

## Density Effect for the Ionization Loss in Various Materials\*

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The reduction in the ionization loss of charged particles due to the polarization of the medium has been calculated using the mean excitation potentials recently determined by Caldwell. Values of the coefficients of an approximate analytic expression for the density effect correction have been obtained for a number of metals, compounds, and gases.

### I. INTRODUCTION

THE density effect for the ionization loss of charged particles<sup>1-4</sup> has been evaluated previously<sup>4</sup> for various materials. The appropriate density effect correction to be applied to the Bethe-Bloch stopping power formula depends directly on the value of the mean excitation potential  $I$  which enters into this formula. The previous calculations<sup>4</sup> were based on the values of  $I$  obtained by Bakker and Segrè<sup>5</sup> from their measurements of the range of 340-Mev protons. Recently the values of  $I$  for nine medium and heavy elements have been determined very accurately by Caldwell<sup>6</sup> from the results of the stopping-power measurements of Sachs and Richardson.<sup>7</sup> Caldwell's values of  $I$  are of the order of  $13Z$  ev and are appreciably larger than those of Bakker and Segrè. In the present paper, we have recalculated the density effect correction  $\delta$  to the ionization loss, using the excitation potentials of Caldwell.<sup>6</sup> The results are given in Table II which lists the coefficients of an analytic expression for  $\delta$  for various substances.

### II. CALCULATIONS

From Eqs. (38) and (46) of A, the density effect correction is given by

$$\Delta \frac{dE}{dx} = \frac{2\pi n e^4}{m v^2} \left\{ \sum_i f_i \ln \left( \frac{l_i^2 + l^2}{l_i^2} \right) - l^2 (1 - \beta^2) \right\}, \quad (1)$$

where  $l$  satisfies the following equation:

$$\frac{1}{\beta^2} - 1 = \sum_i \frac{f_i}{\bar{\nu}_i^2 + l^2}. \quad (2)$$

Here  $n$  = number of electrons per cc,  $v = \beta c$  = velocity

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<sup>1</sup> E. Fermi, Phys. Rev. **57**, 445 (1940).

<sup>2</sup> O. Halpern and H. Hall, Phys. Rev. **57**, 459 (1940); **73**, 477 (1948).

<sup>3</sup> G. C. Wick, Nuovo cimento (9), **1**, 302 (1943).

<sup>4</sup> R. M. Sternheimer, Phys. Rev. **88**, 851 (1952); **91**, 256 (1953). These papers will be referred to as A and B, respectively.

<sup>5</sup> C. J. Bakker and E. Segrè, Phys. Rev. **81**, 489 (1951).

<sup>6</sup> D. O. Caldwell, Phys. Rev. **100**, 291 (1955). I am very much indebted to Dr. Caldwell for showing me this paper before publication.

<sup>7</sup> D. C. Sachs and R. J. Richardson, Phys. Rev. **83**, 834 (1951); **89**, 1163 (1953).

of particle,  $f_i$  is the oscillator strength for the  $i$ th atomic transition,  $\bar{\nu}_i$  is the energy of the transition in units of the plasma energy  $h\nu_p$ , where  $\nu_p = (ne^2/\pi m)^{1/2}$ . The constants  $l_i$  are given by

$$l_i = (\bar{\nu}_i^2 + f_i)^{1/2}. \quad (3)$$

The quantity in the curly bracket of Eq. (1) has been referred to above as  $\delta$ . Equation (1) gives the density effect correction to be applied to the Bethe-Bloch formula when the measured values of the excitation potential  $I$  are used. These values of  $I$  already include the low-energy density effect<sup>8</sup> when the measurement is done in a solid. We have

$$I = h\nu_p \prod_i l_i^{f_i}. \quad (4)$$

In obtaining  $\delta$  the difference between  $\bar{\nu}_i$  and  $l_i$  can be neglected for all transitions,<sup>9</sup> except for those of the conduction electrons in a metal for which  $\bar{\nu}_i = 0$ ,  $l_i = f_i^{1/2}$ .

