Zero-Zero Transition in Carbon-12†*

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The electric-monopole transition density between the ground state and the 7.68-Mev level of carbon-12 is examined on the basis of the nuclear shell model. It is found to vanish for all stages of intermediate coupling if only the $(1p)^8$ configuration is involved. A nonzero value is obtained by including states of the $(1p)^7(2p)$ configuration, brought in by a residual central internucleon interaction. For computational simplicity, both this and the spin-orbit interaction are first treated as perturbations about the *LS* limit. Since the results show that the residual interaction is probably too large for this to be reliable, an attempt is made to diagonalize the Hamiltonian exactly, but the severe restriction on the number of states considered makes the result rather unsatisfactory. It is concluded that if all of the possible states of the low-lying configurations were to be included in the diagonalization, agreement with experiment might result, but that in this case some semicollective model might better be applied to the problem.

I. INTRODUCTION

BECAUSE of their symmetrical structure, the nuclei carbon 12 and oxygen 16 have been examined in terms of a number of nuclear models, which usually regard them as very similar systems. This similarity is exemplified by the fact that they both have low-lying excited states of spin-parity 0+. In O^{16} this 0+ state is the first excited state, and from its decay by pair-emission Devons *et al.*[1] have deduced for the matrix element of the monopole operator $\sum_P r_P^2$ between the ground state and the excited state a value of 3.8×10^{-26} cm^2. In C^{12}, where the 0+ level is the second excited state, the corresponding matrix element has been measured in a different manner: the transition to this state from the ground state is induced by the (inelastic) scattering of high-energy electrons. From the results of this experiment by Fregeau and Hofstadter,[2] Schiff[3] has estimated that the C^{12} matrix element has about the same value as that in O^{16}, although because of the extrapolation of the experimental results required, its value is not known very accurately.

An examination of this electric monopole transition in C^{12} has been made by Schiff[3] from two extreme points of view, using the collective fluid model and the alpha-particle model on the one hand, and a simple *jj*-coupling shell model on the other. For the former case he finds a matrix element too large by about a factor three, and for the shell model a result too small by a considerable factor. The shell-model calculation was performed for the *jj* limit, and if excitation to only the $1p_{\frac{1}{2}}$, $1d_{\frac{5}{2}}$, or $2s_{\frac{1}{2}}$ shells is assumed, then a 0+ excited state of C^{12} must involve the excitation of two nucleons from the filled $1p_{\frac{3}{2}}$ shell. Since, in the first Born approximation, the interaction between the electron and the nucleus is a strictly single-nucleon interaction, Schiff found it necessary in his shell-model calculation to invoke a residual internucleon force in order to obtain a nonzero matrix element. Roughly speaking the mechanism, as he envisaged it, is that the electron excites one nucleon to one of the higher orbits and it, through the internucleon force, pulls up another nucleon to form a two-nucleon excitation. His calculation of the process treated the residual interaction as a perturbation on the *jj* limit, although the use of the Green's function for a bound nucleon means that, to first order, all possible admixed states are included. In view of the success of the more orthodox intermediate-coupling shell-model calculations of Lane[4] and Lane and Radicati[5] in explaining electric-dipole and -quadrupole transitions in the $1p$-shell nuclei, it seemed desirable to apply the same techniques to the C^{12} electric monopole transition.

In this paper intermediate-coupling calculations are performed in an *LS* representation. For such calculations the nucleons outside a closed shell are usually assumed to be in the lowest configuration which can explain the nuclear states involved. Then, neglecting the interaction part of the total nuclear Hamiltonian, there will be a number of possible *LS* states within this one configuration. For instance, for the $(1p)^8$ configuration in C^{12}, the *LS* states with total spin and total isotopic spin both zero are $^{11}S[44]$, $^{13}P[431]$, $^{11}S[422]$, $^{15}D[422]$, and $^{13}P[332]$. The nomenclature is as follows: the numeral superscripts refer to the isotopic spin and ordinary spin multiplicity, respectively; the letter symbol is the usual designation for orbital angular momentum; and the bracketed numbers describe the symmetry character of the *LS* states. These states, being in the same configuration, are all degenerate.

If, now, some residual interaction between particles

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[1] Devons, Goldring, and Lindsey, Proc. Phys. Soc. (London) **A67**, 134 (1954).

[2] J. H. Fregeau and R. Hofstadter, Phys. Rev. **99**, 1503 (1955).

[3] L. I. Schiff, Phys. Rev. **98**, 1281 (1955).

[4] A. M. Lane, Proc. Phys. Soc. (London) **A66**, 977 (1953).

