Semiempirical Model for Direct Nuclear Breakup Reactions*

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A semiempirical model describing the direct breakup of light nuclei by low-energy nucleons is proposed. The model is based upon the assumption that a statistical equilibrium is established with respect to the incident particle and $(\nu-1)$ tightly bound subgroup components of the target nucleus inside of an energy-dependent interaction volume. The interaction volume is taken to be an oblate spheroid which is oriented along the direction of the incident particle path, and which has a minor axis equal to the de Broglie wavelength λ, and a major axis equal to $(R+a\lambda)$, where R and a are parameters characteristic to the particular reaction involved. The direct breakup cross section is calculated for the $Be^9(n,2n)Be^8$ and $C^{12}(n,n')3He^4$ reactions.

I. INTRODUCTION

THE problem considered in this paper is that of the direct breakup of a light nucleus into a particular number of tightly bound subgroup components of that nucleus by a low-energy incident nucleon. The conditions that the target nucleus have a low atomic number, and that the energy of the incident nucleon be small, are imposed in order to assure that the number of energy levels of the compound nucleus available to the incident nucleon is small, and thus the probability of compound nucleus formation is very small.

The model proposed for breakup reactions is based on the assumption that, within a volume of interaction of the projectile nucleon with the $(\nu-1)$-particle subgroups of the target nucleus, a statistical equilibrium occurs with respect to the center-of-mass motion of the $(\nu-1)$-particle subgroups. The projectile nucleon then shares its energy and momentum with the $(\nu-1)$-particle subgroups and, in the final state, ν-free particles emerge isotropically.

The purpose of this calculation is to obtain an expression for the direct breakup cross section without the necessity of an explicit treatment of the details of the nuclear forces which exist between the projectile nucleon and the particles contained in the target nucleus. The basic assumption of statistical equilibrium being established inside of an interaction volume is analogous to that used by Fermi[1] in his treatment of multiple meson production.

II. BREAKUP CROSS SECTION

Let us consider a box of volume V to contain a target nucleus and a projectile nucleon with a relative velocity v_i. The cross section for the breakup process is

$$\sigma_{if}=W_{if}V/v_i, \tag{1}$$

where W_{if} is the transition probability per unit time of going from the initial state $|i\rangle$, which describes the free particle states of projectile and target nuclei, to the final state $|f\rangle$ which describes the free particle states of the ν ejected nuclei.

The probability of going from state $|i\rangle$ to state $|f\rangle$ is $W_{if}dt=P_{if}$. Further, P_{if} is the product of the probability that a nucleon in a volume V will be found in an interaction volume Ω multiplied by the probability of going from state $|\Omega\rangle$ to state $|f\rangle$. Thus

$$W_{if}dt=P_{if}=(\Omega/V)P_{\Omega f}=(\Omega/V)W_{\Omega f}dt. \tag{2}$$

Substituting Eq. (2) into Eq. (1) gives

$$\sigma_{if}=W_{\Omega f}\Omega/v_i. \tag{3}$$

It is shown in S-matrix theory that $W_{\Omega f}$ is given precisely by the expression

$$W_{\Omega f}=(2\pi/\hbar)|\langle f|S|\Omega\rangle|^2\rho_f, \tag{4}$$

where ρ_f is the number of final states per unit energy,

$$|\Omega\rangle=|p_\Omega^{(1)},\alpha_\Omega^{(1)};\cdots p_\Omega^{(\nu)},\alpha_\Omega^{(\nu)}\rangle \tag{5}$$

is the manifold of state vectors comprising the state of ν particles inside of an interaction volume Ω,

$$|f\rangle=|p_f^{(1)},\alpha_f^{(1)};\cdots p_f^{(\nu)},\alpha_f^{(\nu)}\rangle \tag{6}$$

is the manifold of state vectors comprising the final state of ν particles in free space, and S is an operator which projects $|\Omega\rangle$ onto $|f\rangle$. In Eqs. (5) and (6), $p^{(i)}$ represents the energy-momentum four-vector of the ith particle, and $\alpha^{(i)}$ represents all other constants of the motion associated with that particle.