For the metals, the values of  $l_i$  for use in Eq. (1) were obtained as follows. Table I of A<sup>4</sup> lists approximate values for the ionization potentials  $h\nu_i$  of the various shells. These values were used except for the outermost shell which corresponds to the conduction electrons for which  $l_i = f_i^{1/2}$ . The  $h\nu_i$  for the inner shells were multiplied by a factor  $\rho$  which is determined by the requirement that the resulting geometric mean of the frequencies shall give the observed  $I$ . Assuming that there are altogether  $j$  dispersion oscillators,  $\rho$  satisfies the equation

$$\sum_{i=1}^{j-1} f_i \ln(h\nu_i \rho) + f_j \ln(h\nu_p f_j^{1/2}) = \ln I, \quad (5)$$

where  $j$  labels the dispersion oscillator for the conduction electrons. After  $\rho$  has been obtained,  $l_i$  can be calculated from

$$l_i = \nu_i \rho / \nu_p, \quad (i < j) \quad (6)$$

$$l_j = f_j^{1/2}. \quad (7)$$

As an example, for aluminum, the values  $h\nu_1 = 115$  ry,  $h\nu_2 = 6.7$  ry, with  $f_1 = 2/13$ ,  $f_2 = 8/13$  (see Table I of

<sup>8</sup> R. M. Sternheimer, Phys. Rev. **93**, 351 (1954).

<sup>9</sup> Calculations for a typical case (Fe) show that the replacement of  $\bar{\nu}_i$  by  $l_i$  (for  $i < j$ ) in Eq. (2) changes the values of  $\delta$  by less than 0.01 at all energies.

A) give  $\sum_{i=1}^2 f_i \log_{10}(h\nu_i) = 0.825$  (with  $h\nu_i$  in Rydbergs). The excitation energy for the conduction electrons is  $h\nu_p f_3^{\frac{1}{2}} = (2.41)(3/13)^{\frac{1}{2}} = 1.16$  ry, whence  $f_3 \log_{10}(h\nu_p f_3^{\frac{1}{2}}) = 0.015$ . The mean excitation potential  $I$  is 12.0 ry ( $\log_{10} I = 1.079$ ) so that

$$\log_{10} \rho = \frac{1.079 - 0.825 - 0.015}{(10/13)} = 0.3108,$$

giving  $\rho = 2.046$ . Thus  $l_1 = (115)(2.046)/2.41 = 97.6$ ,  $l_2 = (6.7)(2.046)/2.41 = 5.69$ , and  $l_3 = (3/13)^{\frac{1}{2}} = 0.481$ . As noted above, we neglect the difference between  $\bar{\nu}_i$  and  $l_i$  for the transitions from the bound states ( $i < j$ ), so that Eq. (6) was also used for  $\bar{\nu}_i$  in Eq. (2). However, for the conduction electrons ( $i = j$ ) we have  $\bar{\nu}_j = 0$  in Eq. (2).

The resulting values of  $l_i$  and  $\rho$ , together with the oscillator strengths  $f_i$ , are given in Table I. For the metals not listed in Caldwell's table,  $I$  was obtained by linear interpolation of the values of  $I/Z$  given by Caldwell, for the elements between aluminum and gold. For Pb and U, we used the same value of  $I/Z$  as for Au ( $I/Z = 14.4$  ev). For Li, graphite, and Mg, we took  $I/Z = 13$  ev.<sup>10</sup> For Be, the value  $I = 64$  ev of Madsen and Venkateswarlu<sup>11</sup> was used. The values of  $\rho$  for the heavy elements are of order 2. This indicates that the most prominent transitions produced by the electric field of the charged particle go to continuum states with (positive) energy of the same order as the binding energy of the electron in the initial state.