[5] A. M. Lane and L. A. Radicati, Proc. Phys. Soc. (London) **A67**, 167 (1954).

is included (LS limit), these degenerate LS states are split into separate levels. If, instead of a particle interaction, a spin-orbit interaction (jj limit) is included, one obtains separate levels which are linear combinations of the original LS states. The procedure in intermediate coupling is to diagonalize the interaction Hamiltonian at some point between the two limits. As long as only one configuration is considered and a central interaction is used, the interactions within closed shells and between shells can be neglected since these contributions are common to all LS states and diagonal in them.[6] It turns out, however, that the electric-monopole matrix element is only nonzero because of the presence of higher configurations, so that it has been necessary to include some of them in our calculation.

In calculating matrix elements of the interaction Hamiltonian, the coefficients of fractional parentage introduced by Racah[7] are used to reduce an antisymmetric state of n particles in a given configuration to an antisymmetric state of $(n-m)$ particles vector-coupled to an antisymmetric state of m particles. For matrix elements of a single-particle operator, the coefficients of fractional parentage for reduction by one particle are required. These coefficients for n particles in a p shell have been tabulated by Jahn and van Wieringen[8] in an LS representation and by Edmonds and Flowers[9] in a jj representation. For matrix elements of a two-particle operator, the coefficients of fractional parentage for reduction by two particles are required, and these have been tabulated by Elliott, Hope, and Jahn[10] in an LS representation of n particles in a p shell.[11]

For the numerical values of the single-particle matrix elements needed in our calculation, and for the type of coupling that prevails in C^{12}, we refer to a theoretical analysis of the experimental electron-induced transition to the first excited level (2+) in that nucleus.[12] Briefly, the results of that investigation are that the common well is, to a good approximation, parabolic with a length parameter approximately 1.64×10^{-13} cm, and that the coupling is probably close to the LS limit. Since the following investigation has not led to any precise estimate of those parts of the wave functions required for the monopole transition probability, we have made no attempt to fit the detailed shape of the inelastic electron-scattering cross section, but have tried to obtain agreement only with $\sum_P r_P^2$, which involves just the cross section near the forward direction.

II. CALCULATION INVOLVING ONLY THE $(1p)^8$ CONFIGURATION

It is desired to calculate the matrix element of the monopole transition operator Ω between the ground and first excited 0+ state of the C^{12} nucleus, using for the moment only the $(1p)^8$ configuration. Ω is given by

$$\Omega=\sum_P r_P^2=\sum_i(\tfrac{1}{2}-m_i)r_i^2,$$

where m_i is the z component of isotopic spin of the ith nucleon ($+\frac{1}{2}$ for a neutron and $-\frac{1}{2}$ for a proton). The initial and final states of the nucleus are orthogonal and are made up of linear combinations of the five LS states listed in Sec. I. Thus, matrix elements of the type $\langle\psi_J|\Omega|\psi_{J'}\rangle$ will occur, in which

$$\psi(\gamma TSL,M_TJM)=\sum_{\text{mag}}C^{LSJ}\psi(\gamma TSL,M_TM_SM_L),$$

where C^{LSJ} is a vector-coupling coefficient and γ represents any other quantum number necessary to describe the LS state.

Using Eq. (23) of Racah,[7] with obvious modifications to include isotopic spin, there results

$$\begin{aligned}\langle\psi_J|\Omega|\psi_{J'}\rangle=n\sum(\bar\psi\|\psi)(\bar\psi\|\psi')\sum\,&C^{LSJ}C^{L'S'J'}\\&\times C^{T_PtT}C^{T_Pt'T'}C^{S_P\sigma S}C^{S_P\sigma' S'}C^{L_PlL}C^{L_Pl'L'}\\&\times\langle t\sigma lm_tm_\sigma m_l|\,(\tfrac{1}{2}-m_t)r^2\,|t'\sigma'l'm_{t'}m_{\sigma'}m_{l'}\rangle.\end{aligned}\quad(1)$$

In this expression, $(\bar\psi\|\psi)$ is the coefficient of fractional parentage for reduction of the LS state ψ by one particle to obtain the parent state $\bar\psi$; the bare summation sign and the absence of magnetic numbers on the vector-coupling-coefficients mean summation over all magnetic numbers except M and M_T; the subscript P refers to the parent state. In this particular calculation $J=M=T=M_T=T'=M_{T'}=0$, $l=l'=1$, and $\sigma=\sigma'=t=t'=\frac{1}{2}$. The final matrix element in (1) is a single-particle matrix element and may be evaluated as

$$\langle(\tfrac{1}{2}-m_t)r^2\rangle=\delta_t\delta_\sigma\delta_l\delta(m_t,-\tfrac{1}{2})\langle r^2\rangle_{1p,\,1p},$$

where δ is the Kronecker delta symbol for the appropriate magnetic quantum numbers and $\langle r^2\rangle_{1p,\,1p}$ is the mean square radius of the $1p$ orbit $\int_0^\infty[R(r)]^2r^4dr$, where $R(r)$ is the radial wave function for the $1p$ shell. Then taking $T=T'=0$ and using the properties of the vector-coupling coefficients,[13] Eq. (1) becomes

$$\langle\psi_J|\Omega|\psi_{J'}\rangle=\tfrac{1}{2}n\sum(\bar\psi\|\psi)(\bar\psi\|\psi')\delta_{LL'}\delta_{SS'}\langle r^2\rangle_{1p,\,1p}.\quad(2)$$

