spectrum of Re^{188} gives indication[22] of similar concave downward deviations from the allowed shape for the transitions to ground and first excited states of Os^{188}. We are pursuing investigation of the isotope, and of Tm^{170}, searching for smaller deviations than have thus far been sought in the latter. We feel also that a more careful investigation of the shape of the main transition in Au^{198} is merited.

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Beta-Gamma Directional Correlation in Re^{186}†

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The directional correlation between the nonunique first forbidden 934-kev beta group and the 137-kev $E2$ gamma transition in the decay of Re^{186} has been measured as a function of beta energy above 150-kev. Analysis of the experimental data included a correction for the finite energy resolution of the anthracene crystal scintillation spectrometer, which takes into account the effect of the backscattering of electrons from the crystal. The differential anisotropy increases with increasing beta energy to 0.158 ± 0.006. The directional correlation $W(\theta)=1+(0.105\pm0.003)\cos^2\theta$ for beta energies of 257–934 kev, with no evidence for a $\cos^4\theta$ term, in agreement with theoretical expectation for a first forbidden beta transition.

Limited ranges for the ratios of the nuclear matrix elements governing the beta decay were obtained by comparison with theory, assuming the following: the spin and parity sequence $(1-, 2+, 0+)$; the beta interaction mixture scalar plus tensor; point nuclear charge. Higher order terms in $2\rho/\alpha Z$ were included.

I. INTRODUCTION[1]

CONCURRENT with the investigation[2] of the shape of the beta distribution of 89-hr Re^{186}, a study was made of the directional correlation of the 934-kev beta branch with the 137-kev gamma.[3] The primary purpose of this experiment was to secure an independent determination of the nuclear matrix element ratios governing this beta decay, for comparison to those derived from beta spectroscopy. An added motive was to obtain a measurement of the correction to be applied to the magnetic spectrometer coincidence data to derive the "integrated over 4π solid angle" spectral shape. We measured both the differential (with beta energy) and integral correlations.

In the only previously reported[4] measurements of this beta-gamma directional correlation, an upper limit of 0.07 was set on the anisotropy.

The decay scheme[5] of Re^{186} is given in Fig. 1, reference 2. The 137.2-kev gamma ray is known to be $E2$ in character from the L-subshell relative conversion ratios,[6] from the K-conversion coefficient,[2,5] and from the 627-kev gamma–137-kev gamma angular correlation.[4] The lifetime of the lower 2(+) state in Os^{186} is 0.8×10^{-9} seconds.[7] From the first forbidden character of the 934-kev beta transition and the pure $E2$ character of the 137-kev gamma ray, a correlation function $W(\theta)$ of the form

$$W(\theta)=1+A(E)\cos^2\theta \qquad (1)$$

is anticipated, neglecting contributions from third and higher order forbidden terms.

II. EXPERIMENTAL

A. Apparatus

The sources were prepared from the identical supply of neutron activated and aged $KReO_4$ as used for the spectrometer sources,[2] and were similarly volatilized in vacuum onto 200 $\mu g/cm^2$ Al foils. Average source thickness over the $\frac{1}{4}$-inch diameter sources was 12 $\mu g/cm^2$. The foils were mounted on a 2-in. diameter thin aluminum ring to minimize source mount scattering.

The source holder was centered in a chamber[8] 7 in. in diameter and 7 in. high with $\frac{1}{8}$ in. thick Bakelite walls, which was evacuated to about 1 mm Hg. The plane of the source was oriented at 45° to the beta direction.

The $\frac{1}{4}$-in. thick by $1\frac{1}{4}$-in. diameter anthracene beta

† Based on work performed under the auspices of the U. S. Atomic Energy Commission.

[1] For an extended discussion of some of the details of this paper, see Novey, Freedman, Porter, and Wagner, Argonne National Laboratory Report ANL-5523, February, 1956 (unpublished).

[2] Porter, Freedman, Novey, and Wagner, Phys. Rev. **103**, 921 (1956), preceding paper.

[3] Preliminary reports of these experiments have been given; Porter, Freedman, Novey, and Wagner, Phys. Rev. **98**, 214 (1955); and Phys. Rev. **99**, 671(A) (1955).

[4] J. P. Hurley and P. S. Jastram, Phys. Rev. **95**, 627 (1954).

[5] F. R. Metzger and R. D. Hill, Phys. Rev. **82**, 646 (1951); R. M. Steffen, Phys. Rev. **82**, 827 (1951).

[6] J. B. Swan and R. D. Hill, Phys. Rev. **91**, 424 (1953).

[7] F. K. McGowan, Phys. Rev. **81**, 1066 (1951).

[8] T. B. Novey, Phys. Rev. **89**, 672 (1953).

detector crystal was coupled to an RCA 6342 photomultiplier. Resolution for the 624-kev Ba^{137m} conversion electron line was about 18%. The gamma crystal, a right cylinder of NaI(Tl), $2\frac{15}{32}$ in. in diameter by $2\frac{9}{16}$ in. high, was mounted on a DuMont 6363 3-in. cathode photomultiplier at about 1.5% gamma geometry. Resolution for the 661-kev Ba^{137m} gamma ray was 8%.

