investigation of the galvanomagnetic effects at very low temperatures provides a powerful tool to obtain information about surface properties of semiconductors. For very thin specimens or films, surface conduction may already be important at much higher temperatures.

ACKNOWLEDGMENTS

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Study of 1/*f* Noise in Semiconductor Filaments*

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A theory is devised for the generation of 1/*f* noise in semiconductor filaments, involving the diffusion of impurity atoms along the surface of the sample and along edge dislocations within the bulk. A set of six experiments is described which tends to corroborate this theory.

I. INTRODUCTION

ASSUMING that 1/*f* noise in semiconductor filaments is a result of a fluctuation in the sample resistance, it has been possible to construct a logically consistent theory which can explain at least the more obvious empirical features of the noise generating process. Moreover, a set of six experiments (most of which had been suggested from an inspection of the theory) were performed and produced data which, when analyzed, tended to corroborate the theory.

It is felt by the author, that the combined theoretical and experimental study has now developed to the point where a detailed description is justified. This description will, therefore, be presented in the following sections.

II. THEORY

1. Preliminary Considerations

Before the presentation of a specific theory, a general requirement for any 1/*f* noise mechanism will be pointed out. This will be used later to restrict the number of possible noise models. The observed 1/*f* spectrum in semiconductor noise can most probably be accounted for by a process which consists of a large number of independent events. For a constant impressed voltage, the occurrence of each event produces a small change in the flow of current, whose endurance will be the same as the lifetime of the event. The events must then have a very wide range of lifetimes, some extending as long as several hours. One test, therefore, of any suggested noise-generating mechanism is its ability to produce single events causing current changes of very long duration (of the order of hours). Two possible noise models proposed in the past appear to fail this test.

The first is a simple trapping process under the condition of quasi-equilibrium (i.e., where the Fermi levels for holes and electrons are equal) such as considered by van der Ziel.[1] Here a single mobile carrier (either hole or electron) is trapped, for a time depending on the energy depth of the trap involved, and then released. A wide range of trapping times, T, is implicit in the assumption of a spectrum (or band) or energy depths. This type of mechanism, however, cannot give rise to a 1/*f* spectrum, since no single trapping event can cause a current change of long duration. If T is assumed to be a very long time, it would be necessary for the current to be decreased as long as the mobile carrier is trapped. This situation is illustrated in Fig. 1(a). Actually, however, the current alteration caused by the trapping event will be as shown in Fig. 1(b). The reason for this is that a mobile carrier can give rise to current for only as long as it is free. Moreover, when a carrier is trapped for a time T, it can have been in existence only for a time τ (the lifetime of the carrier) before it was trapped, and live only a time τ after it has been released. Since in actual semiconductors $\tau \ll T$, the contribution to the current from the single carrier as a result of being trapped must be as shown in Fig. 1(b). There will effectively be two short positive pulses of duration τ instead of one long negative one of duration T. Thus, in this process, a single event is not capable of producing a conductivity change of long duration and is, as a result, insufficient to account for the shape of the low-frequency end of the 1/*f* spectrum. Apparently, then, a simple trapping process cannot produce 1/*f* noise.

The second type of noise mechanism is a simple modulation of the generation of decay rate of either

* The research in this document was supported jointly by the Army, Navy, and Air Force under contract with the Massachusetts Institute of Technology.

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[1] A. van der Ziel, Physica **16**, 359 (1950).

type of mobile carrier. If the quantities are averaged over a time long compared to τ, a relation that is always valid is

$$n = g\tau, \tag{1}$$

where n is the mobile carrier volume density, g the generation rate per unit volume, and τ the lifetime. In semiconductors in quasi-equilibrium, the mean value of n is known to be a function only of the doping and the temperature. Thus, with a sudden change of the generation or decay rate (i.e., g or τ) there may be a time period (of the order of τ) during which Eq. (1) is violated. In a period long compared to τ, however, g and τ must adjust themselves so that n returns to its value before the change. Since neither the doping (or Fermi level) nor the temperature has been assumed to change in the foregoing process, the long-time average of n cannot change. It follows that simple modulation of the generation or decay rate alone cannot account for the observed spectrum shape of $1/f$ noise.

As noted in the aforementioned, the carrier concentrations when averaged over times long compared to τ, depend on the relative positions of the Fermi level and the band edges. A noise mechanism in which a single event can cause a slight relative shift of these levels produces long lasting changes in carrier concentration (or in current at constant voltage) and is thus able to produce $1/f$ noise.

There are two plausible ways in which the above process may occur. In the first, the Fermi level would be stationary, and a portion of the band edges (corresponding to a limited physical volume) would be shifted by the single noise event. This type of process has been considered briefly elsewhere.[2,3] In the second way, the band edges do not change but the Fermi level on the whole sample is shifted by a single noise event. This latter way will be assumed to occur on $1/f$ noise in the theory that follows.

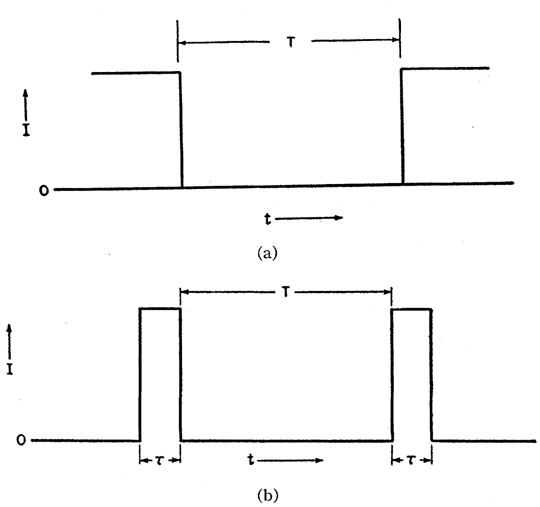


FIG. 1. (a) A long trapping event of a very long lifetime carrier. (b) A long trapping event of a short lifetime carrier.

2. General Physical Model

Before consideration of any specific physical model for noise generation, a general model will be constructed to satisfy three sets of empirical data connected with $1/f$ noise: (a) the observed shape of the noise spectrum, which has a $1/f$ behavior over the order of eight and one-half decades; (b) the observed noise level; and (c) the connection between noise and surface conditions.