But the coefficients of fractional parentage obey an orthonormality relationship given by Eq. (13) of Racah,[7] so that finally

$$\langle\psi_J|\Omega|\psi_{J'}\rangle=\tfrac{1}{2}n\delta_{\psi\psi'}\langle r^2\rangle_{1p,\,1p},$$

[6] J. P. Elliott, Proc. Roy. Soc. (London) **A218**, 345 (1953).
[7] G. Racah, Phys. Rev. **63**, 367 (1943).
[8] H. A. Jahn and H. van Wieringen, Proc. Roy. Soc. (London) **A209**, 502 (1951).
[9] A. R. Edmonds and B. H. Flowers, Proc. Roy. Soc. (London) **A214**, 515 (1952).
[10] Elliott, Hope, and Jahn, Trans. Roy. Soc. (London) **A246**, 241 (1953).
[11] For a discussion of the concept of fractional parentage in nuclear states, see, for example, A. M. Lane and D. H. Wilkinson, Phys. Rev. **97**, 1199 (1955).
[12] D. G. Ravenhall (to be published).
[13] J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952), Appendix A.

where $\delta_{\psi\psi'}$ vanishes unless the initial and final LS states ψ and ψ' are absolutely identical.

Now the initial and final states of the nucleus, being linear combinations of LS states, may be expressed as

$$\psi_{\rm gr}=\sum_{\gamma LS} a_{\gamma LS}\psi(\gamma LST,M_TJM)$$

and

$$\psi_{\rm ex}=\sum_{\gamma LS} b_{\gamma LS}\psi(\gamma LST,M_TJM)$$

where $\sum a_{\gamma LS}b_{\gamma LS}=0$, since the two states are orthogonal. Therefore, the final result for the monopole transition matrix element in the $(1p)^8$ configuration of C^{12} is

$$\langle\psi_J|\Omega|\psi_J'\rangle=\tfrac{1}{2}n\sum_{\gamma LS} a_{\gamma LS}b_{\gamma LS}\langle r^2\rangle_{1p,1p}=0. \quad (3)$$

Thus the conclusion is reached that if no mixing of configurations is considered, the monopole transition matrix element in C^{12} vanishes for all stages of intermediate coupling; Lane[14] has generalized this result and concludes that, where only one configuration is considered, the monopole transition matrix element vanishes, in all stages of intermediate coupling, between any two states of which one has isotopic spin equal to zero. To explain the monopole transition in C^{12}, then, one must consider configuration mixing. This will be discussed in the next section.

III. CALCULATION INCLUDING THE $(1p)^7(2p)$ CONFIGURATION

A. Discussion

In addition to the $(1p)^8$ configuration, the next lowest configurations in which 0+ states of the C^{12} nucleus can be obtained are the $(1p)^7(1f)$ and $(1p)^7(2p)$ configurations. In this section, the residual internucleon interaction will be used as a perturbation to admix states of these other configurations. The angular part of a $(1p)^7(1f)$ wave function is orthogonal to the angular part of a $(1p)^8$ wave function. Therefore, to first order in the perturbation, admixtures of only the $(1p)^7(2p)$ configuration need be considered in calculating the monopole transition matrix element. To this approximation, excitation of the core, described by the configuration $(1s)^3(2s)(1p)^8$, for example, does not contribute. (See the discussion at the end of Sec. IIIC.)

A proper perturbation treatment for admixing the $(1p)^7(2p)$ configuration with the $(1p)^8$ configuration in intermediate coupling should proceed as follows. At some stage of intermediate coupling the interaction Hamiltonian is diagonalized to obtain the ground and first excited states of the nucleus as linear combinations of $(1p)^8$ LS states. A similar diagonalization is carried out to obtain nuclear states made up of linear combinations of $(1p)^7(2p)$ LS states. These latter nuclear states are then admixed with the ground and first excited states by means of perturbation theory. However, in the $(1p)^7(2p)$ configuration there are nineteen possible LS states, which means that a 19 by 19 matrix must be diagonalized. This becomes prohibitively involved; and so, for simplicity, this calculation will first be performed in the LS limit with the spin-orbit interaction treated as an additional perturbation.