In connection with the values of Table I, it may be noted that for Sn, W, Pb, and U, the oscillator strength for the transitions with lowest frequency of Table I of A has been divided into two groups, thereby giving an additional term in Eqs. (1) and (2). For Sn, a single dispersion oscillator was used in A for the four  $n = 5$  electrons. In the present calculations, only the two  $5p$  electrons are assumed to be free [conduction electrons with  $l_6 = (2/50)^{\frac{1}{2}}$ ], whereas the two  $5s$  electrons have  $l_5 = \nu_{5p}/\nu_p$ . Similarly, for W, the two  $6s$  electrons are assumed free [with  $l_6 = (2/74)^{\frac{1}{2}}$ ], and the twelve  $n = 5$  electrons form a separate group. For Pb, the two  $6p$  electrons are taken as free; the  $6s$  electrons have  $l_6 = \nu_{6p}/\nu_p$ . For U, the situation is similar to that for W, with only two electrons assumed to have zero frequency.

For the gases listed in Table I, the factor  $\rho$  by which the ionization potentials  $h\nu_i$  must be multiplied is obtained from

$$\sum_{i=1}^j f_i \ln(h\nu_i \rho) = \ln I. \quad (8)$$

Thus  $\rho$  is given by  $I/h\nu_m$ , where  $h\nu_m$  is the weighted geometric mean of the ionization potentials for the

various shells. The excitation potentials  $I$  for gases with  $Z > 13$  were obtained by interpolation of the values of  $I/Z$  from Caldwell's table. For H, we used  $I = 19$  ev, which is close to the value obtained from Thompson's measurements.<sup>12</sup> For He,  $I = 44$  ev was used.<sup>13</sup> For N<sub>2</sub>, O<sub>2</sub>, and Ne,  $I/Z$  was taken as 13 ev.<sup>10</sup>

It may be noted that in obtaining the  $I$  values for  $Z > 13$ , no correction was made for the difference between the values of  $I$  as determined in solids and gases. It has been shown previously<sup>8</sup> that if the  $h\nu_i$  for all transitions are exactly the same for the gas and the solid,  $I_{\text{gas}}$  will be smaller than  $I_{\text{solid}}$  by a factor  $\exp(D/2)$ , where  $D$  is given by

$$D = \sum_{i=1}^j f_i \ln(1 + f_i/\bar{\nu}_i^2). \quad (9)$$

The most important term of  $D$  is that due to the transitions from the outermost shell ( $i = j$ ). An attempt was made in reference 8 to estimate  $D$  using reasonable values for the  $f_i$  and  $\bar{\nu}_i$  in Eq. (9). When the measurement of  $I$  is made in a metal, the situation is somewhat different because the frequency  $\bar{\nu}_j$  for the outermost shell (which corresponds to the optical transitions in the gas) becomes zero for the conduction electrons in the metal. As a result, the dominant term  $i = j$  of Eq. (9) becomes

$$D_j = 2f_j \ln(h\nu_p f_j^{\frac{1}{2}}/E_j), \quad (10)$$

where  $E_j$  is the excitation energy for optical transitions in the gas. The data of Table I show that  $h\nu_p f_j^{\frac{1}{2}}$  is of the same order as  $E_j$  ( $\sim 1$  ry), so that  $D_j$  should be very small. In connection with this result, it should be noted that  $D_j$  as given by Eq. (10) is the sum of two compensating effects: the low-energy density effect which raises the transition frequency in the solid from  $\bar{\nu}_j$  to  $l_j$ , and the decrease in the frequency in the absence of polarization effects as the electrons of the outermost shell become conduction electrons in the metal. In the calculations, these effects were neglected, in view of the other uncertainties, in particular our lack of knowledge of the exact effective number of free electrons and the excitation bands of the solid. It may also be noted that the interpolation procedure used to obtain  $I/Z$  from the table of Caldwell neglects possible shell effects, i.e., systematic variations of  $I/Z$  as one approaches the end of a closed shell.<sup>14</sup>

For the compounds listed in Table I, there is a separate value of  $\rho$  for each constituent atom, obtained from Eq. (8). The mean excitation potential of the compound is determined by

$$\ln I = \sum_k F_k \ln I_k, \quad (11)$$

where  $F_k$  is the oscillator strength for the atoms of the

<sup>10</sup> D. O. Caldwell (private communication).

