change in the height of the Coulomb barrier, and the distortion of the proton wave function, give essentially negligible changes in the ratio of proton to neutron cross sections at 10 Mev.

It is thus seen that the dependence of the total cross section on energy, as well as the dependence of the differential cross section on angle, indicate the presence at low energies of an additional process, possibly virtual state formation.

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Alpha-Particle Model of C^{12}

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The α-particle model for C^{12} has been re-examined. In addition to correlating the 0^+, 2^+, and 0^+ states at 0, 4.43, and 7.65 Mev, respectively, two possible identifications are given for the 9.61-Mev level: 1^- or 2^+. These levels completely determine the model, and the position and character of all levels up to 15 Mev are given. The main defect of the model is its prediction of a 3^- state at 5.53 Mev which has never been observed. The separation of the α particles in C^{12} is 3.7×10^{-13} cm and the mean zero-point kinetic energy per vibrational degree of freedom is about 2 Mev.

WHEN the α-particle model was first discussed, it was impossible to evaluate in detail its predictions of level schemes for light nuclei because of insufficient experimental information. This situation is now greatly improved. Dennison,[1] for example, has correlated a considerable number of states in O^{16} with this model. To determine whether the agreement is restricted to just this nucleus, the α-particle model for C^{12} has been re-examined. The physical basis of the α-particle model will not be discussed here,[2] although it is certainly open to question, nor will its position in the over-all theory of nuclear structure be evaluated.

In the α-particle model of C^{12} the equilibrium configuration is an equilateral triangle of side s with the α particles at the vertices. Only small displacements from equilibrium are considered and it is assumed that rotation and vibration are separable. The potential energy is

$$V=\tfrac{1}{2}\alpha(Q_1^2+Q_2^2+Q_3^2)+\beta(Q_1Q_2+Q_1Q_3+Q_2Q_3),\quad (1)$$

where the internal coordinates Q_1, Q_2, and Q_3 are length changes of the sides of the triangle. The constants α and β will be determined from the observed energy level spectrum. The frequencies of the familiar normal vibrations are

$$\omega_1^2=3(\alpha+2\beta)/M_\alpha,\quad \omega_2^2=\tfrac{3}{2}(\alpha-\beta)/M_\alpha,\quad (2)$$

where the subscripts specify the degeneracy and M_α is the α-particle mass. The rotational motion is that of a symmetric top ($I_1=I_2=\tfrac{1}{2}I_3=\tfrac{1}{2}M_\alpha s^2$). Only those quantum states are allowed which satisfy Bose statistics for the α particles. Wheeler[3] has listed the number of allowed states as a function of n_1 and n_2, the occupation numbers of the vibrational modes, J, the total angular momentum, and K, its projection on the figure axis. The parity[4] of a level is determined solely by the rotational wave function and is $(-)^K$. Since $|K|\leq J$, 0^- states do not occur. Finally, the excitation energy is

$$E=[J(J+1)-\tfrac{1}{2}K^2]\Delta+n_1\delta_1+n_2\delta_2,\quad (3)$$

with $\Delta=\hbar^2/2I_1$, $\delta_1=\hbar\omega_1$, and $\delta_2=\hbar\omega_2$. As Wheeler pointed out, the requirement of Bose statistics eliminates a considerable number of states, particularly low-lying ones. Thus 1^+ states involve a minimum excitation of the degenerate mode ω_2 of three quanta, and the first state of this type will not be found until the excitation energy is above 20 Mev. Table I gives the eigenvalues for the allowed states of low excitation. The nondegenerate mode ω_1 is not included since its symmetry (even) and parity (even) are independent of n_1. Hence, additional states are obtained from those of Table I by exciting this mode by amounts $n_1\delta_1$, where n_1 is any integer. The present simple description of the α-particle model states of C^{12} is, of course, restricted to low excitation. Above 7.4 Mev the virtual nature of the levels

[1] D. M. Dennison, Phys. Rev. **96**, 378 (1954).

[2] A. Herzenberg [Nuovo cimento **10**, 986 and 1008 (1955)] has recently restudied some of the fundamental problems.

[3] J. A. Wheeler, Phys. Rev. **52**, 1083 (1937).

[4] Professor L. Rosenfeld has kindly informed us that his list of "parities," Table 13.21 in *Nuclear Forces* (North Holland Publishing Company, Amsterdam, 1948), gives the behavior of the wave function under reflections in a side of the equilateral triangle.

has been neglected,[5] and above 15 Mev it will not produce the required $T=1$ states.

A knowledge of the angular momentum and parity of the low-lying states is necessary if the parameters Δ, δ_1, and δ_2 are to be determined from the actual level scheme. Many states have been observed in C^{12} but, of the low-lying levels, the spins and parities of only the ground state and the first excited state are definitely known.[6] The requirement that the model reproduce these states exactly leads to the assignments[7] $|00,00\rangle$ for the 0^+ ground state and $|00,20\rangle$ for the 2^+ first excited state at 4.43 Mev. One parameter is now determined: $\Delta=0.74$ Mev. From Table I it is seen that an unobserved 3^- level, the rotational state $|00,33\rangle$, is predicted at 5.53 Mev. We are aware of no particular reason why this level, if it really exists, should not have been observed.