B. The LS Limit

Following Lane,[4] the residual internucleon interaction used is that given by Rosenfeld's prescription[15]:

$$V_{ij}=-0.13V_W+0.93V_M+0.46V_B-0.26V_H, \quad (4)$$

where W, M, B, and H indicate Wigner, Majorana, Bartlett, and Heisenberg interactions, respectively. The interaction $\sum V_{ij}$ is diagonal in the five LS states of the $(1p)^8$ configuration and the diagonal elements may be computed from the results of Racah.[16] They are (listed in the same order as the five states given in Sec. I) $9.56L+12.92K$, $6.76L+14.02K$, $3.98L+20.06K$, $6.74L+10.78K$, and $3.04L+19.38K$, where L and K are, respectively, the direct and exchange Slater integrals.[17] L will be negative for the type of force we shall use (see Sec. IIIB) and the ratio K/L has a maximum value of $\frac{1}{3}$ (achieved when the force has zero range).[18] It is easy to show that in the LS limit, the ground state is $^{11}S[44]$ and the first excited state is $^{13}P[431]$ for all possible values of K/L.

There is only one of the $(1p)^7(2p)$ states which can be linked to $^{11}S[44]$ by the residual internucleon interaction. This is the state $^{11}S\{^{22}P[43],2p\}$, where the notation means that a $(1p)^7$ state, $^{22}P[43]$, is vector-coupled to a $(2p)$ nucleon to give a ^{11}S state. However, there are ten $(1p)^7(2p)$ states which can be linked to $^{13}P[431]$. To further simplify the calculation, only one of these states will be included. The state $^{13}P\{^{22}P[43],2p\}$ is chosen as being the lowest lying of the ten states, since it has the greatest symmetry and its seven $(1p)$ particles are in a state of least orbital angular momentum.

The first-order perturbed wave function for the ground state is

$$\Psi_{\rm gr;0}=^{11}S[44]+C_{\rm gr}\,^{11}S\{^{22}P[43],2p\}, \quad (5)$$

where the perturbation coefficient $C_{\rm gr}$ is given by

$$C_{\rm gr}=(\Delta E)^{-1}\langle^{11}S\{^{22}P[43],2p\}|\textstyle\sum_{i<j} V_{ij}|^{11}S[44]\rangle. \quad (6)$$

The subscript zero on Ψ indicates that as yet there is no spin-orbit coupling. The matrix element occurring in (6) will be calculated in some detail to show the procedure.

The general formula, Eq. (11) of Elliott,[6] for the

[14] A. M. Lane (private communication).

[15] L. Rosenfeld, *Nuclear Forces II* (North Holland Publishing Company, Amsterdam; Interscience Publisher, Inc., New York, 1949) Sec. 11.33.

[16] G. Racah, Helv. Phys. Acta **23**, Suppl. III, 229 (1950).

[17] E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, England, 1951), Sec. 8[6].

[18] D. R. Inglis, Revs. Modern Phys. **25**, 390 (1953).

matrix element of a two-particle operator may be modified, for instance, by means of Eq. (33b) of Racah,[7] to apply to the matrix element in (6). There results

$$\langle\psi'(l^{n-1},l)|\sum V_{ij}|\psi(l^n)\rangle=n^{\frac{1}{2}}(n-1)U(0S'LJ;SL')\times\sum(\tilde{\psi}\phi]]\psi)(\tilde{\psi}\phi']]\bar{\psi}')U(\tilde{T}tT';\bar{T}'T_2)U(\tilde{S}\sigma S\sigma';\bar{S}'S_2)U(\tilde{L}lL';\bar{L}'L_2)\times\langle\phi'(\gamma_2'T_2'S_2'L_2')\|V(1,2)\|\phi(\gamma_2T_2S_2L_2)\rangle. \quad (7)$$

In this expression, $(\tilde{\psi}\phi]]\psi)$ is the coefficient of fractional parentage for reduction of the state $\psi(l^n)$ by two particles to form the parent state $\tilde{\psi}(l^{n-2})$ vector-coupled to an antisymmetric state of two (l) particles $\phi(\gamma_2T_2S_2L_2)$. Similarly, $(\tilde{\psi}\phi']]\bar{\psi}')$ is the coefficient of fractional parentage for reduction of the state $\bar{\psi}'(l^{n-1})$ by one particle to form the parent state $\tilde{\psi}(l^{n-2})$ vector-coupled to an antisymmetric state of an (l) particle and an (l') particle $\phi(\gamma_2'T_2'S_2'L_2')$. The summation is over $\tilde{\psi}$, ϕ, and ϕ'. The γ_2 and γ_2' are inserted to account for the fact that with inequivalent particles one may have both symmetric and antisymmetric orbital wave functions with the same L_2. The function $U(abcd;ef)$ is defined as

$$[(2e+1)(2f+1)]^{\frac{1}{2}}W(abcd;ef),$$

where $W(abcd;ef)$ is the Racah function,[19] whose properties are listed and numerical values tabulated by Simon, Vander Sluis, and Biedenharn.[20] The last term in (7) is the reduced matrix element as defined by Elliott[6]: for any tensor operator $P_q^{(k)}$,

$$\langle j\|P^{(k)}\|j'\rangle=\langle jm|P_q^{(k)}|j'm'\rangle/C_{mm'q}^{jj'k}.$$