The model assumes that "emission centers" are scattered at random over the surface of a germanium filament. They can emit and absorb large numbers of impurity atoms. As long as they remain on the surface, these impurity atoms act as trapping centers for holes and electrons. A typical noise event is the emission of an impurity atom from an emission center and diffusion along the surface in a quasi-Brownian motion about the center. This event will endure until the impurity atom captured either by its own emission center or by a neighboring center. During the time, T, that it diffuses over the surface, the impurity atom immobilizes by virtue of its trapping action nearly one hole or electron which would otherwise have been free to wander in the bulk. This action would cause a change in sample conductance, and a consequent change in the current through the sample for a constant impressed voltage. A random time sequence of these events produces $1/f$ noise.

From the foregoing, it is apparent that there must be a wide distribution of trapping times, T. The distribution function, $g(T)$, of these times essentially determines the shape of the noise spectrum. In this model $g(T)$ turns out to produce a spectrum with a $1/f^m$ behavior (where m can be nearly, but not necessarily equal to unity) over a very wide range of frequencies.

The detailed treatment of the spectrum shape can be found in a previous work[4] which described, however, a somewhat different model. In making the connection between the two models, one must note that the "vacancies" of the model of (I) play the role of "emission centers" here, and $\rho(t)$ is now the probability of survival of an impurity atom. Summarized, the two most important results derived from (I) are: (a) the spectral exponent, m, is determined by the atomic lattice spacing a and the capture length λ_c of an impurity atom by an emission center [the exact relation is $m = 2 - \lambda_c/(\pi a)$], and (b) the frequency range of the $1/f$ behavior is determined by the surface density of emission centers, M_c. The exact relationship between the upper radian frequency, ω_h, and the lower radian frequency, ω_l, beyond both of which the spectrum

[2] H. C. Montgomery, Bell System Tech. J. **31**, 950 (1952).
[3] W. Shockley, *Electrons and Holes in Semiconductors* (D. Van Nostrand Company, New York, 1950), pp. 342–346.
[4] Leon Bess, Phys. Rev. **91**, 1569 (1953). This will hereafter be referred to as (I).

departs significantly from the $1/f^m$ form is[5]

$$\omega_h/\omega_l=[\pi(2-m)a^2M_c]^{-1}. \qquad (2)$$

To account for the observed spectrum behavior ($\omega_h/\omega_l \cong 3\times10^8$), M_c cannot exceed about 10^6/cm^2. On the other hand (see Sec. II.4), to account for the observed noise level, the mean surface density, M_T, of trapping centers (i.e., the mean number of impurity atoms wandering on a square centimeter of the surface) must be at least of the order of 10^{11}/cm^2. This means that there must be of the order of 10^5 impurity atoms associated with each emission center. The exact relationship between M_T and M_c is given by

$$M_T=M_c g_c \bar{t}_s, \qquad (3)$$

where $\bar{t}_s$ $(=\int_0^\infty g(T)TdT)$ is the mean lifetime of an impurity atom on the surface and g_c is the average rate of emission of impurity atoms per emission center. $\bar{t}_s$ is determined by $g(T)$, the lifetime distribution function, and by using the results of (I) can be shown to be about 5×10^{-4} second. Thus, using the previously stated values of M_c and M_T (10^6/cm^2 and 10^{11}/cm^2, respectively) it is possible to calculate g_c from Eq. (3) and its value turns out to be about 2×10^8/second. It is therefore essential that any physical model for an emission center must be able to account for an emission rate of the order of 10^8 impurity atoms/second.

In order that the spectral exponent, m, be near unity, the capture length, λ_c, must have a value of about πa. Furthermore, it is probable that λ_c^2 will have a magnitude about equal to the area of extension of an emission center. Therefore the average area of extension of an emission center must be somewhere between about $4a^2$ and $10a^2$.

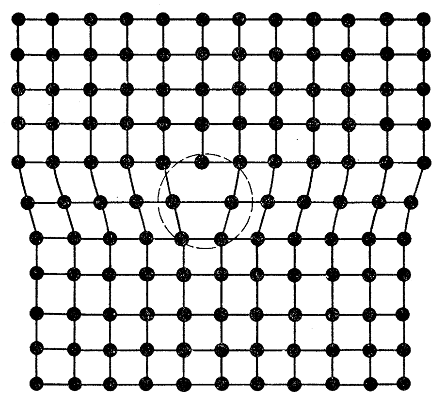
3. Emission Center Model

There are several possible physical models of an emission center which can satisfy the requirements of Sec. (II.2).[6] The particular model described and treated in detail in the remaining sections of this paper seems by far the most plausible on the basis of the following criteria: (a) predicted behavior of noise with temperature variation conforms to observed experimental results and (b) the ability of a single noise event to produce a conductivity change of long duration, a necessary attribute of any $1/f$ noise process (Sec. II.1).

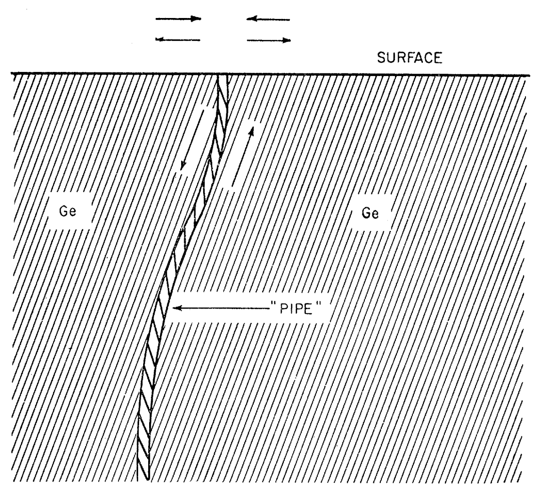
The model assumes that emission centers are connected with edge dislocation singularity lines. These are quite common in metals and have been reported in germanium by Kulin and Kurtz,[7] by Vogel *et al.*[8] and by S. G. Ellis.[9] Figure 2(a) illustrates the atom configurations in an edge-dislocation and Fig. 2(b) shows a cross section of an edge-dislocation singularity line or "pipe" (represented by the heavily shaded region).

Using an etching treatment, investigators[7] have measured the density of points at which the "pipes" come to the surface and have found them to be of the order of 10^6/cm^2. This is the order of density value required of emission centers in the noise model. Thus it may be assumed that an emission center occurs where an edge dislocation singularity meets the surface. The dotted curve of Fig. 2(a) can be considered to enclose the area of extension (roughly equal to $5a^2$) of the emission center.

The $1/f$ noise generation action can now be described in greater detail. An n-type sample will be assumed,



(a)



(b)

FIG. 2. (a) Atom configurations in an edge-dislocation. (b) Cross section of an edge-dislocation singularity line or "pipe."