If N_α represents the total possible number of α states, then the cross section, calculated as an average over α states, is

$$\sigma_{if}=\frac{2\pi\Omega\rho_f}{\hbar v_i}\left\{\frac{1}{N_\alpha}\sum_{\alpha_\Omega^{(i)}}\sum_{\alpha_f^{(i)}}|\langle p_f^{(i)},\alpha_f^{(i)}|S|p_\Omega^{(i)}\alpha_\Omega^{(i)}\rangle|^2\right\}. \tag{7}$$

Following arguments given by Fermi,[1] the matrix element is given by

$$\frac{1}{N_\alpha}\sum_{\alpha_\Omega^{(i)}}\sum_{\alpha_f^{(i)}}|\langle p_f^{(i)},\alpha_f^{(i)}|S|p_\Omega^{(i)},\alpha_\Omega^{(i)}\rangle|^2=D^2\left(\frac{\Omega}{V}\right)^{\nu-1}, \tag{8}$$

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[1] E. Fermi, Progr. Theoret. Phys. (Japan) 5, 570 (1950).

where D is a constant with dimensions of energy, and will be taken to be the potential well depth of the target nucleus. Combining Eqs. (8) and (7), the cross section becomes

$$\sigma_{if}=\frac{2\pi D^2}{\hbar v_i}\frac{\Omega^\nu}{V^{\nu-1}}\rho_f. \tag{9}$$

III. DENSITY OF FINAL STATES

The number of final free-particle states per unit energy interval is

$$\rho_f=V^{\nu-1}\prod_{i=1}^{\nu}(2I_i+1)\frac{d}{dW}\left\{\frac{1}{(2\pi\hbar)^{3(\nu-1)}}\times\int\prod_{i=1}^{\nu-1}d\mathbf{p}_i\delta(\sum\mathbf{p}_i)\delta\left(W-\sum\frac{p_i^2}{2M_i}\right)\right\}, \tag{10}$$

where the product of the two delta functions insures that the laws of conservation of momentum and energy hold, and where $(2I_i+1)$ is the spin degeneracy of the ith emitted particle. The integral appearing in Eq. (10) has been evaluated by Milburn[2] and the following result is obtained for the density of final states:

$$\rho_f=\frac{V^{\nu-1}\prod_{i=1}^{\nu}(2I_i+1)(2\pi)^{3(\nu-1)/2}}{(2\pi\hbar)^{3(\nu-1)}\Gamma[3(\nu-1)/2]}\frac{\prod_{i=1}^{\nu}M_i^{\frac{3}{2}}}{\left(\sum_{i=1}^{\nu}M_i\right)^{\frac{3}{2}}}W^{(3\nu-5)/2}. \tag{11}$$

W is the total kinetic energy in the final state, and by conservation of energy is just $E\pm\mathcal{E}$, where E is the initial relative kinetic energy, $\mathcal{E}$ is the Q value, and the plus and minus signs refer to exothermic and endothermic reactions, respectively. Substituting Eq. (11) into Eq. (9), and expressing the initial relative velocity as $[2E/\mu]^{\frac{1}{2}}$, where μ is the reduced mass of the projectile nucleon, the cross section is

$$\sigma=\Omega^\nu\frac{\prod_{i=1}^{\nu}(2I_i+1)}{2^{(3\nu-4)/2}\pi^{(3\nu-5)/2}\Gamma[3(\nu-1)/2]}\times\frac{\mu^{\frac{1}{2}}}{\hbar^{3\nu-2}}\frac{\prod_{i=1}^{\nu}M_i^{\frac{3}{2}}}{\left(\sum_{i=1}^{\nu}M_i\right)^{\frac{3}{2}}}\frac{D^2(E\pm\mathcal{E})^{(3\nu-5)/2}}{E^{\frac{1}{2}}}. \tag{12}$$

