\!\!^{-2}A_0}{(2J_0+1)(2s_1+1)};\quad A_0=\sum(2J_1+1)|S|^2,$$

and the relative intensity of observed radiation with j_3,l_3 is $|\gamma(j_3l_3)|^2$. The B_ν coefficients have been given explicitly in an earlier publication[6]; with $\nu+l+l'$ even only,

$$\begin{aligned}B_\nu(jlj'l'J;J_1J_1')&=[(2j+1)(2j'+1)(2J_1+1)(2J_1'+1)]^{\frac{1}{2}}(-)^{J_1-J+s}\times C(jj'\nu;s-s)W(jj'J_1J_1';\nu J)\\&=(2J_1+1)^{\frac{1}{2}}a_\nu(jj'JJ_1)\quad\text{if}\quad J_1=J_1',\\B_0&=(2J+1)^{\frac{1}{2}}\delta(jj')\delta(J_1J_1'),\end{aligned}$$

where $s=1$ for photons, $s=\frac{1}{2}$ for spin $\frac{1}{2}$ particles, and $s=0$ for alpha particles. The C's are Clebsch-Gordan coefficients, and the W's are Racah coefficients. The a_ν are extensively tabulated: $a_\nu=\eta_\nu$ [7] for spin $\frac{1}{2}$, $a_\nu=F_\nu$ [8] for photons.

The I_ν describes the unobserved outgoing radiation, depending only on the total angular momentum j it carries away, and the nuclear spins, but not the nature of the radiation.[1] No interference terms appear between different j.

$$I_\nu(jJ_1J_1'J_2J_2')=[(2J_1+1)(2J_1'+1)]^{\frac{1}{2}}(-)^{J_1'+J_2-j_1-\nu}\times W(J_1J_1'J_2J_2';\nu j),$$
$$I_0=[(2J_1+1)/(2J_2+1)]^{\frac{1}{2}}\delta(J_1J_1')\delta(J_2J_2').$$

Should the observed radiation be preceded by other unobserved transitions, an additional factor I_ν for each has to be included in A_ν.

The triple correlation between two reaction products relative to the incident beam is a similar generalization of the formula given in reference 1.

The continuum theory treatment of the scattering amplitudes S consists of two steps. First we make the statistical assumption that a sufficient number of compound nuclear states are involved for all interference terms from states of different spin and parity, and from different incoming and outgoing partial waves, effectively to average to zero.

$$\langle S(J_1;j_2l_2;j_1l_1)S^*(J_1';j_2'l_2';j_1'l_1')\rangle_{\text{Av}}=\bar{S}(J_1;j_2l_2;j_1l_1)\bar{S}^*(J_1;j_2'l_2;j_1'l_1)\times\delta(J_1J_1')\delta(l_1l_1')\delta(l_2l_2').$$

Then to obtain the magnitude of these average transition amplitudes $\bar{S}$ we assume they depend only on the penetrability or transmission coefficients $T_l(E)$, of the partial waves, at energy E. These may be calculated either on the basis of a "black" nucleus,[2] or the "cloudy crystal ball" model.[3] The $\bar{S}$ matrix elements then become

$$\bar{S}(J_1;j_2l_2;j_1l_1)\bar{S}^*(J_1;j_2'l_2;j_1'l_1)\simeq T_{l_1}(E_1)T_{l_2}(E_2)/{\sum}'_{jl}T_l(E),$$

independent of j_1 and j_2. The denominator, omitted in reference 1, is summed only over channels j, l, E by which that particular compound state could decay. To this extent, the transition amplitudes are still dependent on the J_1, parity, and energy of the compound nuclear state. Introduced into the expression for the total cross section, this leads immediately to the result of Hauser and Feshbach.[2] Inclusion of spin-orbit coupling[6] in the calculation of the T_l would make the $\bar{S}$ elements depend on j_1 and j_2 also, but this dependence is expected to be weak.

Of particular interest is the angular distribution of γ rays following inelastic neutron scattering. If one uses the continuum theory, this reduces to

$$W(\theta)=\sum(2J_1+1)\eta_\nu(j_1j_1'J_0J_1)F_\nu(j_3j_3'J_3J_2)\times I_\nu(j_2J_1J_1J_2J_2)P_\nu(\cos\theta)\gamma(j_3)\gamma(j_3')\times T_{l_1}(E_1)T_{l_2}(E_2)/{\sum}'_{jl}T_l(E),$$

where, as before, the primed sum in the denominator is only over those channels open to the particular compound state. The experiments have been done with even-even target nuclei, for which $J_0=0$, exciting the first excited state with $J_1=2$, and observing the ensuing $E2$ γ ray, so that $j_3=2$, $J_3=0$. If one considers only s, p, and d waves, the angular distribution is

$$W(\theta)=\frac{2T_0T_2'}{T_0+2T_2'}+\frac{T_1T_1'}{T_1+T_1'}+\frac{T_1T_1'}{T_1+2T_1'}P_2(\cos\theta)+\frac{T_2T_0'}{T_2+T_0'+2T_2'}[5+2.714P_2(\cos\theta)-1.715P_4(\cos\theta)]+\frac{T_2T_2'}{T_2+T_0'+2T_2'}[10+0.714P_2(\cos\theta)+0.857P_4(\cos\theta)].$$

T_l is evaluated at the energy of the incident neutrons, T_l' at that of the inelastic neutrons, these being the only channels open at reasonably low energies.

The corresponding expression in reference 1, apart from omission of the penetrability denominators, contains a numerical error: the constant in the first square bracket above appears as 2, not 5. Using the T_l calculated on the optical model for medium-weight nuclei[4] and for energies of 1 to 2 Mev, we find that correction of this numerical error leads to anisotropies roughly half those previously predicted. This removes most of the discrepancy between theory and experiment.[4,5] Omission of the penetrability denominators appears to have little effect.

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