The electronic system[1] consisted of a fast-slow coincidence arrangement with single-channel pulse-height analyzers for energy selection, similar to that used in the beta coincidence spectroscopy.[2] Two distributed line amplifiers were used in both beta and gamma channels, and amplified pulses were fed into the pulse limiter of a Bell, Graham, and Petch[9] diode coincidence circuit, modified by McGowan.[10] This circuit uses a common delay-line pulse shaper and a biased G7A diode detector of coincident pulses.

The common delay-line pulse-shaping time used in these experiments was 20 mμsec. With independent sources, the resolving time 2τ was measured as 30–35 mμsec instead of the anticipated 40 mμsec. For this range of resolving times, a coincidence efficiency of >90% was realized. The Oak Ridge type single-channel pulse analyzers have a dead-time of about one microsecond.

Angular positioning and recording of data were accomplished automatically, usually at 20–40 minute intervals.

Analyzed beta counts (B), gammas (G), and triple coincidences (T) are recorded. No observable instrumentally introduced angular asymmetries greater than one percent are present, based on previous experience with the apparatus.

B. Treatment of Data

(1) Calculation of Real Coincidence Rate

Real coincidence rates (R) were derived from the formula

$$R=(T-2\tau GB)e^{-\lambda(t-t_0)}/GB. \quad (2)$$

The absolute value of the coincidence efficiency is not material for these experiments, insofar as the measurement of anisotropy is concerned, as in the comparison of real coincidences at different angles, the efficiency cancels out, if it is constant during the measurement over angles.

In the measurement of 2τ with independent sources of Re^{186} the circuits saw the identical pulse-height distributions and counting rates as those seen in the actual measurements. During the experiments the resolving time was measured daily. No dependence of 2τ on beta energy was observed.

Correction for the small variation in gamma solid angle with the beta-gamma detection angle, θ (resulting from imperfect location of the source on the rotation axis), and for radioactive decay of the sample was made by referring observed coincidences to unit analyzed gamma recorded. Drifts in the analyzed beta (B) count amounting to about 1.5% deviation from the value calculated from the 88.9-hr half-life were corrected by dividing by $Be^{\lambda(t-t_0)}$.[1]

In the experiments, maximum analyzed beta (B) and gamma (G) rates were each 200 000/min. In order to assess any rate effects, the beta spectrum of Tl^{204} was measured by using strong and weak sources so as to give maximum beta integral counting rates of 2.5×10^6 counts/min and 2×10^5 counts/min; the spectrum shape was seen to be independent of the rate over this range and to have the expected $[C_{1T}{}^{(2)}]$ form.

(2) Geometrical Corrections

For the 1.5% geometrical efficiency used, the correction for angular resolution is important. Correction formulas for circular detectors and a line source are given by Walter *et al.*,[11] by Frankel,[12] and by Feingold and Frankel.[13] Only a very small error is introduced by applying these formulas to the case of a circular source $\frac{1}{4}$ in. in diameter. The geometrical correction amounts to 10.0% of the anisotropy.

(3) Correction for Finite Beta-Energy Resolution

A major instrumental distortion of the true anisotropy arises from well-recognized effects, which in scintillation spectroscopy distort the beta spectrum distribution. These are mainly the following: (1) the statistical response of the phosphor-photomultiplier combination; (2) the finite width of the pulse analyzer "window" employed; and (3) the most important effect, backscattering of electrons from the phosphor. In principle all of these distortions can be taken into account in one procedure[14] by applying a "finite energy resolution correction" based on a manifold of line shapes of monoenergetic conversion line sources, including the backscattering "tails." Using this procedure, we have computed the true composite beta spectrum of Re^{186} from scintillation spectrometry data; the corrected spectrum matches the total spectrum observed on the magnetic lens spectrometer[2] (corrected for its resolution width, of course) to <2% above 200 kev, whereas the uncorrected spectrum deviates above the true spectrum by 20% at 200 kev.

To compute the energy resolution correction to our data, we obtained the true spectrum distribution of the main inner group of Re^{186} from the magnetic coincidence spectrometer.[2] From this spectrum and the manifold of line shapes for the scintillation coincidence apparatus, one can derive the true anisotropy $A(E)$ by a process of successive approximations. The validity of

[9] Bell, Graham, and Petch, Can. J. Phys. **30**, 35 (1952).
[10] F. K. McGowan (private communication).
[11] Walter, Huber, and Zunti, Helv. Phys. Acta **23**, 697 (1950).
[12] S. Frankel, Phys. Rev. **83**, 673 (1951).
[13] A. M. Feingold and S. Frankel, Phys. Rev. **97**, 1025 (1955).
[14] Freedman, Porter, Novey, and Wagner (to be published).

the procedure is attested to, at least insofar as the accuracy and applicability of the manifold of line shapes is concerned, by the excellence of the match of the corrected scintillation spectrum to the true spectrum. We took the coincidence efficiency as independent of beta energy for the purposes of the correction.