[5] In deriving $\omega_h(=1/t_0)$ from (I), this will follow from Eq. (1) of (I) if it is assumed that $\langle r^2\rangle=a^2$ when $t=0$.

Instead of using the exact spectrum, $G(\omega)$, developed in (I), the following simplified approximation will be used in the calculations that follow:

$$\begin{aligned} G(\omega) &= C\omega_h^{m-1}/\omega_l^m \text{ for } \omega<\omega_l \\ &= C\omega_h^{m-1}/\omega^m \text{ for } \omega_l>\omega>\omega_l \\ &= C\omega_h/\omega^2 \text{ for } \omega>\omega_l, \end{aligned}$$

where C is a constant.

[6] Leon Bess, Technical Memorandum No. 59 of Massachusetts Institute of Technology (unpublished).

[7] S. A. Kulin and A. D. Kurtz, Acta Metallurgica **2**, 354 (1954).
[8] Vogel, Pfann, Corey, and Thomas, Phys. Rev. **90**, 489 (1953).
[9] S. G. Ellis, J. Appl. Phys. **26**, 1140 (1955).

although an analogous argument holds for a p-type sample. It has been proposed[7] that donor impurities will tend to cluster about edge dislocations and form a Cottrell atmosphere.[10] It will be postulated here, in addition, that the impurity atoms will diffuse up and down the edge dislocations with a diffusion constant, D_l. [This action is indicated in Fig. 2(b) by the arrows going up and down along the "pipe."] Those impurity atoms going in the upward direction [Fig. 2(b)] will emerge on the surface and diffuse out (with a diffusion constant, D_s) in the manner described in (I). These atoms can be considered to be "emitted" from a center as described previously. It would also follow that impurity atoms already present on the surface migrate into a "pipe" dislocation and then diffuse downward into the bulk. This is equivalent to the "capture" by a center of a surface diffusing atom. Thus, after a time, there must be an equilibrium condition in which the average rate of atoms emitted from a "pipe" must equal the average rate captured. In this condition the relationship of Eq. (3) will be valid.

That impurity atoms can diffuse from the surface into dislocations and vice versa has been found experimentally by Hendrickson and Machlin[11] using radioactive impurities in silver.

The next question to be considered is just how the impurity atoms coming to the surface will affect the concentrations of mobile carriers. It has been shown above that $1/f$ noise may be caused by a shifting Fermi level. This means that the Fermi level must change slightly whenever an impurity atom (here assumed to be an ordinary donor atom) is moved from the bulk to the surface. This shift would result if the ionization energy (E_c-E_d) of the donor atoms in the bulk were small and the ionization energy (E_c-E_{ds}) of these atoms on the surface were large as is illustrated in Fig. 3. The energy level, E_d, of the bulk donors would then be far enough above the Fermi level, E_F, so that their state of occupation, $\rho_T=\{1+\exp[(E_d-E_F)/kT]\}^{-1}$, would be very nearly zero and they would be nearly completely ionized. The energy level, E_{ds}, of the donor atoms on the surface, however, would be far enough below the Fermi level, E_F, so that their state of occupation, ρ_T, is very nearly unity and these atoms would be nearly neutral. Since the Fermi level depends principally on the temperature and the number of empty (or ionized) donors, it is apparent that it would undergo the required shift whenever a donor atom migrated from the bulk to the surface. When an atom diffused from the surface to the bulk, the shift of the Fermi level would, of course, be in the opposite direction.

The assumption that the ionization energy of a donor atom on the surface is much greater than such an atom in the bulk appears to be a very plausible one. This can

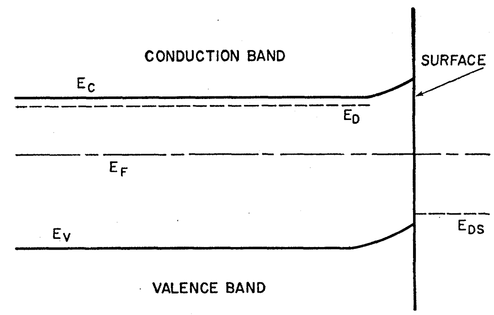


FIG. 3. Bulk and surface energy levels for donor atoms.

be supported by the following rough, simplified analysis (a reasonably exact treatment would probably be very complicated). The conventional model for a donor will be assumed here. That is, a donor can trap an electron in a hydrogen-like orbit whose ionization energy is the Rydberg constant with the nuclear charge, Z, replaced by $1/\epsilon$, where ϵ is the dielectric constant of the semiconductor. Since the ionization energy of the donor would then vary as $1/\epsilon^2$, it would be much greater on the surface than in the bulk if the effective ϵ were significantly less on the surface than in the bulk. This would be reasonable to expect since, first, there would be incomplete shielding at the surface and, secondly, the surface oxide layer on which the atom would rest would very likely have a smaller dielectric constant than the pure bulk material.

Incidentally, shifting the Fermi level will affect not only the electron concentration but also the hole (minority carrier) concentration. This can be seen from the following relation:

$$np=n_i^2. \tag{4}$$

The quantities here must be considered to be long-time averages, and the relationship is valid for all Fermi level shifts. Differentiating Eq. (4) results in the following relationship:

$$\delta p=-(p/n)\delta n. \tag{5}$$

Thus Eq. (5) shows that with every change of electron concentration, δn, there will be an opposite change in holes, δp, which will be smaller since (p/n) is less than unity.

4. Noise Output Level

If M_T is known, the noise output level can be determined. The fluctuation (or variance) $(\langle\Delta N^2\rangle)^{\frac{1}{2}}$ in the number of trapping atoms from the mean value must first be calculated. This can be obtained from simple probability theory if it is noted that the probability at any time that a given atom is on the surface is $\bar{t}_s/(\bar{t}_s+\bar{t}_l)$, while the probability that it is in a "pipe" dislocation is $\bar{t}_l/(\bar{t}_s+\bar{t}_l)$. ($\bar{t}_l$ is the mean lifetime of an impurity atom along a "pipe.") Thus from the standard binomial

[10] A. H. Cottrell and B. A. Bilby, Proc. Roy. Soc. (London) **A62**, 49 (1949).
[11] A. A. Hendrickson and E. S. Machlin, J. Metals **6**, 1035 (1954).

distribution of probability theory

$$\langle \Delta N^2 \rangle = M_T A \bar{t}_l / (\bar{t}_s + \bar{t}_l), \tag{6}$$

where A is the total surface area of the filament.

