In the second experiment, the bismuth activities resulting from the Pb(d,xn)Bi reactions produced by 15-Mev deuterons in the MIT cyclotron were adsorbed on Dowex-1 anion resin and the lead was quickly eluted to look for a very short half-life. The eluate flowed rapidly into the well of a NaI scintillation counter. An increase in counting rate was observed, but the activity had a relatively long half-life ($t_{\frac{1}{2}}$ $\sim$several hours) and could be attributed to the equilibrium concentration of Bi activities in the eluate. A half-life as short as 2 seconds could have been detected. No Pb activity, the initial concentration of which fell with the 14-day half-life of the Bi^{205} parent, could be observed in washings of the Dowex-1 column which would have removed any Pb decay daughters.[9–11]

These data indicate that Pb^{205} does not decay by K capture with a half-life in the range of 2 sec to 10^{10} years. A partial K-capture half-life of less than 2 sec can be ruled out on the basis of the observed[1] L-capture half-life of 5×10^7 years. Hence it is concluded that if Pb^{205} decays by K capture at all the half-life for this decay mode must be greater than 10^{10} years.

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[9] E. C. Campbell and F. Nelson, Phys. Rev. **91**, 499 (1953).

[10] F. Nelson and K. A. Kraus, J. Am. Chem. Soc. **76**, 5916 (1954).

[11] *Note added in proof.*—From the negative results of this experiment, nothing can be concluded regarding the existence of a short-lived Pb^{205} isomer, except that Bi^{205} electron capture does not heavily populate the $i_{13/2}$ level in Pb^{205}. In the analogous Bi^{207} decay, 87% of the e.c. (electron capture) transitions go to the $i_{13/2}$ state of the 0.8 sec Pb^{207m}. The half-life of this $h_{9/2}$ to $i_{13/2}$ transition is 9.2 years (8 y/0.87) and the competing $h_{9/2}$ to $f_{7/2}$ transition in B^{207} has an energy of only 50±40 kev. In the decay of Bi^{205}, this latter transition probably has considerably more energy, and may be the transition most influencing the half-life. The $h_{9/2}$ to $i_{13/2}$ decay may be roughly estimated by the ratio $14d$ to 9 years or 0.004, and this low rate of population of the $i_{13/2}$ level of Pb^{205m} may well be the reason that we do not observe this isomer.

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Nucleon-Hole Interaction in *jj* Coupling*

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A theorem connecting the energy levels in jj coupling of a nucleon-nucleon and nucleon-hole system is derived, and applied in particular to Cl^{38} and K^{40}.

THE interaction of a nucleon with a hole in a closed shell has been considered by Racah and others[1] by an application of tensor algebraic methods. An alternate approach, independent of the nature of the two-nucleon interaction, is possible for the jj-coupling case. It is based on the property that the coefficients of fractional parentage (c.f.p.) connecting the states of one- and two-hole systems have a particularly simple analytical form.[2] This approach yields a theorem which gives the energy levels of a nucleon-hole system directly in terms of the energy levels of the corresponding nucleon-nucleon system.

Consider a proton (or a neutron) hole in shell j, and a neutron (or a proton) in shell j' coupled to a resultant spin J. The $2j$ protons are antisymmetrized with respect to each other, but not with respect to the neutron. (Note, however, that if, as we assume, all the lower-lying proton and neutron subshells are filled, this state does have a specified isotopic spin.) The interaction Hamiltonian for the system, $\sum_{i<j}H_{ij}$, may then be written as a sum of two parts: (i) The interactions among the identical nucleons giving the energy of the hole, $E[j^{-1}]$, independent of J, and (ii) the neutron-proton interactions. To evaluate the second part, c.f.p. are used to write the state of the hole $\langle j^{-1}|$ in terms of $2j-1$ antisymmetrized protons coupled to a single proton; then the neutron and the proton are recoupled by means of a Racah coefficient. Now, using the explicit analytical form for the c.f.p., the sum over the parent states of the hole can easily be carried out, leaving only a sum over the states J_0, of a proton in shell j coupled to a neutron in shell j'. Finally, we obtain for the relative energies $E_J[j^{-1}j']$ of the neutron-proton hole configuration,[3]

$$E_J[j^{-1}j']=-\sum_{J_0}(2J_0+1)W(jj'j'j;JJ_0)E_{J_0}[jj'], \quad (1)$$

and the reciprocal relation,

$$E_J[jj']=-\sum_{J_0}(2J_0+1)W(jj'j'j;JJ_0)E_{J_0}[j^{-1}j']. \quad (1')$$

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[1] G. Racah, Phys. Rev. **62**, 438 (1942); Marty, Nataf, and Prentki, J. phys. radium **15**, 134 (1954).

[2] G. Racah, Phys. Rev. **63**, 367 (1943), Eq. (19); C. Schwartz and A. de-Shalit, Phys. Rev. **94**, 1257 (1954).

[3] For detailed derivation of (1) and (1′) see S. P. Pandya, U. S. Atomic Energy Commission Report NYO-7590 (unpublished), Appendix.

We note, then, that by substituting on the right side of (1) explicit expressions for the matrix elements of a two-nucleon interaction Hamiltonian, the matrix elements for the nucleon-hole system may be directly obtained.