The second excited state at 7.65 Mev must have spin and parity both even or both odd because it decays into Be^8 and an α particle.[8] Ajzenberg and Lauritsen[6] had tentatively listed a 0^+ assignment on the basis of the observation of pairs corresponding to a level at 7.0 ± 0.6 Mev, the absence of 7-Mev γ rays, and the detection of cascade γ rays through the first excited state. The angular correlation of the cascade radiation is in agreement with this assignment.[9] Some recent work has failed to find evidence for the pairs[10] and shown that the predominant mode of decay is α emission.[10–12] The assignment 0^+ is still the most likely at this stage of the experimental investigation. Adopting this assignment, the characterization $|10,00\rangle$ determines the second parameter: $\delta_1=7.65$ Mev.

The third excited state of C^{12} also decays into Be^8 and an α particle[6] so that its spin and parity are both even or both odd. Inspection of Table I indicates that this state then involves excitation of the degenerate vibrational mode; otherwise the next appropriate correlation does not occur until 12.1 Mev. There are two possible identifications: (a) a 1^- state, $|01,11\rangle$, or (b) a 2^+ state, $|01,22\rangle$. Scheme (a) has the advantage of greater simplicity in that it introduces no additional states of excitation lower than 9.61 Mev. Scheme (b) involves

TABLE I. Allowed states of C^{12} according to the α-particle model. From each of the states listed another can be formed by excitation of the nondegenerate vibrational mode by an amount $n_1\delta_1$, where n_1 is any integer. For each value of n_2 the states are given in order of increasing rotational energy.

n_2	J	K	$J\pi$	$E-n_1\delta_1$
0	0	0	0^+	0
0	2	0	2^+	6Δ
0	3	3	3^-	$(15/2)\Delta$
0	4	3	4^-	$(31/2)\Delta$
0	4	0	4^+	20Δ
0	6	6	6^+	24Δ
0	5	3	5^-	$(51/2)\Delta$
1	1	1	1^-	$\delta_2+(3/2)\Delta$
1	2	2	2^+	$\delta_2+4\Delta$
1	2	1	2^-	$\delta_2+(11/2)\Delta$
1	3	2	3^+	$\delta_2+10\Delta$
1	3	1	3^-	$\delta_2+(23/2)\Delta$
1	4	4	4^+	$\delta_2+12\Delta$
1	4	2	4^+	$\delta_2+18\Delta$
1	5	5	5^-	$\delta_2+(35/2)\Delta$
1	4	1	4^-	$\delta_2+(39/2)\Delta$
2	0	0	0^+	$2\delta_2$
2	1	1	1^-	$2\delta_2+(3/2)\Delta$
2	2	2	2^+	$2\delta_2+4\Delta$
2	2	1	2^-	$2\delta_2+(11/2)\Delta$
2	2	0	2^+	$2\delta_2+6\Delta$
2	3	3	3^-	$2\delta_2+(15/2)\Delta$
2	3	2	3^+	$2\delta_2+10\Delta$
2	3	1	3^-	$2\delta_2+(23/2)\Delta$

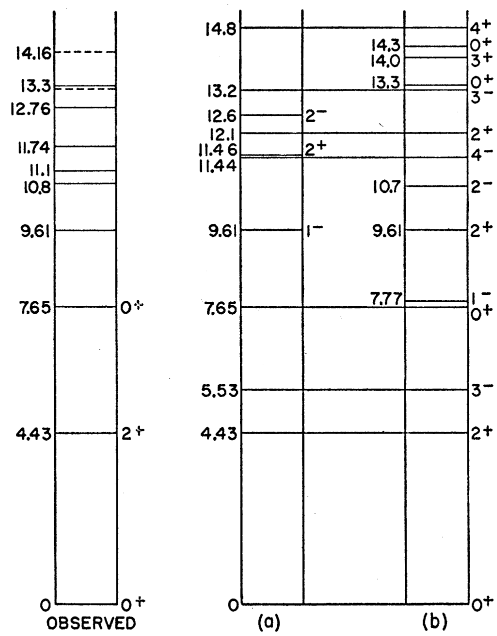

FIG. 1. Comparison of observed level structure for C^{12} and predictions of α-particle model. The determination of the parity of the 9.61-Mev state will distinguish between the two possible correlations (a) and (b).

[5] According to Professor Rosenfeld, this problem is now being investigated at Manchester.

[6] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).

[7] The quantum numbers are indicated by $|n_1n_2,JK\rangle$.

[8] The α particles have recently been observed directly [Fowler, Cook, Lauritsen, Lauritsen, and Mozer, Bull. Am. Phys. Soc. Ser. II, **1**, 191 (1956)].