(4) *Beta-Energy Calibration*

For the measurement of the differential correlation (as a function of beta energy), the energy calibration of the beta pulse analyzer was based on the 624-kev conversion line of Ba^{137m}, on the L-conversion line of the 137.2-kev gamma of Re^{186} (125 kev), and on an extrapolation of the high-energy end of the Re^{186} spectrum.[1] In agreement with the results of Hopkins[15] and of Taylor *et al.*,[16] the calibration curve was linear above 100 kev with low-energy intercept of 22 kev. The calibration was checked before and after each run at a given energy.

C. Extranuclear Field Effects

For the purpose of correcting the magnetic spectrometer coincidence data, the correlation results from the solid sources of volatilized $KReO_4$ are, in principle, directly applicable, even if the anisotropy were found to suffer attenuation due to crystalline field effects. The sources used were prepared identically, and the resolving time used in either experiment (35 mμsec or 120 mμsec) is very long in comparison to the lifetime of the intermediate state, 0.8 mμsec.[7] However, if a fitting to β-γ angular correlation theory is to be attempted, some consideration must be given to the magnitude of such attenuation effects.

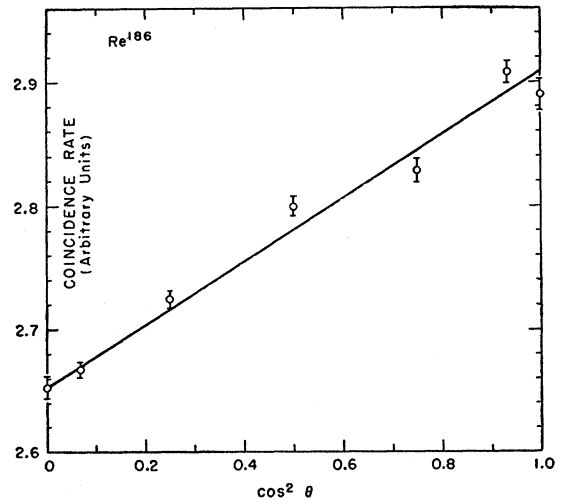

FIG. 1. Integral beta-gamma directional correlation in Re^{186} *vs* $\cos^2\theta$, for beta energies in the range 257–934 kev. The straight line is the least squares fit to the data assuming no $\cos^4\theta$ term contribution. The slope of this line is 0.0955; when corrected for finite angular resolution this corresponds to an anisotropy of 0.105 ± 0.003.

Only small attenuations are expected for lifetimes of intermediate states of the order of 1 mμsec in solid sources. Fraser and Milton[17] have measured the attenuations as a function of delay time in In^{111} and Am^{241} solid sources, from which one can deduce a half-life for attenuation of these correlations of the order of 5 mμsec. From the experimental lifetime of the intermediate state of Re^{186}, 0.8 mμsec, one would estimate an attenuation of about 5–10%, making the crude assumption that the combined influence of crystalline field gradients and nuclear electric quadrupole moments are similar in our case and theirs.

As a check on the possibility that large attenuations might be occurring in our volatilized solid sources, we examined the anisotropy for beta energies of 700–934 kev, using as a source an aqueous solution of $KReO_4$. One microliter of this solution was spread over 1 cm^2 as a thin film (1 mg/cm^2) between 0.5-mg/cm^2 rubber hydrochloride film and a 1-mg/cm^2 mica square; the anisotropy was measured in an atmosphere of helium saturated with water vapor to prevent the source from drying out. The observed anisotropy was 0.17 ± 0.015. A second experiment for beta energies of 500–934 gave an anisotropy of 0.092 ± 0.015. From the $A(E)$ curve obtained with solid sources [Fig. 3(b)] one obtains, for the average anisotropy over these energy ranges, 0.147 and 0.130, respectively. The agreement is quite poor, but perhaps indicates that severe attenuation does not occur in the solid sources. This point requires more detailed investigation. We assume in making the comparison to theory, that the results with the solid sources truly represent the unattenuated directional correlation.

D. Inner Bremsstrahlung Effect

In reference 2, it was argued that the influence of inner bremsstrahlung (I.B.) on the determination of beta spectral shapes, even for such a case as Re^{186}, is negligible because of the small I.B. intensity and the strong angular correlation of β-I.B. which discriminates against the angles used in the magnetic spectrometer. For the angular correlation experiments, however, the gamma discrimination is differential rather than integral above a lower limit, and for the case of the 137-kev photopeak of Re^{186}, only about $\frac{1}{3}$ of the I.B. spectrum above 100 kev is accepted, giving a contribution of 0.0025 for the ratio of β-I.B. coincidences to β-γ coincidences over all angles. Figure 12 of reference 8 demonstrates that about 80% of this contribution (or 0.002) will be seen at 90° and none at 180°, leading to a 0.2% reduction of the β-γ anisotropy. In comparison to the approximately 10% β-γ anisotropy, this effect was neglected.