<sup>11</sup> C. B. Madsen and P. Venkateswarlu, Phys. Rev. **74**, 648 (1948).

<sup>12</sup> T. Thompson, University of California Radiation Laboratory Report UCRL-1910, 1952 (unpublished).

<sup>13</sup> E. J. Williams, Proc. Cambridge Phil. Soc. **33**, 179 (1937).

<sup>14</sup> Green, Cooper, and Harris, Phys. Rev. **98**, 466 (1955).

TABLE I. Data used to calculate the density effect. The values of  $I$  and  $h\nu_p$  are in Rydberg units.

Material	Li	Be	Graphite	Mg	Al	Fe	Cu
$l_1$	6.22	8.78	16.5	91.3	97.6	235	305
$l_2$	0.577	0.707	2.94	5.71	5.69	26.7	33.2
$l_3$			0.577	0.408	0.481	2.41	2.48
$l_4$						0.279	0.186
$f_1$	2/3	2/4	2/6	2/12	2/13	2/26	2/29
$f_2$	1/3	2/4	2/6	8/12	8/13	8/26	8/29
$f_3$			2/6	2/12	3/13	18/26	18/29
$f_4$						2/26	1/29
$I$	2.87	4.71	5.74	11.5	12.0	24.8	27.7
$\rho$	1.20	1.52	1.36	1.87	2.05	1.96	1.97
$h\nu_p$	1.02	1.89	1.89	1.96	2.41	4.05	4.27
Material	Ag	Sn	W	Au	Pb	U	
$l_1$	803	1181	1521	2073	2969	2913	
$l_2$	111	167	242	340	484	488	
$l_3$	15.5	25.5	46.7	67.4	98.2	105	
$l_4$	2.05	3.52	6.78	8.37	12.9	17.7	
$l_5$	0.146	0.714	1.07	1.43	2.66	3.89	
$l_6$		0.200	0.164	0.113	0.690	0.996	
$l_7$					0.156	0.147	
$f_1$	2/47	2/50	2/74	2/79	2/82	2/92	
$f_2$	8/47	8/50	8/74	8/79	8/82	8/92	
$f_3$	18/47	18/50	18/74	18/79	18/82	18/92	
$f_4$	18/47	18/50	32/74	32/79	32/82	32/92	
$f_5$	1/47	2/50	12/74	18/79	18/82	18/92	
$f_6$		2/50	2/74	1/79	2/82	12/92	
$f_7$					2/82	2/92	
$I$	48.5	52.1	72.9	83.5	86.8	97.4	
$\rho$	1.93	2.04	1.75	2.05	2.06	1.95	
$h\nu_p$	4.52	3.71	5.89	5.88	4.48	5.68	
Material	H <sub>2</sub>	He	N <sub>2</sub>	Ne	A	Kr	Xe
$l_1$	70	162	704	1951	5701	24 284	42 335
$l_2$			81.3	122	523	2987	6202
$l_3$			53.4		70.3	259	1016
$l_4$						67.0	207
$l_5$							31.6
$f_1$	1	1	2/7	2/10	2/18	2/36	2/54
$f_2$			2/7	8/10	8/18	8/36	8/54
$f_3$			3/7		8/18	18/36	18/54
$f_4$						8/36	18/54
$f_5$							8/54
$I$	1.40	3.24	6.69	9.56	16.8	36.3	55.7
$\rho$	1.40	1.80	1.23	1.37	1.46	1.97	1.73
$h\nu_p$	0.020	0.020	0.053	0.045	0.060	0.085	0.104
Material	Anthracene	Toluene	H <sub>2</sub> O	AgCl	AgBr	NaI	
$l_1$	15.5	18.4	33.3	1076	1007	1682	
$l_2$	2.75	3.27	3.15	149	139	245	
$l_3$	0.872	1.04	2.28	20.8	19.5	38.9	
$l_4$	0.814	0.966	0.886	2.75	2.57	7.43	
$l_5$				0.346	0.323	0.962	
$l_6$				84.0	460	55.9	
$l_7$				5.97	55.6	2.91	
$l_8$				1.62	7.42	0.234	
$l_9$					0.927		
$f_1$	28/94	14/50	2/10	2/64	2/82	2/64	
$f_2$	28/94	14/50	2/10	8/64	8/82	8/64	
$f_3$	28/94	14/50	4/10	18/64	18/82	18/64	
$f_4$	10/94	8/50	2/10	18/64	18/82	18/64	
$f_5$				1/64	1/82	7/64	
$f_6$				2/64	2/82	2/64	
$f_7$				8/64	8/82	8/64	
$f_8$				7/64	18/82	1/64	
$f_9$					7/82		
$I$	4.94	4.58	5.45	36.1	42.2	41.3	
$h\nu_p$	1.72	1.45	1.58	3.33	3.56	2.65	