To evaluate the reduced matrix element in (7), the usual assumption is made that all the terms in (4) have the same radial dependence, which we take to be of Gaussian form. Then

$$V(1,2)=-V_0\exp(-r_{12}^2/r_0^2)\times\tfrac{1}{4}[0.01+0.01\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+0.41\mathbf{t}_1\cdot\mathbf{t}_2+0.93\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\,\mathbf{t}_1\cdot\mathbf{t}_2].$$

The spin and isotopic spin parts of the reduced matrix element can be evaluated easily by making use of the properties of Pauli spin operators. Then, applying (7) to (6), and consulting tables of coefficients of fractional parentage,[7–9] we find that

$$C_{\rm gr}=(\Delta E)^{-1}\{-3.02\langle S\|V(r)\|S\rangle-3.42\langle P\|V(r)\|P\rangle-7.16\langle D\|V(r)\|D\rangle\}, \quad (8)$$

where only the orbital parts of the reduced two-particle matrix elements, remain to be evaluated. The radial and angular parts of these may be separated by using the expansion

$$V(r_{12})=\sum_{k=0}^{\infty}V_k(r_1,r_2)P_k(\cos\omega),$$

where ω is the angle between $\mathbf{r}_1$ and $\mathbf{r}_2$. The angular integrals may now be performed by the method of Racah,[19] with the result that

$$\langle 11L\|V(r_{12})\|11L\rangle=\sum_{k=0}^{\infty}\{(-1)^L(9/2)W(1111;Lk)C_{1k1}\}\times\sqrt{2}F^k[(1p)^2;(1p)(2p)], \quad (9)$$

where $F^k[(1p)^2;(1p)(2p)]$ is the radial integral

$$\langle R_{1p}(1)R_{1p}(2)|V_k(r_1,r_2)|R_{1p}(1)R_{2p}(2)\rangle,$$

and the factor $\sqrt{2}$ arises from proper symmetrization of the radial wave functions. As is physically plausible, C_{1k1} is nonzero only for $k=0$ or $k=2$, so that there are only two terms in the summation in (9). Numerical values of C_{1k1} have been tabulated by Shortley and Fried.[21]

The choice of the Gaussian form for the radial dependence of V_{ij} and the use of harmonic-oscillator wave functions for R_{1p} and R_{2p}, which is suggested by the analysis of the electron-induced transition to the lowest excited level,[12] enable us to evaluate the radial integrals F^k by the method of Talmi.[22] Talmi's method involves transforming from the coordinates $\mathbf{r}_1$, $\mathbf{r}_2$, and ω to the coordinates $\mathbf{r}$, $\mathbf{R}$, and α, where $\mathbf{r}=\mathbf{r}_2-\mathbf{r}_1$, $\mathbf{R}=(\mathbf{r}_1+\mathbf{r}_2)/2$, and α is the angle between r and R. The radial wave functions to be used, then, are

$$R_{1p}(r)=(8/\pi^{\frac{1}{2}}a^5)^{\frac{1}{2}}r\exp(-r^2/2a^2),$$

$$R_{2p}(r)=(20/3\pi^{\frac{1}{2}}a^5)[1-(2r^2/5a^2)]\exp(-r^2/2a^2).$$

Talmi's result is that

$$F^k=\tfrac{1}{2}(2k+1)\int_{-1}^{+1}\int_0^{\infty}\int_0^{\infty}V(r)\phi_k(R,r,\alpha)R^2dRr^2drd(\cos\alpha),$$

where $\phi(R,r,\alpha)$ is a function formed by expressing the product of the radial wave functions with $P_k(\cos\omega)$ in terms of R, r, and α. Inserting $V(r)=V_0\exp(-r^2/r_0^2)$ and performing the integrations results in

$$F^0=[(10)^{\frac{1}{2}}/48][I_0-I_1+5I_2-7I_3], \quad (10)$$

and

$$F^2=[5(10)^{\frac{1}{2}}/48][3I_0-13I_1+17I_2-7I_3];$$

where

$$I_m=V_0[\lambda^2/(1-\lambda^2)]^{m+\frac{3}{2}},\quad \lambda^2=r_0^2/2a^2. \quad (11)$$

It is now necessary to decide on a value for r_0. In view of the great uncertainty in our knowledge of V_{ij},