[15] J. I. Hopkins, Rev. Sci. Instr. **22**, 29 (1951).
[16] Taylor, Jentschke, Remley, Eby, and Kruger, Phys. Rev. **84**, 1034 (1951).
[17] J. S. Fraser and J. C. D. Milton, Phys. Rev. **94**, 795 (1954).

III. RESULTS

A. Integral Anisotropy

The integral β-γ anisotropy in Re^{186} was observed by accepting beta particles of energy ≥ 257 kev. The 137.2-kev gamma peak was selected differentially so as to reduce the x-β coincidences to $<1\%$.[18] Data were taken every 15° from $\pm 90°$ to 180°; the real coincidence rate (per analyzed gamma) always exhibited symmetrical behavior about 180°. Chance rates averaged about 1% of true coincidence rates.

In Fig. 1 the correlation function $W(\theta)$ is plotted *vs* $\cos^2\theta$. The straight line, a least squares fit to the data using only the $\cos^2\theta$ term, as expected for the spin sequence involved, $1(-) \rightarrow 2(+) \rightarrow 0(+)$, corresponds

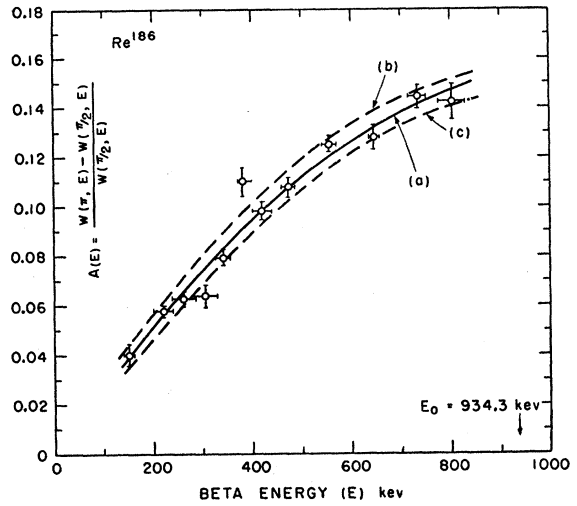

FIG. 2. The beta-gamma directional anisotropy in Re^{186} as a function of beta particle energy. The vertical flags on the experimental points represent errors due only to counting statistics. The horizontal flags represent uncertainty in the energy. All corrections except that for the energy resolution of the β detector have been applied. Curve (a) is the "best fit" to the data. Curves (b) and (c) give our estimate of extreme limits of error taking into account the counting statistics and uncertainty in the energy.

to a value for the integral anisotropy $A = 0.105 \pm 0.003$, for beta energies of 257–934 kev. A least squares fit of the data to a series including a $\cos^4\theta$ term gave the result, $W = 1 + (0.127 \pm 0.016)\ \cos^2\theta - (0.025 \pm 0.018) \times \cos^4\theta$. We do not believe that this represents a significant detection of a $\cos^4\theta$ term.

B. Differential Anisotropy

The differential correlation was examined in 100-kev increments of beta energy at angles 180° and 90° on both sides. Several Re^{186} sources were used so as to maintain counting rates within preset limits. The ratio of true/chance coincidences exceeded 7 at all energies. About 200 000 coincidences were obtained at each

[18] The 307 β-γ and γ-γ coincidences were completely negligible ($<0.001\%$).

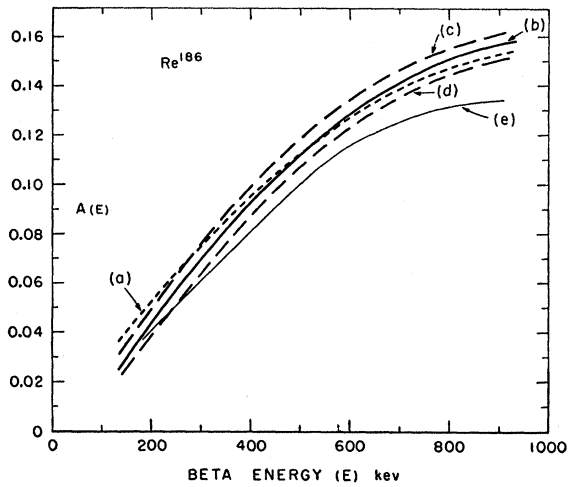

FIG. 3. The beta-gamma directional anisotropy in Re^{186} with correction for finite energy resolution of the beta scintillation spectrometer. Curve (a) repeats the central curve of Fig. 2, and transforms into curve (b) on application of the energy resolution correction. Curves (c) and (d) define the error band about (b) including a small contribution from uncertainty in the energy resolution correction. Curve (e) is that function $A(E)$ which was used in correcting the magnetic spectrometer coincidence data; it is based on preliminary measurements and is shown here for completeness.

energy in many runs. The gamma analyzer was adjusted as in the integral correlation experiment.