$k$ th species and  $I_k$  is the corresponding atomic excitation potential. In each case  $\rho$  is given by  $I/h\nu_m$ , where  $h\nu_m$  is the mean of the ionization potentials  $h\nu_i$  listed in Table I of A. We have neglected possible

deviations from the additivity given by Eq. (11) arising from the molecular binding of the compound.<sup>12</sup> For anthracene and toluene, the constants  $l_1$ ,  $l_2$ , and  $l_3$  listed in Table I pertain to C, and  $l_4$  pertains to H. For

C, the mean of the ionization potentials given in Table I of A is  $h\nu_m=5.00$  ry. Thus the value of  $I$  assumed in the calculations,  $I=5.74$  ry gives  $\rho=1.15$ . Similarly,  $\rho=1.40$  for H. For  $H_2O$ ,  $l_1-l_3$  correspond to the dispersion oscillators representing the oxygen atom, and  $l_4$  refers to H. For O,  $I=7.65$  ry and  $h\nu_m=6.14$  ry, so that  $\rho=1.25$ . For AgCl,  $l_1-l_5$  pertain to Ag, while  $l_6-l_8$  pertain to Cl. For Ag we have  $h\nu_m=25.2$  ry and  $I=48.5$  ry, so that  $\rho=1.92$ . For Cl,  $h\nu_m$  is 11.8 ry. The excitation potential  $I=15.9$  ry derived from Caldwell's table gives  $\rho=1.35$ . For AgBr,  $l_1-l_5$  refer to Ag, and  $l_6-l_9$  refer to Br. For Br, the values  $h\nu_m=21.1$  ry and  $I=35.0$  ry give  $\rho=1.66$ . For NaI,  $l_1-l_5$  pertain to I, and  $l_6-l_8$  pertain to Na. For I, we have  $h\nu_m=29.7$  ry,  $I=54.8$  ry, so that  $\rho=1.85$ . For Na, we used  $I=10.5$  ry ( $=13Z$  ev<sup>10</sup>) so that  $h\nu_m=6.73$  ry gives  $\rho=1.56$ .