[19] G. Racah, Phys. Rev. **62**, 438 (1942).
[20] Simon, Vander Sluis, and Biedenharn, Office of Naval Research (Washington) Report No. ONRL-1679, 1954 (unpublished).
[21] G. H. Shortley and B. Fried, Phys. Rev. **54**, 739 (1938), Table III.
[22] Igal Talmi, Helv. Phys. Acta **25**, 185 (1952).

we have contented ourselves with choosing $\lambda^2=\frac{1}{2}$. This value then leads to the ratio $L/K=\frac{1}{6}$ for the Slater integrals, the same as has been assumed, for want of a better value, by other authors.[4,18] For later use we note that with $\lambda^2=\frac{1}{2}$, $K=0.0214V_0$. Combining (10) and (11) in (9), with $\lambda^2=\frac{1}{2}$, we obtain

$$\begin{aligned}\langle P\|V(r)\|P\rangle&=+0.0476V_0,\\ \langle S\|V(r)\|S\rangle&=+0.0635V_0,\\ \langle D\|V(r)\|D\rangle&=+0.0540V_0.\end{aligned}\tag{12}$$

The energy denominator ΔE in (8) is taken to be the difference in energy of a $(1p)$ particle and a $(2p)$ particle. This neglects the interaction between shells, which is expected to have a small effect. In a harmonic well ΔE is given by $\Delta E=-(\hbar^2/2M)(4/a^2)$, and with $a=1.64\times10^{-13}$ cm,[12] this is -30.7 Mev. The first-order perturbed wave function for the ground state, Eq. (5), then becomes

$$\bar{\psi}_{\mathrm{gr};0}={}^{11}S[44]+2.41\times10^{-2}V_0\,{}^{11}S\{{}^{22}P[43],2p\},\tag{13}$$

where V_0 is measured in Mev. By a similar calculation the first-order perturbed wave function for the first excited state is found to be

$$\bar{\psi}_{\mathrm{ex};0}={}^{13}P[431]+8.02\times10^{-4}V_0\,{}^{13}P\{{}^{22}P[43],2p\}.\tag{14}$$

At this stage we note that in the LS limit, if only a scalar interaction V_{ij} is used, then, because of the factor $U(0S'LJ,SL')$ in (7), the ground state must consist entirely of ^{11}S states and the first excited state entirely of ^{13}P states. Then, since the operator Ω links only states of the same L and same S [see Eq. (2)], the monopole transition matrix element vanishes in the LS limit to all orders of the perturbation V_{ij}.

The Coulomb interaction may also be used to admix other configurations. Although it is expected to have much less effect than the residual central interaction, it can admix states having isotopic spin $T=0$, 1, or 2.[23] This, however, does not change the sense of the previous paragraph.

C. Perturbation by the Spin-Orbit Interaction

The approximate calculation of the previous section will have no meaning except near the LS limit, since in intermediate coupling all five of the states of the $(1p)^8$ configuration should be included. Therefore, the spin-orbit term in the Hamiltonian will also be treated as a perturbation. The perturbation is used to mix a small amount of the ground state, Eq. (13), with the excited state, Eq. (14), and vice versa.

The spin-orbit is given by $\alpha\sum\boldsymbol{\sigma}_i\cdot\mathbf{l}_i$, and it is seen that states of different configurations are not linked by this term. To first order in both perturbations, then, the sole contribution to the mixing comes from the matrix element

$$\langle{}^{11}S[44]|\alpha\textstyle\sum\boldsymbol{\sigma}_i\cdot\mathbf{l}_i|{}^{13}P[431]\rangle=-(8/3)^{\frac{1}{2}}\alpha.$$

The energy denominator for this perturbation is taken to be the difference between the residual interaction energies of a $^{11}S[44]$ state and a $^{13}P[431]$ state with $L=6K$, i.e., $\Delta E=\pm15.70K$. Finally the wave functions for the ground and first excited nuclear states, including both perturbations to first order, are

$$\begin{aligned}\Psi_{\mathrm{gr};\alpha}&=\Psi_{\mathrm{gr};0}-0.104(\alpha/K)\Psi_{\mathrm{ex};0},\\ \Psi_{\mathrm{ex};\alpha}&=\Psi_{\mathrm{ex};0}+0.104(\alpha/K)\Psi_{\mathrm{gr};0}.\end{aligned}\tag{15}$$

As in Sec. II, the contribution to the monopole transition matrix element arising entirely from $(1p)^8$ states vanishes. There is left (to first order in both perturbations):

$$\begin{aligned}\langle\psi_{\mathrm{gr}}|\Omega|\psi_{\mathrm{ex}}\rangle=&\,5.02\times10^{-3}V_0(\alpha/K)\\ &\times\langle{}^{11}S[44]|\Omega|{}^{11}S\{{}^{22}P[43],2p\}\rangle\\ &-1.67\times10^{-4}V_0(\alpha/K)\\ &\times\langle{}^{13}P[431]|\Omega|{}^{13}P\{{}^{22}P[43],2p\}\rangle.\end{aligned}\tag{16}$$