Figure 2, a plot of the anisotropy $A(E)$ *vs* the beta energy in kev, presents the experimental results with the vertical flags representing statistical errors only. The error contributed by uncertainty in 2τ is negligible in comparison. Horizontal flags represent uncertainty in energy calibrations. All corrections, except that for the energy resolution, have been applied to the data. The central curve (a) is drawn to represent the best fit to the data, and the upper (b) and lower (c) curves give our estimate of the extreme edges of an "error band" within which the true experimental curve is judged to lie, taking into account both the uncertainties in statistics and in energy.

Curve (a), Fig. 3 reproduces the central curve of Fig. 2; application of the correction for the finite energy resolution of the beta scintillation spectrometer transforms it into curve (b). A comparison of the curves shows that the correction is small ($<4\%$) and positive above about 550 kev and rises to about 37% (negative) at 150 kev. With a small additional contribution to the total error to account for the estimated inaccuracies in the resolution correction, the error band curves of Fig. 2 transform into those of Fig. 3, curves (c) and (d).

Curve (c) of Fig. 3 presents that function $A(E)$ that was used to compute the correction for the β-γ anisotropy to be applied to the magnetic spectrometer coincidence spectrum.[19] This curve is based on earlier measurements, with poorer statistics, and a more approximate energy resolution correction. The final

[19] See reference 2, footnote 51.

"best" curve (b) lies within the error band (not shown) of curve (e).

Integration of the corrected curve (b) from 257–934 kev leads to a value for the anisotropy of 0.101, in comparison to 0.105 ± 0.003 obtained in the integral experiment.

IV. COMPARISON OF THEORY AND EXPERIMENT

A. Theoretical Formulation

The observed β-γ directional correlation as a function of β energy in Re^{186} is here compared with the theoretical expression for this correlation, from which one obtains the ratios of nuclear matrix elements effective in the β transition.

The treatment of β-γ directional correlation as first given by Falkoff and Uhlenbeck[20] has been extended to give formulations not restricted by the approximation $\alpha Z \ll 1$ and to include the cross terms associated with a combination of the five beta-decay interactions.[21–23] Several attempts have also been made to take into account the effects of finite nuclear size.[24] In the present work we assume point nuclear charge.

The theory for the β-γ angular correlation involving first-forbidden beta decay is given by Yamada and Morita[25] and by I. Hauser.[23] In the present section we have followed the notation and formulation of Hauser as his work contains a complete formulation of the problem of interest, in particular, explicit expressions for the calculation of the quantities L_{10} and N_{10} which enter into the cross term coefficients. Specifically, we consider the case of a first-forbidden β transition followed by an $E2$ gamma transition in which the sequence of nuclear spins and parities is $[1(-), 2(+), 0(+)]$.

The β-γ correlation is expressed as

$$W(\theta,E)=\sum_k A_k(E)B_kP_k(\cos\theta), \quad k=0, 2, \qquad (3)$$

where W is the relative probability of emission of beta rays with energy E at angle θ to the gamma direction, and the P_k are Legendre polynomials. The coefficients $A_k(E)$ depend only on those variables which describe the β decay, and the B_k[26] depend only on the gamma transition ($B_0=1$).

It can be shown[1] that

$$B_2A_2(E)/A_0(E)=2A(E)/[3+A(E)], \qquad (4)$$

where $A(E)$ is the experimentally measured anisotropy defined in Eq. (1). Finally the quantities $A_2(E)$ and $A_0(E)$ must be specified. $A_0(E)/D(E)$ is just the familiar angle-independent beta-spectrum shape "correction factor," while $A_2(E)/D(E)$ is the beta-spectrum "shape factor" of the angular dependent term in Eq. (3), and $D(E)$ is the allowed beta-energy distribution $[D(E)=pEK^2F_0(Z,E)]$. In measurement of the directional correlation between beta and gamma rays only the ratio $B_2A_2(E)/A_0(E)$ is obtained.

For the currently accepted beta-decay interaction combination scalar plus tensor, as was used in fitting the spectrum shape,[2] and for a first forbidden transition from $1(-)$ to $2(+)$,[23,25]

$$\begin{aligned}A_0(E)/D(E)&=x^2(M_0+\tfrac{1}{3}K^2L_0+\tfrac{2}{3}KN_0+2L_1)+v^2L_0\\&+u^2(\tfrac{1}{8}K^2L_0+9L_1/8)+2xv(N_0+\tfrac{1}{3}KL_0)\\&+2x(M_0-L_1)+2v(N_0-\tfrac{1}{3}KL_0)\\&+(M_0+\tfrac{1}{6}K^2L_0-\tfrac{2}{3}KN_0+\tfrac{1}{2}L_1), \qquad (5)\end{aligned}$$