For the metals, for which  $\bar{v}_j=0$ , Eq. (2) shows that even at very low velocities  $l>0$ , so that there is a finite  $\delta$  at all energies in this case. However, as will be seen below, the resulting values of  $\delta$  are very small ( $\lesssim 0.05$ ) for nonrelativistic particles. For the gases and compounds,  $\delta$  vanishes for velocities  $\beta$  less than  $\beta_0$ , where  $\beta_0$  is given by

$$\beta_0 = (1 + \sum_i f_i / \bar{v}_i^2)^{-1/2}. \quad (12)$$

### III. RESULTS

It has been shown in A that an approximate analytic expression for  $\delta$  is given by

$$\delta = 4.606X + C + a(X_1 - X)^m, \quad (X_0 < X < X_1) \quad (13)$$

$$\delta = 4.606X + C, \quad (X > X_1) \quad (13a)$$

where  $X \equiv \log_{10}(p/\mu c)$  [ $p$ =momentum,  $\mu$ =mass of the charged particle],  $a$ ,  $m$ , and  $X_1$  are constants which must be chosen appropriately for each substance to obtain an adequate fit for  $\delta$ , and  $C$  is given by

$$C = -2 \ln(I/h\nu_p) - 1. \quad (14)$$

For the gases and compounds,  $X_0$  is the value of  $X$  corresponding to  $\beta_0$ . For the metals,  $X_0$  is appropriately chosen as a value of  $X$  (generally near 0) for which  $\delta$  is very small.  $X_1$  is such that  $\delta$  has reached its asymptotic behavior [Eq. (13a)] in which case  $dE/dx$  no longer depends on the ionization potentials, but only on the electron density<sup>1</sup>  $n$ . For each substance, after a suitable value of  $X_1$  is chosen,  $a$  and  $m$  are obtained by requiring that Eq. (13) give the correct value of  $\delta$  at  $X_0$  and at a value of  $X \equiv X_2$  in the range  $(X_0, X_1)$ . In the calculations, at least three values of  $X_2$  were tried; for each  $X_2$ ,  $a$  and  $m$  were obtained and  $\delta$  was calculated from Eq. (13) for 4-8 values of  $X$ . The absolute values of the deviation of Eq. (13) from the actual  $\delta$  were then averaged, and in general those values of  $a$  and  $m$  were chosen which give the smallest average deviation. In most cases, the maximum deviation was less than 0.1; for a few substances, Eq. (13) gave deviations of

TABLE II. Values of the ionization potential  $I$  and the coefficients for the ionization loss.  $I$  is in Rydberg units,  $A$  is in units Mev/g cm<sup>-2</sup>.