[9] J. Seed, Phil. Mag. **46**, 100 (1955).

[10] Bent, Bonner, McCrary, and Ranken, Phys. Rev. **100**, 771 (1953). Using a magnetic lens pair spectrometer, these authors failed to observe pairs from this level on bombarding a *thick* Be target with 4.3-Mev α particles. They concluded that more than 96% of the decays are by α-particle emission. It should be noted, however, that for the relative populations of the 4.43- and 7.65-Mev states in the $Be^9(\alpha,n)C^{12*}$ reaction they used the only available data of Guier, Bertini, and Roberts, Phys. Rev. **85**, 426 (1952) for 5.3-Mev α particles bombarding a *thin* target.

[11] Rasmussen, Miller, and Sampson, Phys. Rev. **100**, 181 (1955). In the absence of any evidence for C^{12*} recoils corresponding to α particles inelastically scattered from this level, these authors conclude that more than 80% of the decays proceed by α-particle emission.

[12] W. F. Hornyak, Bull. Am. Phys. Soc. Ser. II, **1**, 197 (1956).

a 1^- state, $|01,11\rangle$, at 7.77 Mev. Such a level has a higher barrier against α-particle decay than the 0^+ state nearby, and the main mode of γ decay, $E1$ radiation, is greatly inhibited for nuclei containing α particles.[13] Their proximity may make it difficult to distinguish between this state and the 0^+ level at 7.65 Mev where, as discussed above, there does seem to be contradictory experimental evidence. In either case, the third parameter is determined by the position of the 9.61-Mev level: $\delta_2^{(a)}=8.50$ Mev, or $\delta_2^{(b)}=6.66$ Mev. The two alternative schemes and the observed levels are presented in Fig. 1 for excitations up to 15 Mev. The parameters for the two correlations are listed in Table II. The position of the levels is not to be taken too literally since the three parameters have been chosen to reproduce the first three observed states exactly.

It is of interest to compare the parameters obtained here with those given by Dennison's analysis[1] of O^{16}. The ratio of the potential parameters, β/α, which measures the ratio of three-body to two-body forces, is -0.25 for correlation (a) and -0.13 for correlation (b). Thus, in this respect there is a real distinction between the two level schemes. A similar situation exists in O^{16}, where the two almost equally successful correlations have values for this ratio about equal to those used here. However, the mean zero-point kinetic energy per vibrational degree of freedom is 2.05 Mev for (a) and 1.75 Mev for (b). These values are significantly greater than those in O^{16}, which are 1.4 or 1.2 Mev depending on which of Dennison's identifications is used. The rotational parameter Δ determines the separation of the α particles in C^{12} to be 3.7×10^{-13} cm, compared with 4.6×10^{-13} cm in Be^8 [2] and 3.2×10^{-13} cm in O^{16}. The expectation value of r^2 in the ground state is

$$\langle 00,00|r^2|00,00\rangle=\tfrac{1}{3}s^2\left[1+\frac{1}{2}\left(\frac{\Delta}{\delta_1}+2\frac{\Delta}{\delta_2}\right)\right]. \quad (4)$$

Let R be the radius of the "equivalent" spherical constant mass density, which is defined to have the same root-mean-square radius as the actual ground state. Then (4) is set equal to $\frac{3}{5}R^2$, and R may be evaluated with the parameters in Table II. Using the usual radius formula $R=r_0A^{\frac{1}{3}}$, $r_0=1.3\times10^{-13}$ cm for both identifications (a) and (b). This is in agreement with the size determined by elastic electron scattering. A similar analysis for O^{16} leads to a value somewhat closer to $r_0=1.2\times10^{-13}$ cm. The Δ/δ terms in (4) represent the ratio of the mean square amplitude of the zero-point oscillations to the square of the equilibrium separation. This parameter measures the corrections due to rotation-vibration interaction; in C^{12}, $\Delta/\delta\simeq\frac{1}{10}$.

In conclusion, it is seen that the α-particle model can correlate the ground and first two excited states of C^{12}. It also gives two possibilities for the third excited state. Future experiments will have to distinguish between these identifications. The parameters used in the model have reasonable magnitudes. Possibly its weakest point at present is an unobserved 3^- state at 5.53 Mev. No comparisons can be made above 10 Mev since the spins and parities of the observed levels are not known.

TABLE II. Parameters in the α-particle model of C^{12}. The energies Δ, δ_1, and δ_2 are in Mev, s is in 10^{-13} cm, and the potential constants α and β are in units of (Mev^2 $M_\alpha/\hbar^2$).

	Δ	δ_1	δ_2	s	α	β
(a)	0.74	7.65	8.50	3.75	39	−10
(b)	0.74	7.65	6.66	3.74	26	− 3

[13] H. Bethe, Revs. Modern Phys. 9, 69 (1937), ¶87B.