IV. INTERACTION VOLUME

The assumption is made that the incident particle enters into a volume of interaction with the target nucleus and instantaneously shares its energy and momentum with the $(\nu-1)$ tightly bound subgroup components of the target nucleus. Classically, the assumption is made that the interaction takes place at a given point along the incident particle path. However, if one invokes the uncertainty principle, it turns out that the smallest segment of the incident particle path along which the interaction can take place is $2\lambda\!\!\!^{-}$. It is further assumed that as the initial energy increases the cross-sectional area of the interaction volume, as seen by the incident nucleon, approaches a constant value. The shape of the interaction volume chosen to satisfy these two assumptions is an oblate spheroid oriented along the direction of the incident particle, with the minor axis equal to the de Broglie wavelength $\lambda\!\!\!^{-}$, and the major axis equal to $(R+a\lambda\!\!\!^{-})$, where R and a are parameters which are characteristic of the particular breakup reaction involved. Thus, the interaction volume is taken to be

$$\Omega=(\tfrac{4}{3}\pi)(R+a\lambda\!\!\!^{-})^2\lambda\!\!\!^{-}. \tag{13}$$

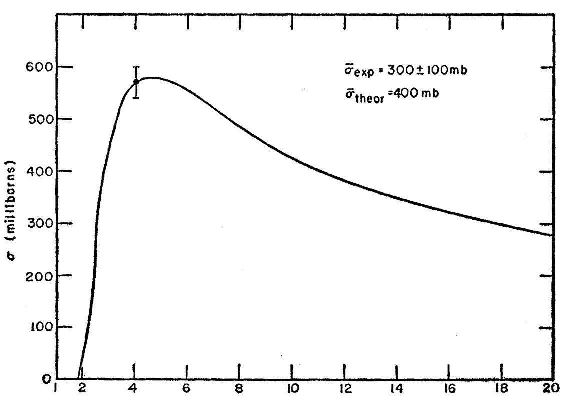

FIG. 1. The cross section for the direct breakup reaction $Be^9(n,2n)Be^8$. The circle represents the $(n,2n)$ cross section obtained from the experimental values of the reaction cross section and the (n,He^4) cross section. $\bar{\sigma}$ represents the cross section averaged over the Ra−Be source of neutrons.

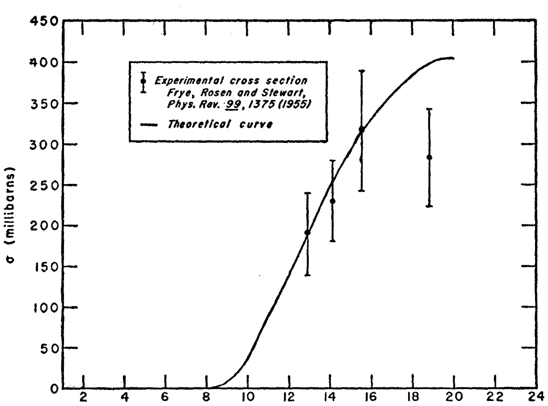

FIG. 2. The cross section for the direct breakup reaction $C^{12}(n,n')3He^4$.

[2] R. H. Milburn, Revs. Modern Phys. 27, 1 (1955).

The assumption of the establishment of statistical equilibrium inside of an interaction volume, together with the spheroidal shape of the interaction volume, constitutes the description of the proposed semi-empirical model for nuclear breakup reactions.

In order to illustrate the energy dependence of the cross section and verify it with some experimental data, the two-body breakup cross section for the $Be^9(n,2n)Be^8$ reaction, and the three-body breakup cross section for the $C^{12}(n,n')3He^4$ reaction will be calculated in the next sections.

V. TWO-BODY BREAKUP CROSS SECTION

The cross section for a two-body breakup ($\nu=3$) is

$$\sigma=\prod_{i=1}^{3}(2I_i+1)\frac{2\pi}{27\hbar^4}\frac{\prod_{i=1}^{3}M_i^{\frac{3}{2}}}{\mu\left(\sum_{i=1}^{3}M_i\right)^{\frac{3}{2}}}\times D^2\left(1\pm\frac{\mathcal{E}}{E}\right)^2\left(R+\frac{a\hbar}{(2\mu E)^{\frac{1}{2}}}\right)^6 \text{ cm}^2. \quad (14)$$

$Be^9(n,2n)Be^8$ Reaction

Substituting the values $\prod_{i=1}^{3}(2I_i+1)=2\times2\times1=4$, $\mathcal{E}=1.656$ Mev, and $D=40$ Mev; and expressing $R+[a\hbar/(2\mu E)^{\frac{1}{2}}]$ in units of 10^{-13} cm, the $Be^9(n,2n)Be^8$ direct-breakup cross section becomes

$$\sigma=6.777\left(1-\frac{1.840}{\epsilon}\right)^2\left(R+\frac{5.080a}{\epsilon^{\frac{1}{2}}}\right)^6 \text{ mb}, \quad (15)$$

where ϵ is the incident neutron energy in the laboratory system of coordinates, in units of Mev.