As an application of (1), we shall consider the energy levels of Cl^{38} and K^{40}.[4] It is known[5] that in the neighborhood of the $f_{7/2}$ shell, jj coupling is a good approximation for low-lying nuclear states. Such states for Cl^{38} and K^{40} may then be described in terms of the configurations $(d_{3/2})\ (f_{7/2})$ and $(d_{3/2})^{-1}\ (f_{7/2})$, and are expected to have spin values 2,3,4,5. The level scheme for K^{40} is fairly well known. The ground state has spin 4, and excited states are observed at 0.032, 0.8, and 0.89 Mev.[6] The results of the $K^{39}(n,\gamma)K^{40}$ experiment[7] suggest strongly that the spins of the 0.032- and 0.8-Mev states are 2 or 3, whereas the spin of the 0.89-Mev state should be >3. It seems reasonable then, to assume this latter spin value to be 5. The two alternate level schemes possible for K^{40}, and the corresponding level schemes for Cl^{38} derived by using (1′) are given in Table I. The experimental results[6,8] show that the ground state has spin 2, and indicate a $J=5$ state at 0.672 Mev. The results, with suggested spin assignments, are shown in the last column of Table I. It is clear that the scheme II is in excellent agreement with the experimental results. Thus jj coupling seems to be a good approximation for these nuclei,[9] and enables us to assign tentative spin values for the various levels in Cl^{38} and K^{40}.[10]

It is possible to apply the above analysis to other pairs of odd-odd nuclei, which can be described as nucleon-nucleon and nucleon-hole systems, provided the validity of jj coupling is reasonably assured, and enough knowledge of the energy levels is experimentally available. In actual practice not many such cases are now known. It is of some interest, however, to consider Cl^{34} and Cl^{36} nuclei in the light of this theorem, even though at present there is little evidence of the validity of the jj coupling description for them. In Cl^{34}, the ground state and the 0.142-Mev state are known to have spin values 0+ and 3+ respectively.[6] Ajzenberg *et al.*[11] have reported further levels at 1.1, 1.9, 2.7, 3.7 Mev, etc. In view of the occurence of a 2+ state at about 2-Mev excitation in S^{34} and A^{38}, the 1.9-Mev state may be assigned spin 2+. A knowledge of the location of the remaining 1+ state of the $(d_{3/2})\ (d_{3/2})$ configuration would be useful for further analysis of nuclei in the $d_{3/2}$ shell. Assigning the 1+ value to various remaining states in Cl^{34}, and evaluating the level scheme of Cl^{36} by means of (1), we find that for the 1+ state at 1.1 Mev, we obtain in Cl^{36}, 2+ as the ground state and excited states 3+ at 0.66 Mev, 1+ at 1.62 Mev, and 0+ at 3 Mev. Levels are in fact known near these energies,[6,8] but the correlation is doubtful because there are other low-lying levels. If the 1+ state is assumed to be at 2.7 Mev or higher, the 3+ level in Cl^{36} is obtained too close to the ground state and eventually as the ground state itself, in contradiction with experiment. This then suggests the identification of 1.1-Mev state in Cl^{34} as 1+, unless there remains an as yet undiscovered level in Cl^{34} below 2.5 Mev, or it turns out that jj coupling is an unsatisfactory approximation for this case.

TABLE I. Energy levels of Cl^{38} as predicted by Eq. (1′) from two alternate level schemes for K^{40}. The last column lists the experimental levels of Cl^{38} against suggested spin values.

J	Energy levels of K^{40} (Mev) I	II	Energy levels of Cl^{38} (Mev) I	II	Experiment (Mev)
2	0.032	0.8	0	0	0
3	0.8	0.032	1.28	0.748	0.762
4	0	0	0.684	1.324	1.312
5	0.89	0.89	1.062	0.696	0.672

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[4] Since this work was completed, there has appeared an explicit calculation by S. Goldstein and I. Talmi [Phys. Rev. **102**, 589 (1956)] for the particular case of K^{40} and Cl^{38}. However, they do not use the theorem derived above, but make use of the actual numerical values of the c.f.p.

[5] C. Levinson and K. W. Ford, Phys. Rev. **100**, 13 (1955); B. J. Raz and J. B. French (to be published).

[6] See the review article by P. M. Endt and J. C. Kluyver, Revs. Modern Phys. **26**, 95 (1954).

[7] G. A. Bartholomew and B. B. Kinsey, Can. J. Phys. **31**, 927 (1953).

[8] Paris, Buechner, and Endt, Phys. Rev. **100**, 1317 (1955).

[9] This result differs from that of G. E. Tauber and T. Y. Wu [Phys. Rev. **94**, 1307 (1954)] for K^{40}. However, their analysis indicates the $J=5$ level to be below the $J=2$ level and is apparently inconsistent with the $K^{39}(n,\gamma)K^{40}$ experiment. We believe that the reason for the magnetic moment anomaly is not necessarily found in intermediate coupling, particularly near the *LS* limit.

[10] The level scheme II coincides with that suggested by Endt and Kluyver[6] on the basis of relative cross sections at 90° of the $K^{39}(d,p)K^{40}$ experiment. Their argument would be satisfactory in this case if the cross sections were available at peak values.

[11] Ajzenberg, Rubin, and Likely, Phys. Rev. **99**, 654 (1955).