The matrix elements appearing in (16) may be evaluated by using Eq. (27) of Racah[6] with a calculation similar to that in Sec. II. Using $a=1.64\times10^{-13}$ cm,[12] the quantity $\langle r^2\rangle_{1p,\,2p}$ is 4.5×10^{-26} cm^2. The final result is that

$$\langle\psi_{\mathrm{gr}}|\Omega|\psi_{\mathrm{ex}}\rangle=-0.032V_0(\alpha/K)\times10^{-26}\ \mathrm{cm}^2.\tag{17}$$

This result needs some discussion with regard to the validity of the perturbation treatment. The parameter (α/K) can be made as small as one wishes, so that the spin-orbit perturbation can certainly be made valid. However, there is some question as to what value should be chosen for V_0. If one were to assume that V_{ij} is the interaction obtained from the triplet neutron-proton system,[24–27] then with the Gaussian form we have used for the radial dependence and with $r_0=1.64\times10^{-13}$ cm (which follows from taking $\lambda^2=\frac{1}{2}$) the value of V_0 would be about -60 Mev. With this value the numerical coefficient in Eq. (13) would be greater than unity, and the perturbation treatment of V_{ij} would be invalid. In fact, however, V_0 must obviously be considerably smaller than this, since most of the internucleon interaction presumably goes into the potential well in which all the particles move. Since, however, the value of V_0 is in doubt, a more exact treatment is attempted in the next section.

It should be pointed out here that core excitation has been neglected. Such effects can easily be included in the calculation if interaction between shells is

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neglected. With the perturbation approximation used in this section, however, the contribution to the final matrix element (16) from such states vanishes in the same way as that from states in the $(1p)^8$ configuration.

IV. DIAGONALIZATION OF THE INTERACTION

Since there is some question as to what value of V_0 should be used, it seems worthwhile to carry out a semiexact treatment by diagonalizing the interaction for a limited number of possible states. In this way one would hope to find some values of the parameters V_0 and (α/K) for which agreement with experiment could be obtained. The experimental facts to be matched are the energy separation of the ground and excited states (7.65 Mev)[28] and the monopole transition matrix element (3.8×10^{-26} cm^2).[3]

This diagonalization has been carried out for a group of three states. The state $^{13}P\{^{22}P[43],2p\}$ was neglected since the perturbation treatment showed its effect to be very small. The states used then are $^{11}S[44]$, $^{13}P[431]$, and $^{11}S\{^{22}P[43],2p\}$. It should be emphasized that this treatment is far from exact since so many states have been neglected. In addition to other possible states in the $(1p)^8$ and $(1p)^7(2p)$ configuration there are many possible states in the $(1p)^7(1f)$ configuration. States in this last configuration are not linked to states in the $(1p)^8$ configuration by the monopole operator Ω, but can contribute to the final matrix element and should be included in an exact treatment, as should, also, states arising from core excitation.

The results of the diagonalization treatment are unsatisfactory. It appears that the experimental energy separation can be obtained for a wide range of values of V_0 and (α/K) but that a large enough monopole transition matrix element cannot be obtained for any combination of these parameters. Lane[3] and Lane and Radicati[4] have obtained reasonable agreement with experiment in other light elements with a value of (α/K) equal to 5. With this value the correct energy separation can be obtained by taking $V_0\approx-13$ Mev; but with this combination of the parameters the monopole transition matrix element is only about one-third of the experimental value. However, no great significance should be attached to these figures because so many states have been ignored.

V. CONCLUSION

It has been shown that in order to obtain a nonzero value for the electric monopole transition matrix element in C^{12} it is necessary to include, in the description of the states involved, higher configurations of the type $(1s)^4(1p)^7(2p)$, besides the lowest configuration $(1s)^4(1p)^8$. Thus the electric monopole transition is a sensitive detector of small admixtures of higher configurations. The treatment of Sec. III, in which both the residual internucleon force and the spin-orbit interaction are treated as perturbations about the LS limit, yields for this matrix element the value $0.032V_0(\alpha/K)\times10^{-26}$ cm^2, compared with the observed value of about 4×10^{-26} cm^2. It is seen that with reasonable values for V_0 and (α/K) there is still a considerable discrepancy between this theory and experiment. It is possible that if the exact diagonalization procedure of Sec. IV could be extended to include more states this discrepancy could be removed. In that case, however, a model embodying some collective motion of the nucleons would probably be more convenient. Our calculations thus confirm Schiff's conclusion,[3] that an explanation of this transition requires a model less collective than the elastic-fluid or alpha-particle models, but more collective than the shell model.