Beyster *et al.*[3] have measured the total reaction cross section for 4.07-Mev neutrons on Be^9, and they obtain the result $\sigma_r=620\pm30$ mb. Allen *et al.*[4] have measured the cross section for the $Be^9(n,He^4)He^6$ reaction, and find that at 4.07 Mev, $\sigma(n,He^4)=50$ mb. If one assumes that the total reaction cross section is the sum of the $(n,2n)$ plus the (n,He^4) reaction cross sections, then $\sigma(n,2n)=570\pm30$ mb.

The mean cross section for the $Be^9(n,2n)Be^8$ reaction has been measured for the Ra−Be neutron energy spectrum.[5] The result obtained is $\bar{\sigma}=300\pm100$ mb. The Ra−Be neutron spectrum has been measured by Teucher.[6]

The following choice of parameters in Eq. (15):

$$R=1.39, \quad a=0.463,$$

gives a cross section which is in reasonable agreement with the above data (see Fig. 1). The cross section weighted with the Ra−Be neutron spectrum[6] gives a mean cross section $\bar{\sigma}=400$ mb.

Owing to the lack of experimental data on this reaction, the important thing to note about Fig. 1 is the shape of cross-section curve, which is the general shape of the two-body breakup cross section predicted by this model.

VI. THREE-BODY BREAKUP CROSS SECTION

The cross section for a three-body breakup ($\nu=4$) is

$$\sigma=\frac{\prod_{i=1}^{4}(2I_i+1)}{35\hbar^6}\left(\frac{2}{3}\right)^6\frac{\prod_{i=1}^{4}M_i^{\frac{3}{2}}}{\mu^{\frac{3}{2}}\left(\sum_{i=1}^{4}M_i\right)^{\frac{3}{2}}}\times D^2E\left(1\pm\frac{\mathcal{E}}{E}\right)^{7/2}\left(R+\frac{a\hbar}{(2\mu E)^{\frac{1}{2}}}\right)^8 \text{ cm}^2. \quad (16)$$

$C^{12}(n,n')3He^4$ Reaction

Substituting the values $\prod_{i=1}^{4}(2I_i+1)=1\times1\times1\times2=2$, $\mathcal{E}=7.278$ Mev and $D=40$ Mev; and expressing $R+[a\hbar/(2\mu E)^{\frac{1}{2}}]$ in units of 10^{-13} cm, the $C^{12}(n,n')3He^4$ direct breakup cross section becomes

$$\sigma=0.376\left(1-\frac{7.86}{\epsilon}\right)^{7/2}\epsilon\left(R+\frac{4.96a}{\epsilon^{\frac{1}{2}}}\right)^8 \text{ mb}, \quad (17)$$

where ϵ is the incident neutron energy in the laboratory system of coordinates in units of Mev.

In Fig. 2 the cross section is plotted for the following choice of parameters:

$$R=0.77, \quad a=1.17.$$

The experimental cross sections shown in Fig. 2 were observed by Frye *et al.*[7]

It is noted that the agreement between the theoretical curve and the experimental points is fairly good up to the vicinity of 20 Mev. This can be explained in terms of a higher probability for compound nucleus formation in this energy region. The formation of C^{13*} and its subsequent modes of decay then compete with the direct breakup process, and the model is no longer applicable.

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[3] Beyster, Henkel, Nobles, and Kister, Phys. Rev. **98**, 1216 (1955).
[4] Allen, Burcham, and Wilkinson, Proc. Roy. Soc. (London) **A192**, 114 (1947).
[5] F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **27**, 77 (1955).
[6] M. Teucher, Z. Physik **126**, 410 (1949).
[7] Frye, Rosen, and Stewart, Phys. Rev. **99**, 1375 (1955).