Material	$I$	$A$	$B$	$-C$	$a$	$m$	$X_1$	$X_0$
Li	2.87	0.0664	19.63	3.07	0.374	3.05	2	-0.05
Be	4.71	0.0681	18.64	2.83	0.413	2.82	2	-0.10
Graphite	5.74	0.0768	18.25	3.22	0.531	2.63	2	-0.05
Mg	11.5	0.0758	16.86	4.54	0.0938	3.56	3	0.10
Al	12.0	0.0740	16.77	4.21	0.0906	3.51	3	0.05
Fe	24.8	0.0715	15.32	4.62	0.127	3.29	3	0.10
Cu	27.7	0.0701	15.09	4.74	0.119	3.38	3	0.20
Ag	48.5	0.0669	13.98	5.75	0.251	2.88	3	0.20
Sn	52.1	0.0647	13.83	6.28	0.404	2.52	3	0.20
W	72.9	0.0618	13.16	6.03	0.0283	3.91	4	0.30
Au	83.5	0.0615	12.89	6.31	0.0436	3.62	4	0.30
Pb	86.8	0.0608	12.81	6.93	0.0652	3.41	4	0.40
U	97.4	0.0594	12.58	6.69	0.0652	3.37	4	0.30
Anthracene	4.94	0.0810	18.55	3.11	0.420	2.86	2	0.11
Stilbene	4.81	0.0818	18.60	3.12	0.423	2.86	2	0.12
Polystyrene	4.69	0.0826	18.65	3.15	0.429	2.85	2	0.13
Polyethylene	4.04	0.0876	18.95	2.94	0.393	2.86	2	0.12
Lucite	5.08	0.0829	18.49	3.21	0.456	2.78	2	0.14
Toluene	4.58	0.0834	18.70	3.30	0.454	2.83	2	0.17
Xylene	4.50	0.0839	18.73	3.25	0.444	2.84	2	0.16
H <sub>2</sub> O	5.45	0.0853	18.35	3.47	0.519	2.69	2	0.23
AgCl	36.1	0.0686	14.57	5.77	0.0177	4.21	4	0.33
AgBr	42.2	0.0671	14.25	5.95	0.0235	4.03	4	0.30
Emulsion	27.4	0.0698	15.12	5.55	0.0220	4.01	4	0.23
LiI	46.8	0.0643	14.05	6.66	0.525	2.32	3	0.08
NaI	41.3	0.0656	14.30	6.49	0.452	2.44	3	0.18
H <sub>2</sub>	1.40	0.1524	21.07	9.50	0.505	4.72	3	1.85
He	3.24	0.0767	19.39	11.18	2.13	3.22	3	2.21
N <sub>2</sub>	6.69	0.0768	17.94	10.68	0.125	3.72	4	1.86
O <sub>2</sub>	7.65	0.0768	17.67	10.80	0.130	3.72	4	1.90
Ne	9.56	0.0761	17.23	11.72	0.258	3.18	4	2.14
A	16.8	0.0692	16.09	12.27	0.0255	4.36	5	2.02
Kr	36.3	0.0661	14.56	13.12	0.0771	3.57	5	2.12
Xe	55.7	0.0632	13.70	13.57	0.150	3.07	5	1.90
CH <sub>4</sub>	3.27	0.0958	19.37	9.56	0.0852	4.22	4	1.55
(CH <sub>2</sub> ) <sub>2</sub>	4.04	0.0876	18.95	9.52	0.0700	3.94	4	1.54
(CH) <sub>2</sub>	4.69	0.0826	18.65	9.95	0.0841	3.91	4	1.61
CO <sub>2</sub>	7.08	0.0768	17.82	10.32	0.0865	4.03	4	1.72

$\sim 0.12$  at some energies. These maximum deviations are probably of the same order as the uncertainty in  $\delta$  which is due to our lack of knowledge of the exact values of  $\bar{v}_i$  and  $l_i$  to be used in Eqs. (1) and (2). The values of  $a$ ,  $m$ ,  $C$ ,  $X_0$ , and  $X_1$  are given in Table II.

As mentioned above, the value of  $X_0$  chosen for the metals is arbitrary to some extent. For the  $X_0$  listed in the table,  $\delta(X_0)$  has the following values: for Li:  $\delta(-0.05)=0.05$ ; for Be:  $\delta(-0.10)=0.05$ ; for graphite:  $\delta(-0.05)=0.06$ ; for Mg:  $\delta(0.10)=0.06$ ; for Al:  $\delta(0.05)=0.06$ ; for Fe:  $\delta(0.10)=0.04$ ; for Cu:  $\delta(0.20)=0.04$ ; for Ag:  $\delta(0.20)=0.02$ ; for Sn:  $\delta(0.20)=0.04$ ; for W:  $\delta(0.30)=0.05$ ; for Au:  $\delta(0.30)=0.02$ ; for Pb:  $\delta(0.40)=0.04$ ; for U:  $\delta(0.30)=0.04$ . As is seen, these values of  $\delta$  are quite small. For  $X < X_0$ ,  $\delta$  can be calculated from Eqs. (1) and (2) using the constants given in Table I. For the gases and compounds,  $X_0$  is determined by  $\beta_0$  [Eq. (12)], and  $\delta(X_0)=0$ . Concerning the fit obtained from Eq. (13), it may be noted that for a few compounds for which  $\delta$  remains close to zero up to  $X \sim 0.5$ , Eq. (13) gives values which are slightly negative ( $\sim -0.02$ ) just above  $X_0$ . However, this result is not expected to be misleading. For the corresponding values of  $X$ , it can be assumed that the actual  $\delta$  is less than 0.1.