Since this work was completed we have learned of similar suggestions by other authors. Redmond[29] has proposed that in both C^{12} and O^{16} the 0+ excited state is composed mainly of the configuration $(1s)^3(2s)(1p)^8$. Elliott[30] has suggested that in O^{16} this state is a mixture of a number of configurations involving excitation from either the $(1s)$ or $(1p)$ closed shells. The simple argument advanced by these authors is that radial matrix elements involving excitation to higher configurations are sufficiently larger than experiment (by roughly a factor of two) that only about fifty percent of these configurations would be required to yield agreement with experiment. Unfortunately the numerical values for the matrix elements used by these authors are reduced by about a factor two if the well size determined from the elastic electron-scattering is used.[12] Insertion of the latter, more correct values lessens considerably the force of their arguments. As regards the actual configurations suggested, we have found that in C^{12} configurations involving excitation of nucleons from the unfilled shell play a much stronger role than those involving excitation from the closed shell. Insofar as our calculation, which assumes only small admixtures of the higher configurations, is comparable with theirs, it indicates firstly that Redmond's choice of higher configuration in C^{12} may not be the best one, and secondly that the mechanism will yield much smaller values for the monopole matrix element in O^{16} than those it gives for C^{12}, because both shells in O^{16} are closed. We feel that the true explanation must involve some collective motion of the nucleons involved. We have received a paper by Ferrell and Vischer[31] in which excitations of the kind considered here are combined to make an 0+ excited state in O^{16} with collective properties. It appears likely from their work that such a state can be made to have the correct

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energy and perhaps the correct monopole matrix element with the ground state. It would be interesting to see the same method applied in detail to C^{12}.

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Probable Absence of K Capture in the Decay of Lead-205†

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A number of experiments designed to detect the K-capture decay of Pb^{205} are discussed. No K x-ray activity which could be ascribed to Pb^{205} was observed in deuteron-bombarded natural lead or neutron-irradiated 25.7% Pb^{204}. From these results, the L-capture half-life of Pb^{205}, and its absence in natural lead, it is concluded that Pb^{205} does not decay by K capture with a half-life in the range 2 seconds $<t_{\frac{1}{2}}<10^{10}$ years.

IN view of the recent determination[1] of the L-capture half-life of Pb^{205}, it seemed appropriate to present evidence for the lack of K capture. These data extend the K-capture half-life limits given in previous communications.[2,3]

The abundance of Pb^{205} occurring naturally has been reported to be less than 0.001%[4] of natural lead. The age of the solid shell of the earth has been estimated to be 4.5×10^9 years.[5] The abundance of Pb^{204} is reported to be $1.45\pm0.007\%$.[6] If it is assumed that the primorbial abundances of Pb^{204} and Pb^{205} were equal, one calculates from the present upper limit of 1 part Pb^{205} per 1450 parts of Pb^{204} that the half-life of Pb^{205} is $\leq5\times10^8$ years.

A K-capture half-life in the region 5 days to 10^{10} years was excluded on the basis of two experiments. In the first of these, 0.400 g of lead, enriched to 25.7% in Pb^{204},[7] was exposed to an integrated flux of 3×10^{20} thermal neutrons/cm^2 in the Materials Testing Reactor (MTR). If the thermal-neutron capture cross section of Pb^{204} is taken to be 0.9 barns,[8] one calculates that 8.0×10^{16} atoms of Pb^{205} were present at the end of the irradiation if no decay occurred during the irradiation. After suitable radiochemical purification, the sample in the form of $PbSO_4$ was counted on a NaI scintillation spectrometer. The energy scale was calibrated with the 74.7-kev Pb x-ray of 6.4-day Bi^{206}. No activity (<1 count/min) was detected in this energy region (the Tl K x-ray energy is 72.7 kev). Taking 10 disintegrations/min as the upper limit of the Pb^{205} activity, one calculates the K-capture half-life to be longer than 10^{10} years. Since the irradiated sample was counted approximately one month after the irradiation, a K-capture half-life of the order of 5 days would have been detectable.

The second experiment involved the radiochemical separation of bismuth isotopes from a 0.005-in. lead foil bombarded with 1500 microampere-hours of 21-Mev deuterons. Using the same procedure for milking lead from the separated bismuth sample as was used in the L-capture experiment,[1] we found less than 0.2 count/min of activity due to Pb^{205} in the Tl K x-ray region, corresponding to a K-capture half-life longer than 5×10^8 years.

K-capture half-lives in the region of a few seconds to several days were investigated in two separate experiments. In the first of these, two samples of lead, one of natural abundance and the other enriched to 25.7% in Pb^{204}, were irradiated in the thermal flux region of the Brookhaven reactor. The two samples were counted within 4 minutes after the end of the irradiation in a single-channel pulse analyzer—NaI(Tl) arrangement which had been preset to detect Tl x-rays. No activity was observed that satisfied the 1.45 to 25.7 abundance of Pb^{204} in the two samples.

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