and

$$\begin{aligned}A_2(E)/D(E)&=4(\tfrac{3}{2})^{\frac{1}{2}}w_2(11)[x-\tfrac{1}{2}][x(N_{10}-\tfrac{1}{3}KL_{10}+\tfrac{1}{2}L_1)\\&+(N_{10}+\tfrac{1}{3}KL_{10}-\tfrac{1}{4}L_1)-vL_{10}]+3(\tfrac{3}{2})^{\frac{1}{2}}w_2(21)u\\&\times[x(N_{10}-\tfrac{1}{3}KL_{10}-L_1)+(N_{10}+\tfrac{1}{3}KL_{10}+\tfrac{1}{2}L_1)-vL_{10}]\\&-(9/8)(7/2)^{\frac{1}{2}}w_2(22)u^2L_1. \qquad (6)\end{aligned}$$

The real parameters x, v, and u, which involve the ratios of nuclear matrix elements, are to be determined from the comparison with the experimental data. They are defined as

$$\begin{aligned}x&\equiv\left(iG_S\int\beta\mathbf{r}\right)\Big/\left(G_T\int\beta\boldsymbol{\sigma}\times\mathbf{r}\right),\\v&\equiv\left(\int\beta\boldsymbol{\alpha}\right)\Big/\left(\int\beta\boldsymbol{\sigma}\times\mathbf{r}\right), \qquad (7)\\u&\equiv(i\beta B_{ij})\Big/\left(\int\beta\boldsymbol{\sigma}\times\mathbf{r}\right).\end{aligned}$$

The electron momentum is p, the total electron energy is E (mc^2 units), and $K=E_0-E$ is the neutrino energy. The atomic number of the daughter nucleus is Z. The quantities L_λ, M_λ, N_λ, P_λ, Q_λ are combinations of electronic radial wave functions as given by Konopinski and Uhlenbeck[27] and by Pursey.[28] The quantities L_λ, M_λ, etc., were evaluated from the tables of Rose, Perry,

[20] D. L. Falkoff and G. E. Uhlenbeck, Phys. Rev. **79**, 323, 334 (1950).

[21] A review of work up to 1954 is contained in the article by H. Frauenfelder, *Beta and Gamma-Ray Spectroscopy*, edited by K. Siegbahn (Interscience Publishers, Inc., New York, 1955), Chap. XIX.

[22] H. Takebe, Progr. Theoret. Phys. (Japan) **12**, 561 (1954); **12**, 574 (1954); **12**, 747 (1954).

[23] I. Hauser, doctoral thesis, State University of Iowa (unpublished).

[24] M. Yamada, Progr. Theoret. Phys. (Japan) **10**, 241 (1953); M. E. Rose and D. K. Holmes, Phys. Rev. **83**, 190 (1951).

[25] M. Yamada and M. Morita, Progr. Theoret. Phys. (Japan) **8**, 431 (1952); M. Morita, Progr. Theoret. Phys. (Japan) **10**, 363 (1953).

[26] B_k is equivalent to the symbol $A_k^{(2)}$ of Ferentz and Rosenzweig, whose tables cover the cases of transitions of pure or mixed multipolarity. [M. Ferentz and N. Rosenzweig, Argonne National Laboratory Report ANL–5324 (unpublished).] For our case of a pure $E2$ transition, B_k reduces to $F_k(202)=-0.5976$ as given by L. C. Biedenharn and M. E. Rose, Revs. Modern Phys. **25**, 729 (1953).

[27] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **69**, 308 (1941).

[28] D. Pursey, Phil. Mag. **42**, 1193 (1951).

and Dismuke[29] extended to five significant figures[30] and hence the point nuclear charge approximation is implicit in the results. Further, these tables at $Z=76$ are computed for mass number 189 rather than 186 (our case).

The notation $w_k(JJ')$ is defined as

$$w_k(JJ')\equiv(-1)^{J_b-J_a}(2J_b+1)^{\frac{1}{2}}W(J_bJ_bJJ';kJ_a), \quad (8)$$

where W is a Racah coefficient and J_a and J_b are the initial and final nuclear spins, respectively in the beta transition.

The quantities L_{10} and N_{10} are given by Hauser[31] in a form convenient for computation.

$$L_{10}=\tfrac{1}{2}\sum_{\kappa=\pm1}(L_1-\kappa P_1)^{\frac{1}{2}}(L_0+\kappa P_0)^{\frac{1}{2}}\cos[\delta_1(-\kappa E)+\delta_0(-\kappa E)+\tfrac{1}{2}\pi(\gamma_1-\gamma_0+1)+\sum_{n=1}^{\infty}S_n(g)], \quad (9)$$

$$N_{10}=\tfrac{1}{2}\sum_{\kappa=\pm1}(L_1-\kappa P_1)^{\frac{1}{2}}(M_0-\kappa Q_0)^{\frac{1}{2}}\cos[\delta_1(-\kappa E)+\delta_0(-\kappa E)+\tfrac{1}{2}\pi(\gamma_1-\gamma_0+1)+\sum_{n=1}^{\infty}S_n(g)]; \quad (10)$$

for positron emission, reverse the sign of N_{10}.