If $1/f$ noise is assumed due to a fluctuation of resistance caused by a fluctuation of surface traps as described in Sec. (II.3), the noise in any filament may be given by:

$$[\langle e_N{}^2(t) \rangle]^{\frac{1}{2}} / V_0 = \rho_T [\langle \Delta N^2 \rangle]^{\frac{1}{2}} / V_0, \tag{7}$$

where $e_N(t)$ is the noise voltage, V_0 is the dc bias impressed on the filament and N_0 is the total number of mobile carriers in the filament bulk. In general, $\rho_T \cong 1$ and $\bar{t}_l > \bar{t}_s$. Thus, with the aid of Eq. (6), Eq. (7) simplifies to

$$[\langle e_N{}^2(t) \rangle]^{\frac{1}{2}} / V_0 = (M_T A)^{\frac{1}{2}} / N_0. \tag{8}$$

If ϕ_1 is the observed noise figure of $1/f$ noise at a frequency f_1, the rms noise voltage can be shown to be given by:

$$\langle e_N{}^2(t) \rangle = 4kTR(f_1\phi_1)\Omega(2\pi f_1/\omega_h)^{m-1} \tag{9a}$$

$$\Omega \equiv [m/(m-1)][(\omega_h/\omega_l)^{m-1} + 1 - 2/m]. \tag{9b}$$

On the basis of known experimental data,[2] a typical n-type filament of resistivity 20 ohm-cm can be expected to have a noise figure, ϕ_1, of about 10 at 1000 cps, with 10 volts/cm bias and dimensions of $0.05 \times 0.05 \times 0.7$ cm. Noting that the sample resistance, R, is about 6000 ohms, $m \cong 1.1$, and $\omega_h \cong 2 \times 10^6$ sec^{-1}, the rms noise voltage can be calculated from Eq. (9a) and is found to be about 10^{-6} volt. A straightforward calculation will show that N_0 of the filament is about 6×10^{11}.

Thus, using the above values of $\langle e_N{}^2(t) \rangle$, N_0, and V_0, in Eq. (8), it appears that M_T must be about 10^{11}/cm^2 to account for the observed noise levels.

5. Variation of Noise with Temperature

In the past, one of the principal arguments against a diffusion mechanism for $1/f$ noise has been that it would necessitate a very rapid variation of noise with temperature ranging from 300°K to 100°K, whereas experimentally[2] no such variation is observed. In the following section it will be shown that, at least for the physical model considered here, a diffusion mechanism need not cause a rapid variation of noise with temperature.

First, Eqs. (2) and (6) must be studied in greater detail. It can be shown that the quantities g_c and $\bar{t}_s$ will have the following dependence on various physical parameters:

$$\bar{t}_s \cong (a^2/4D_s)\Omega, \tag{10}$$

$$g_c \cong (2D_l/a)(N_l/L), \tag{11}$$

where $N_l (= g_c \bar{t}_l)$ is the average number of atoms diffusing along a "pipe." From Eqs. (3), (10), and (11), the following explicit expression for M_T can be obtained:

$$M_T = [M_c N_m][1 + F\exp((E_l - E_s)/kT)]^{-1}, \tag{12a}$$

$$F \equiv 2L/a\Omega. \tag{12b}$$

$N_m [= g_c(\bar{t}_s + \bar{t}_l)]$ is the average maximum number of atoms clustering about a "pipe" and is assumed to be a constant.

From Eqs. (6), (7), and (12a), the rms noise voltage takes on the following form:

$$\langle e_N{}^2(t) \rangle = (V_0/N_0)^2 (M_T A)[1 - M_T/(M_c N_m)]. \tag{13}$$

The behavior of $\langle e_N{}^2(t) \rangle$ with temperature may be deduced roughly from Eq. (13). It will be assumed that the variation of N_0 and V_0 with temperature is small compared to the variation in M_T. Since the values of g_c and ω_h are known to be approximately 2×10^8/sec and 2×10^6/sec, respectively, D_s and D_l can be calculated from Eqs. (11) and (14). The results of this calculation are that $D_s = 5 \times 10^{-10}$ cm^2/sec and $D_l = 10^{-7}$ cm^2/sec. Knowing D_s and D_l and assuming a diffusion coefficient of 10^{-13} sec^{-1}, one may calculate the activation energies, E_s and E_l. The results are $E_s = 0.385$ ev and $E_l = 0.271$ ev. Thus $E_l - E_s = -0.114$ ev. Some rough representative values for the parameters F, N_m, and L can be estimated to be $F = 10^4$, $N_m = 3 \times 10^6$, and $L = 10^{-2}$ cm.

From the above set of values in Eqs. (12a) and (13), rough plots were made in Fig. 4 of M_T/M_c and $\langle e_N{}^2(t) \rangle$ *vs* temperature in the range of 100°K to 300°K. The dashed line illustrates the variation of M_T with temperature. At room temperature (300°K), M_T/M_c is about 10^5, and it increases to its maximum value, $N_m (= 3 \times 10^6)$ as the temperature decreases to 100°K. This means that all the impurity atoms associated with the "pipe" dislocations migrate to the surface as the temperature decreases.

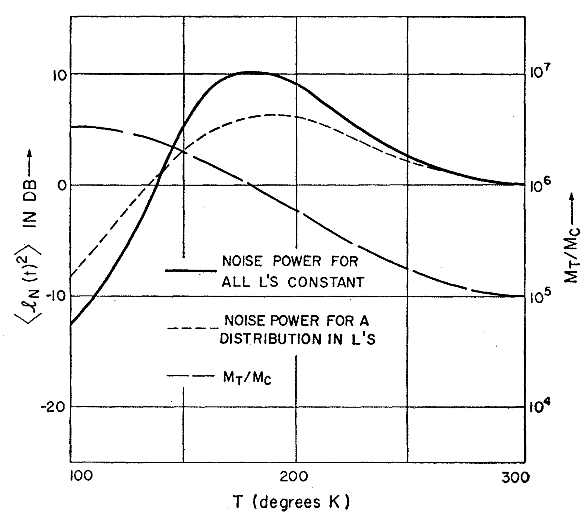


FIG. 4. Theoretical temperature dependence of noise power and surface trap density.

The solid line in Fig. 4 illustrates the behavior of the noise power with temperature; the length, L, of all the "pipes" is assumed constant. Actually, it seems reasonable to suppose that the L's for different "pipes" vary according to a distribution of lengths. Now, it should be noted that for a given L, the noise *vs* temperature characteristic will be a maximum when $M_T=\frac{1}{2}M_cN_m$. The temperature at which this is so will depend on L. Thus, if the length of the "pipes" varied over a wide range (say, from 10^{-1} to 10^{-3} cm), the variation of noise power with temperature would probably resemble that shown by the dotted line in Fig. 4. Although the detailed shape of the characteristic would depend on the distribution function of the L's, the general effect of a variation in the "pipe" lengths may be said to reduce the variation of noise power with temperature in the low-temperature region.