$$\delta_\lambda(\kappa E)=\arctan\{[g/(\gamma_\lambda+\lambda+1)][1-(\kappa/E)]\},$$
$$g=\alpha ZE/p, \quad \alpha=e^2/\hbar c, \quad (11)$$
$$\gamma_\lambda=[(\lambda+1)^2-(\alpha Z)^2]^{\frac{1}{2}},$$

and

$$S_n(g)=\arctan\{[g(\gamma_{\lambda+1}-\gamma_\lambda-1)]/[(\gamma_\lambda+n)(\gamma_{\lambda+1}-1+n)+g^2]\}.$$

For the calculation of $\sum_{n=1}^{\infty}S_n(g)$, the sum of the remainder of the terms past the Nth is given by

$$\sum_{n=N+1}^{\infty}S_n(g)\simeq\tfrac{1}{2}S_{N+1}(g)+(\gamma_{\lambda+1}-\gamma_\lambda-1)\times\arctan\left[\frac{g}{\frac{1}{2}(\gamma_{\lambda+1}+\gamma_\lambda+1)+N}\right] \quad (12)$$

within an error which is less than $\frac{1}{2}S_{N+1}(g)$.

B. $B_{ij}=0$

From Eqs. (5) and (6) it is seen that Eq. (4) is a quadratic in x, v, and u. We consider first the case $u=0$ ($B_{ij}=0$). Then, for any energy E and the corresponding experimental value, $A(E)$, a conic section in the x, v plane can be plotted. This graphical method of determining values for the nuclear parameters x and v is illustrated in Fig. 4. The ordinate in this figure is the same y which was introduced in reference 2. The necessary transformation is as follows:

$$y=\tfrac{1}{2}(1+S)(2\rho/\alpha Z)v-1-x, \quad (13)$$

where $S=(1-\alpha^2Z^2)^{\frac{1}{2}}$ and with Rose *et al.*[29] we take $\rho=\frac{1}{2}\alpha A^{\frac{1}{3}}$, $A=189$. Since the experimental values $A(E)$ have uncertainty, two conics are generated at each energy corresponding to the upper and lower experimental error limits [Fig. 3(c), (d)]. In Fig. 4 the bands whose boundaries are these conics are shown for energies 368 and 818 kev ($p=1.4$ and 2.4).

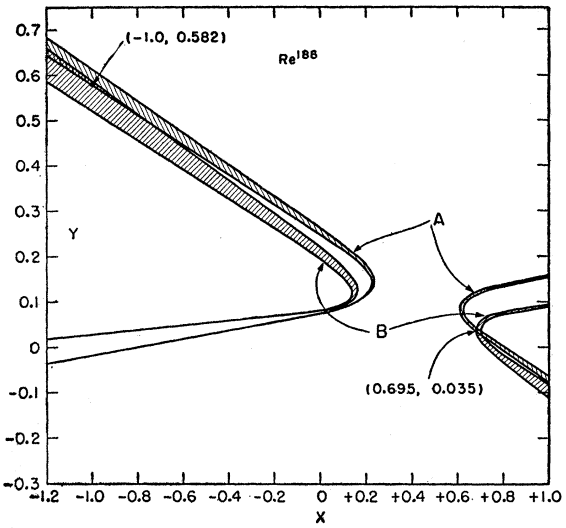

FIG. 4. Conic sections obtained in the graphical solution for the parameters x and y [Eq. (4)]. Because of experimental uncertainty a band (hatched) is generated rather than a single conic. Band A is for $p=2.4$; band B is for $p=1.4$. Points in the cross-hatched region may produce good fits to the experimental β-γ anisotropy. The theoretical anisotropies for the points indicated are shown in Fig. 5.

It is seen that the conic bands overlap in two regions within which (x,y) values lie which may give good fits to the angular correlation. Representative points from these regions are chosen to compute the anisotropy; dashed curves I, and II, Fig. 5 correspond to the points (0.695, 0.035) and (−1.0, 0.58), respectively. The solid curves reproduce the experimental error limit curves [Fig. 3(c), (d)]. It is seen that we are able to obtain an excellent comparison of experiment to theory without B_{ij} contributions.

The spectrum shape fitting for the inner group beta transition also selects (x,y) parameter values. A discussion of attempts to fit both shape and angular correlation simultaneously is given in Sec. VII of reference 2. It is shown there that only points in the neighborhood of (−1.0, 0.58) even approximately fit the spectrum shape.

[29] M. E. Rose and C. L. Perry, Phys. Rev. **90**, 479 (1953); Rose, Perry, and Dismuke, Oak Ridge National Laboratory Report ORNL-1459 (unpublished).

[30] These extended tables were derived from the raw data of the four-figure tables, ORNL-1459, by Rose, Perry, and Dismuke. The data were kindly supplied us by Dr. M. E. Rose. The necessity for the 5-figure accuracy in computing A_0 is discussed in reference 2.