It can be seen here that $\langle e_N^2(t)\rangle$, the noise over the total frequency band, was plotted in Fig. 4, whereas experimentally only the noise in a narrow frequency band is measured. In attempting to compare Fig. 4 with the experiment, it should be noted that the noise in any narrow frequency band will nearly be proportional to $\langle e_N^2(t)\rangle$ as long as the frequency about which the band is centered is less than the upper turnover frequency, ω_h. The proportionality factor will actually be slightly temperature-dependent, but this can be neglected in this rough treatment.

For sake of simplicity, the above development has omitted one phenomenon which probably affects the behavior of noise power with temperature variation. This effect will be described qualitatively here. There is good reason to believe that the activation energy, E_s, for the surface diffusion constant, D_s, will decrease with decreasing temperature. This is because E_s is a function of the concentration[12] of surface diffusing atoms, decreasing its value with increasing concentration. For concentrations of the order of $10^{13}/\text{cm}^2$, the decrease can be as much as 10%. In the diffusion situation here, the atom concentration will vary inversely with the distance to an emission center. The variation in concentration in going from a region immediately surrounding an emission center to a region in between centers will be of the order of 20 to 1. Thus, if the average concentration is of the order of $10^{11}/\text{cm}^2$, the concentration right next to an emission center (where D_s is mostly determined) will be about $2\times10^{12}/\text{cm}^2$. As the temperature decreases, the concentration as determined from Eq. (12a) will tend to go up (by a factor of about 30 in the case of Fig. 4). Thus at 100°K the concentration right next to an emission center (where D_s is mostly determined) can be expected to be of the order of $10^{14}/\text{cm}^2$. It is therefore possible for E_s to have undergone a sizable decrease (perhaps of the order of 50%). The decrease in E_s tends to oppose the increase in concentration, as an inspection of Eq. (12a) will show, and will consequently affect the variation of concentration with temperature.

The variation of E_s with temperature, besides being noticeable in the temperature variation of noise, will have an even more marked influence on the variation of the upper turnover frequency, ω_h, with temperature. Instead of having an $\exp(E/kT)$ type of dependence on temperature, as might be expected from an inspection of Eq. (14), the variation of ω_h with temperature will be much slower, although still sizable.

The result of all of the above discussion is to show that, although an exact analysis of noise variation with temperature is very complicated, it is possible to obtain a rough, first-approximation solution which admits of the following statements. First, the variation of $1/f$ noise power with temperature (in the range of 100°K to 300°K) need not be very great (certainly within a range of 20 db). Second, the general shape of the characteristic curve will be as shown by the dotted line in Fig. 4 (assuming that the variation of N_0 with T is small). These results seem to agree reasonably well with experiment.[2]

III. EXPERIMENTAL CORROBORATION

1. List of Experiments

The following experiments provide information related to the validity of the noise theory described above.

Experiment No. 1

With regard to the theory of Sec. II, one of the most important findings by experimental investigators[2,13,14] is that the spectral exponent m (i.e., the slope of the spectrum on a log log plot) is not exactly equal to unity but can vary between samples from about 0.95 to about 1.33 at room temperatures. At lower temperatures, m has been observed to go as low as 0.80. To the present time, only the theory of Sec. II seems to explain in the simplest way the variation of m. As has been shown above, m is a function of two physical parameters, a and λ_c. It seems reasonable that the capture length λ_c, in particular, vary from sample to sample and probably be dependent on temperature.

Experiment No. 2

Figure 5[15] shows the spectra of a germanium filament at lower temperatures. It will first be noted by referring to (I) that the upper turnover frequency $\omega_h(=1/t_0)$ can be brought into the following form:

$$\omega_h=4D_s/a^2. \tag{14}$$

Thus, from an inspection of Eq. (14) it would be ex-

[12] R. M. Barrer, *Diffusion in and Through Solids* (The Macmillan Company, New York, 1941), p. 373.

[13] B. F. Rollin and I. M. Templeton, Proc. Phys. Soc. (London) **B66**, 259 (1953).

[14] K. M. van Vliet *et al.*, Physica **20**, 481 (1954).

[15] The data for Fig. 5 was obtained by H. A. Gebbie and appeared in the Group 35 Lincoln Laboratory Quarterly Progress Report of February 1, 1955 (unpublished).

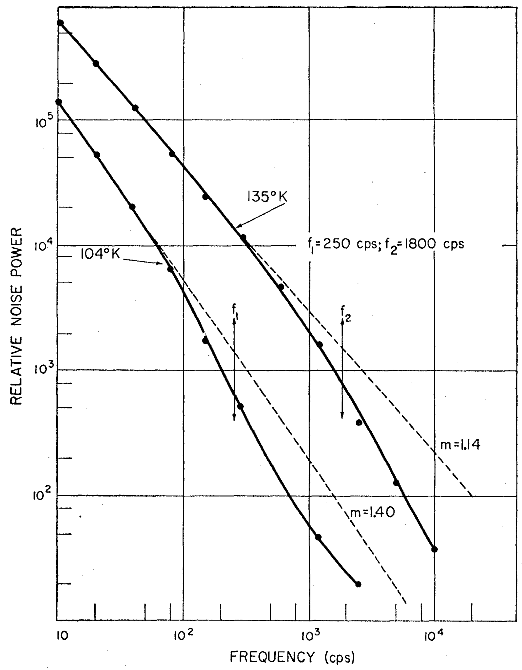


FIG. 5. Noise spectra of an etched n-type (about 20 ohm-cm) germanium filament at lower temperatures.

pected that ω_h should vary rapidly with temperature, decreasing as the temperature decreased, since D_s has this kind of behavior and a is insensitive to temperature. However, as pointed out in Sec. (II-5), the variation of ω_h with temperature need not be as rapid as that expected from an $\exp(E/kT)$ behavior.

The two curves in Fig. 5 both have straight portions (whose extensions are shown by the dotted lines) at the low end of the spectrum. Going toward the upper end of the spectrum, the curves start bending below the dotted extensions with a curvature that is always concave downward. One should note that this type of shape cannot be obtained by adding a shot-noise type of spectrum (or a finite number of shot-noise types of spectra) to a $1/f$ type of curve. Here, in deviating from the $1/f$ form, the resultant curve would first have a concave upward portion rising above the straight-line extension followed by a concave downward portion falling below the straight-line extension. (There would be a number of such portions if more than one shot-noise spectrum were added.)

The most plausible explanation of the shape of the curves of Fig. 5 seems to be that the deviation from the $1/f$ form (i.e., the deviation of the solid curves from the dotted lines) is due to the fact that the upper turnover frequency, ω_h, has been exceeded. A study of the spectrum form as developed in (I) reveals that in the frequency region above ω_h, the spectral curve would always be concave downward and fall below the straight line as shown in Fig. 5.

The upper turnover frequencies will be taken as the point where the actual curve falls 3 db below its straight line extension and are indicated in Fig. 5, by the double arrows.

Thus, Fig. 5 reveals that at 135°K the upper turnover frequency is about 1.8 kc/sec, and at 104°K it is about 250 cps. An indication of what the upper turnover frequency is at room temperature can be obtained by referring to Fig. 1 and Fig. 5 of reference 2, where curves are exhibited with the characteristic concave downward shapes at their upper ends. These figures seem to show that at room temperature upper turnover frequency is about 300 kc/sec.

The data of Fig. 5 thus seems to indicate that in going from about 300°K to 135°K and 104°K the upper turnover frequency has continuously decreased by a large amount (a factor of about 1000 for the case of 104°K). Moreover, there is a substantial decrease (a factor of 7.2) in going from 135°K to 104°K.

As already stated previously, this is the kind of behavior with temperature that would be expected of ω_h in the form given by Eq. (14) which has been derived from the theory of Part II.

Experiment No. 3

The information pertinent to this experiment is all shown in Fig. 6.[16] Here the solid lines represent the spectra of a germanium filament in two different ambients. As can be seen, the noise spectrum of the filament changed greatly not only in level but also in shape when the ambient was changed from dry nitrogen gas to liquid carbon tetrachloride. The temperature in both cases was about 300°K.

The spectral curve for the dry N_2 ambient can be explained fairly readily as a resultant of the addition of a shot-noise spectrum to a $1/f$ spectrum. In order to explain the spectral curve for the CCl_4 ambient, it must first be noted that shot-noise component of the total noise will not change very greatly with the change in ambient. This is because a change in ambient can only be expected to affect the surface of the filament. According to van der Ziel's[17] theory of shot noise, the only physical parameter depending on surface conditions and influencing shot noise was the minority carrier lifetime. This was measured with the changing of ambients and was found to vary only about 30%. Thus, for the purposes here, it can be said that the shot-noise component in the CCl_4 curve was the same as that of the N_2 curve. The shot noise of the N_2 curve can readily be evaluated and subtracted from the CCl_4 curve to give a resultant curve shown by the heavy dashed line. It is this resultant curve of the heavy dashed line which represents the spectrum connected with the $1/f$ noise process.

An inspection of this resultant curve shows that it has a much higher level than the $1/f$ component of

[16] The data for Fig. 6 were taken from Maple, Bess, and Gebbie, J. Appl. Phys. **26**, 490 (1955).
[17] A. van der Ziel, J. Appl. Phys. **24**, 1063 (1953).

the N_2 curve and that it has the characteristic concave downward shape at the upper end of the spectrum which was described in Experiment No. 2. The dotted line is the extension of the straight line portion at the lower end. The explanation of the CCl_4 curve, therefore, appears to be that in immersing the filament in liquid CCl_4 the level of the $1/f$ component was greatly raised while at the same time the upper turnover frequency was greatly decreased. The double arrow marking the 3 db point below the dotted extension indicated an upper turnover frequency of about 700 cps. An inspection of the curves show that the noise power level has been raised by a factor of about 200 at 10 cps. Assuming again that the upper turnover frequency at room temperature (and an ambient of gaseous N_2) is about 300 kc/sec, it appears that the upper turnover frequency has been decreased by a factor of about 400.

A study of Eqs. (14), (2), and (10) reveals that the theory of Sec. II predicts that, if all other parameters are constant, ω_h should vary directly as D_s, while M_T should vary inversely as D_s. Moreover, Eq. (8) shows that the noise power should be roughly proportional to M_T.

Thus, the CCl_4 curve of Fig. 6 can be explained if it is assumed that in immersing the germanium filament in liquid CCl_4 its surface diffusion constant, D_s, was lowered by a factor of the order of several hundred.

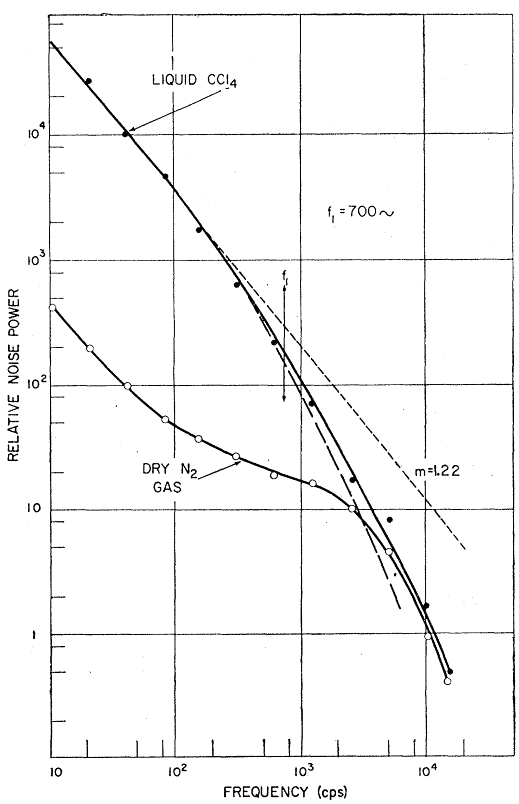


FIG. 6. Noise spectra of an etched n-type (roughly 30 ohm-cm) germanium filament for different ambients.

This should then cause the noise level to be raised by this factor while the lower turnover frequency would be decreased by the same factor. In the actual measurement the increase in noise level was reasonably close (considering the roughness of the measurements) to being the same as the decrease in turnover frequency (a factor of 200 as compared to 400). The fact that immersing a germanium filament should greatly decrease its surface diffusion constant is very reasonable on other physical grounds.[18]

Experiment No. 4

An experiment was devised[19] to show relative influence of the fluctuations of minority and majority carrier concentrations on excess noise in semiconductors. The experiment consisted of noise voltages being measured along various directions in a germanium ribbon carrying a current parallel to the direction of the ribbon. The ribbon was placed in a strong magnetic field whose lines of force are perpendicular to its face. A plot was made of the noise voltage *vs* the angular position of the line connecting the two measuring probes. The noise measurements were performed at the low frequency of 80 cps where $1/f$ noise predominated in the excess noise and at 10 kc/sec where the shot noise predominated.