[31] The L_{10} of Hauser (reference 23) is defined with the sign opposite to that of the L_{12} of Yamada. They are otherwise identical.

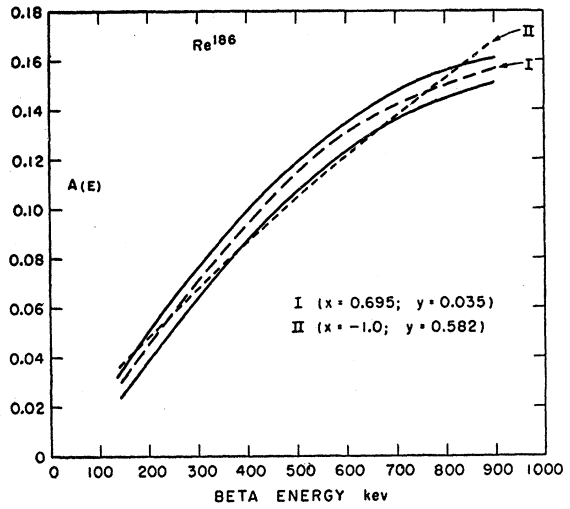

FIG. 5. Comparison of the experimental and theoretical β-γ anisotropy in Re^{186}. The solid lines are the error limits on the experimental data. The dashed curve I is for the (x,y) point (0.695, 0.035); dashed curve II is for the point (−1.0, 0.582). The parameter u is zero ($B_{ij}=0$). The parameters x, y, u are defined in Sec. IV.

C. $B_{ij}\neq 0$

An attempt was made to investigate the sensitivity of the position of the overlap regions of the conics to small admixtures of B_{ij}. Values of $u=\pm 0.1$ were assumed. The resulting conic sections for $p=0.8$ and $p=2.4$ are shown in Fig. 6. It is apparent that the regions of overlap are very sensitive to small B_{ij} contributions. For $u=-0.1$, the overlap region in the neighborhood of $x=-1.0$, $y=0.58$ is moved perhaps to give better agreement with the beta spectrum results while for $u=+0.1$, the overlap moves to regions of the x, y plane not giving agreement with the beta shape [see Fig. 17(a), reference 2]. It does not follow from this, however, that large positive values of u will not give suitable overlap regions for both experiments. Values of u of 0.1 correspond to a ratio of $|i\beta B_{ij}/\int\beta\alpha|$ of about 0.01, whereas from ft values for decays where $\int\beta\alpha$ and B_{ij} are the largest contributors one would expect values of about $\frac{1}{3}$ for this ratio. Thus one may reasonably expect even larger contributions from B_{ij} than were tried here. For $|u|<1$, the directional correlation is more affected by B_{ij} contribution than is the spectrum shape as u appears to the first power in $A_2(E)$ but only as u^2 in $A_0(E)$.

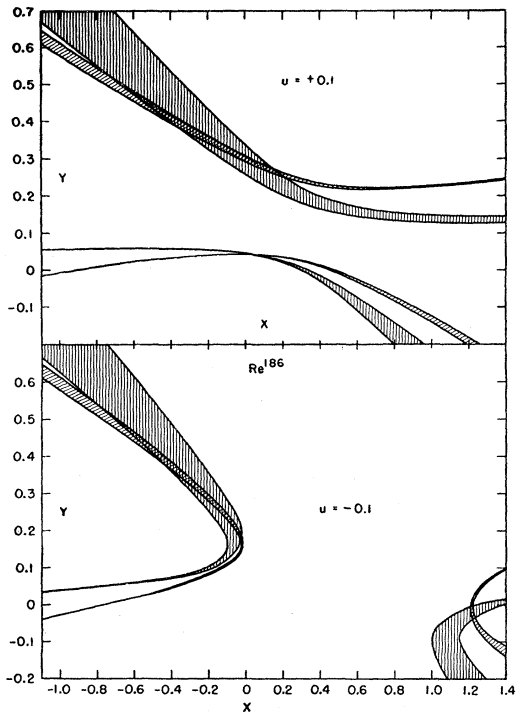

FIG. 6. Conic sections obtained in graphical solution of Eq. (4) for the Re^{186} beta-gamma correlation. Because of experimental uncertainty, bands rather than single conics are generated. These show the effects of including small contributions from βB_{ij} matrix element, i.e., the parameter $u=\pm 0.1$. The parameters x and y are defined in Sec. IV. Vertically hatched bands are for $p=0.8$; diagonally hatched bands are for $p=2.4$.

Rose and Holmes[24] have shown how extremely sensitive the important quantities L_λ, M_λ, etc., are to the nature of the nuclear charge distribution. The most obvious theoretical refinement seems to require the evaluation of these functions to the required 5-figure accuracy for various reasonable nuclear charge distributions. Were these available one might pursue with more confidence as to their physical significance the determination of the nuclear parameters.

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