When the germanium ribbon was inserted in the magnetic field, the Hall effect took place and the directions of the electron current, the hole current, the total current, and the resultant electric vector were all different. By observing the angular position for which the noise voltage is a minimum, in the plot of noise voltage *vs* angular probe line position, and by knowing the directions of the four vectors listed previously, it was possible to calculate the relative proportions of minority and majority carrier fluctuations in the total noise. It can be shown, for example, that when the noise is mostly majority carrier fluctuation the noise voltage will be a minimum when the probe line direction is perpendicular to the resultant electric field vector. As the influence of minority carrier fluctuation begins to be felt more in the total noise, the angular position of the noise voltage minimum shifts toward the direction of minority carrier current.

When the foregoing experiment was performed, it was found that at 80 cps, where $1/f$ noise predominated, the total noise was due almost entirely to majority carrier concentration, while at 10 kc/sec it was due partly to majority carrier fluctuation and partly to minority carrier fluctuation as would be expected from van der Ziel's[15] theory. The fact that $1/f$ noise was caused mostly by majority carrier fluctuation is consistent with the theory of Sec. II which predicts this in Eq. (5).

[18] This was revealed in a private discussion with Professor C. W. Wagner of Massachusetts Institute of Technology.

[19] L. Bess, J. Appl. Phys. 26, 1377 (1955).

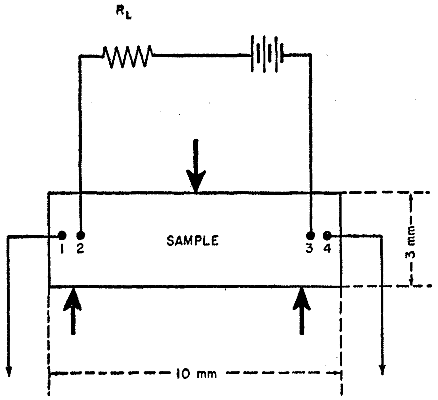


FIG. 7. Geometry and wiring arrangement of a germanium sample.

Experiment No. 5

In this experiment, an n-type germanium slab 10 mm $\times$3 mm$\times$0.25 mm had four contacts soldered to it as shown in Fig. 7. Current was sent through contacts number 2 and 3, and the noise voltage at 80 cps was to be measured between contacts number 1 and 4. The object of the experiment was to see if $1/f$ noise varied with mechanical stress on the sample. A special vise was constructed so that a uniformly distributed compressive force could be applied to the long thin faces of the sample (as shown by the heavy arrows in Fig. 7). When pressure was applied, the stress in the sample was about 800 lb/sq in.

In performing the experiment, noise voltage was measured first with no pressure and then with pressure applied. This procedure was repeated over about seven cycles. To insure the fact that pressure on the solder contacts was not causing a significant part of the effect, a test was devised where noise measurements were made between the contact pairs 1 and 2 and also between 3 and 4 both with the pressure on and with the pressure off. The noise voltage between both of these contact pairs was to be negligible to that between contacts number 1 and 4 before the measurement was accepted.

The experiment was performed for four different samples. In each case there was a change of noise voltage with mechanical stress along with a change of resistance. Before the measurement was accepted, the change of sample resistance with mechanical stress must have been less than 1%. (All of the four samples mentioned above met this test. In general the change of sample resistance with mechanical stress was greater than 1%, being sometimes as high as 10%.) The sample resistance would always increase with applied pressure, whereas the noise voltage could either increase or decrease. The change in noise power was, on the average, about 20%. This value was about 5 times too great to be accounted for by just a resistance change even when sign of the change was in the right direction.

In going from cycle to cycle, the change of noise power with pressure was about the same for three samples, whereas in the other it decreased to less than the accuracy of the measurement (which is about 5%) by the time the seventh cycle was reached. In two samples there was a progressive change in noise power (at no pressure) from cycle to cycle so that at the seventh cycle a change of about 25% had taken place. In the other two samples there was no significant change of noise power from cycle to cycle.

Table I shows the data for sample number 512 which, as can be seen, was a sample where the noise change with pressure was about constant from cycle to cycle and where there was no significant progressive change of noise power from cycle to cycle.

In analyzing the foregoing data, it would seem safe to conclude the mechanical stress can produce a change in $1/f$ noise which cannot be accounted for by just a change in resistance. Moreover, it is almost certain that the noise change is due to some bulk effect, since the fact that no pressure is ever applied to the large sample surfaces and also the fact that the applied stress is relatively so small makes the existence of a surface effect very improbable.

It is suggested here that the main effect of the mechanical stress is to change the length and configuration of the edge dislocations in the sample. An examination of Eqs. (8), (12a), and (12b) in the theory of Part II will reveal that the noise level is dependent on the length of the edge dislocations and thus any effect which causes their length to change will also change the noise level.

However, it is to be emphasized that the most important result that is claimed for this experiment is that it shows that a bulk effect in the sample (other than a simple conductivity change) can influence $1/f$ noise.

Experiment No. 6

In this experiment, an attempt was made to find a direct connection between edge dislocation density and $1/f$ noise level. In the first phase of the experiment, an n-type sample of the shape shown in Fig. 7 had its two large surfaces freshly ground and then connected to four leads by solder contacts (also as shown in Fig. 7).

TABLE I. Data for the variation of noise with mechanical pressure in an n-type germanium filament.

Sample No. 512	$I=10.5$ ma		$R=510$		Pressure $=800$ lb/in.^{2}		
Cycle No.	1	2	3	4	5	6	7
Noise voltage (in 10^{-7} volt)	1.13	1.09	1.20	1.18	1.18	1.20	1.16
Percent change of noise power with applied pressure	−23%	−17%	−25%	−28%	−28%	−27%	−30%
Percent change of resistance with applied pressure	1%	2%	−1%	0%	0%	0%	0%

TABLE II. Data for the variation of noise with heating and plastic deformation in germanium filaments.

	Sample No. 520 $I=10.5$ ma				Sample No. 537 $I=10.5$ ma			
Phase No.	Heating temp.	Noise voltage (10^{-7} volt)	dc voltage (volts)	M_T	Heating temp.	Noise voltage (10^{-7} volt)	dc voltage (volts)	M_T
I		1.3	7.8	3.8×10^{11}		0.33	5.7	8.0×10^{10}
II	650°C	1.03	8.3	1.9×10^{11}	600°C	0.30	6.8	3.7×10^{10}
II′					650°C	0.25	6.7	1.9×10^{10}
III	800°C	1.48	5.7	1.7×10^{12}	700°C	1.39	5.7	1.3×10^{12}

A given value of dc current, I, was sent through the probes 2 and 3. Noise voltage and dc voltage measurements are then made between the probes 1 and 4. The noise voltage is measured at 80 cycles where $1/f$ noise dominates in the excess noise. To ensure that no significant part of the measured noise arose from the contacts, the contact test described in Experiment No. 5 was performed.

In the second phase of the experiment, the sample was heated for about 30 minutes in an oven at a temperature between 600°C and 700°C and then rapidly cooled. After the large surfaces had been freshly ground, measurements of noise voltage and dc voltage were taken on the sample in exactly the same way as in the first phase.

The third phase of the experiment consisted of heating the sample to a temperature between 700°C and 800°C and at the same time subjecting it to a bending stress. The bending stress was created by supporting the sample at its two ends and applying a weight of about 3 ounces in the middle. Three forces would then be acting on the sample as shown by the heavy arrows in Fig. 7. The sample was heated for about 30 minutes and then rapidly cooled and should at this time have been plastically deformed. The large surfaces were then again freshly ground and measurements were made of noise voltage and dc voltage just as in the two previous phases.

The preceding procedure was performed on two samples and the results are summarized in Table II. The phase II′ in Table II is where the sample was heated again without plastic deformation being produced. In order to obtain a meaningful physical parameter for comparison, M_T, the trap density on the surface, was calculated from Eq. (15) below which in turn was derived from Eq. (8).

$$M_T \cong [I^2\langle e_N{}^2(t)\rangle/2QV_0{}^4][n_i\rho_i]^2[l^3/w][\tfrac{1}{2}(1+1/b)]^2, \quad (15)$$

where

$$Q=(\Delta\omega/\omega)(\omega_h/\omega)^{m-1}/\Omega; \quad (16)$$

l and w are the length and width of the sample; n_i and ρ^i are the carrier concentration and resistivity for intrinsic germanium; Q is the factor introduced because the measurement occurs over a limited band width, $\Delta\omega/2\pi$ (4 cycles), instead of the whole frequency range; b is the usual mobility ratio.

In studying Table II, it can be seen that heating the samples without deforming them seemed to lower the effective M_T about a factor of 2. On the other hand, heating the samples and producing a plastic deformation seemed to increase the effective M_T enormously (a factor of about 9 for sample No. 520 and about 50 for sample No. 537).

This behavior can be understood in light of the theory of Sec. II. Heating the samples without deformation would, in general, introduce no new dislocations but would tend to drive away some of the impurities from the Cottrell atmospheres of the dislocations already present. The critical temperature at which this happens is about 500°C in germanium, and the samples in this case were always heated beyond this point. The cooling was so rapid (less than 3 minutes) that the impurities driven away had no chance to diffuse back. Thus, at the end of the heating cycle, the density of the Cottrell atmospheres were decreased. This would cause a lowering of the quantity g_c in Eq. (2) which would then cause M_T (and consequently the noise power) to be lowered. When the samples were heated and caused to undergo plastic deformation, a great many new dislocations were introduced [thus increasing M_c in Eq. (2)]. The density of the Cottrell atmosphere would not have changed much here since it had already been depleted in the previous phase; thus g_c would be about the same as for the previous phase. The net effect of this is to cause a marked increase of M_T (which should be about the same as the increase in dislocation density).

A rough measurement was made of the radius of curvature of the bends associated with the plastic deformations of the samples. These radii turned out to be about 3 cm for sample No. 520 and about 7 cm for sample No. 537 (which had buckled and bent on an axis parallel to the large faces instead of perpendicular to them).

It has been pointed out that heating a sample and causing it to undergo plastic deformation could introduce a polygonization process where the dislocations meet the surface in an orderly line of points instead of points scattered at random. This should not change any of the results described above. The main result of this sort of an effect would be to make the lower turnover frequency, ω_l, higher than would otherwise be the case since the average distance between points of emergence would be less.

Although a formula for calculating the dislocation density introduced in a plastic bend has been developed by Cottrell,[20] it would not be too meaningful here to try to calculate the dislocation density increase and correlate it to the noise increase. In the first place, Cottrell's formula only gives the increase in dislocation density so that the initial dislocation density would have to be known. Secondly, the ratio M_T/M_c is a complicated function of many factors and is thus very difficult to estimate accurately.

However, assuming the ratio, M_T/M_c, to be of the order of 10^5 (as developed in Part II) the M_T's as calculated from Cottrell's formula are certainly within a factor of 2 or 3 in magnitude of the M_T's obtained from Table II.

The most important result[21] claimed for Experiment No. 6 is the qualitative fact that a plastic deformation in a sample will markedly increase $1/f$ noise.

2. Discussion of Results

In assessing the merit of the list of experiments described above, a few qualifying remarks need to be made. It is realized that some of the experiments considered by themselves are somewhat incomplete either because their data are just from one sample (such as Experiment Nos. 3) or because they are more or less qualitative in nature (such as Experiments No. 5 and 6). It is hoped that these experiments may prove to be an inspiration for further work by other investigators. Moreover, the author is aware of the fact that any one experiment may have alternative explanations other than those supplied by the theory of Part II. However, the alternatives for one experiment will probably not be the same as the alternatives for another; and, in all of the six experiments, some consideration on the matter has shown that it will be at least unlikely that there will be an alternative explanation which is common to the set. Thus the omission of alternative explanations is felt justified.

The set of experiments must be considered as a unit. Each of six diverse experiments pointing in the same direction should have a cumulative effect in which a number of single evidences adds up to a strong total evidence.

In summary, Experiments Nos. 1, 2, and 3 tend to confirm the validity of the surface diffusion action described in the general physical model of Part II. Experiment No. 4 tends to verify the assumption that $1/f$ noise is caused by a shifting of the Fermi level. Experiments Nos. 5 and 6 would seem to lend weight to the view that $1/f$ noise has a direct connection with the edge dislocation densities within the bulk.

Taken together, the conclusion seems warranted that the six experiments provide evidence for the validity of the essential features of the $1/f$ noise theory described in Part II.

IV. ACKNOWLEDGMENTS

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[20] A. H. Cottrell, *Dislocation and Plastic Flow in Crystals* (Oxford University Press, Oxford, 1953), p. 29.

[21] This result has also been obtained by J. J. Brophy [Phys. Rev. **100**, 1261 (1955)].