For the case of emulsion, the procedure of the calculation of  $\delta$  was essentially the same as in A. The composition used was that of Ilford G5 emulsion. The dispersion properties of the light elements (excluding H) were taken to be those of nitrogen. A separate term in Eqs. (1) and (2) was used for the hydrogen. Thus

it was assumed that 46% of the oscillator strength is due to Ag, 33% to Br, 18% to N, and 3% to H.

The coefficients for gases given in Table II pertain to normal pressure. In order to obtain  $\delta$  for any other pressure, one can use the relation

$$\delta_P(\not{p}) = \delta_1(\not{p}P^{1/3}), \quad (14a)$$

where  $P$  is the pressure in atmospheres, and  $\delta_P(\not{p}')$  is the density effect correction for momentum  $\not{p}'$  at a pressure  $P$ . Equation (14a) states simply that  $\delta$  at a pressure  $P$  and momentum  $\not{p}$  is equal to  $\delta$  at normal pressure for momentum  $\not{p}P^{1/3}$  [see Eq. (10c) of A].

The constants  $A$  and  $B$  given in Table II are defined by

$$A \equiv 2\pi ne^4 / (mc^2 \rho_0), \quad (15)$$

$$B \equiv \ln[mc^2(10^6 \text{ ev})/I^2], \quad (15a)$$

where  $\rho_0$  is the density. In terms of  $A$  and  $B$ , the average energy loss for particles heavier than electrons can be written

$$-\frac{1}{\rho_0} \frac{dE}{dx} = \frac{A}{\beta^2} \left[ B + 0.69 + 2 \ln \frac{\not{p}}{\mu c} + \ln T_{\text{Mev}} - 2\beta^2 - \delta \right], \quad (16)$$

where  $T_{\text{Mev}}$  is the maximum energy transfer in a single collision (in Mev). The corresponding expressions for the average energy loss of electrons, the ionization loss restricted to energy transfers less than  $T_0$ , and the most probable loss in a thin absorber, are given by Eqs. (49), (50), and (52) of B.

Equation (16) includes the energy loss due to Čerenkov radiation. It has been shown previously<sup>4</sup> that the Čerenkov loss  $W_b$  is very small for condensed materials ( $\lesssim 10^{-3}$  Mev/g cm<sup>-2</sup>). However,  $W_b$  may be appreciable for light gases.<sup>15</sup> An estimate of  $W_b$  is given by Eqs. (35) and (36) of B, in which the quantity

$b_p$  should be taken as<sup>16</sup>  $2\pi\nu_p b/c$ . The asymptotic value of the Čerenkov loss for very large momenta is given by

$$W_b(\infty) \cong \frac{2f_j A}{3\beta^2} \ln \left( \frac{E_j c}{4\pi\nu_p f_j^{1/2} w_j b} \right), \quad (17)$$

where  $f_j$  is the oscillator strength for the optical transitions to the first excited state,  $E_j$  is the corresponding excitation energy,  $w_j$  is the half-width of the lines of the optical spectrum, and  $b$  is the radius of the cylinder for which the Čerenkov loss is evaluated, i.e., it is assumed that the energy deposited at distances larger than  $b$  from the passing particle does not contribute to the droplet count in the gas. Upon taking<sup>4</sup>  $b=0.1$  cm,  $w_j=0.5 \times 10^{-3}$  ev,  $f_j=0.55$ , with  $E_j=0.75$  ry for H<sub>2</sub> and 1.56 ry for He, one finds  $W_b(\infty)=0.130$  Mev/g cm<sup>-2</sup> for H<sub>2</sub> and 0.085 Mev/g cm<sup>-2</sup> for He, at normal pressure. It is expected that  $W_b$  will decrease with increasing  $Z$ , because the oscillator strength  $f_j$  of the outermost shell decreases as the total number of shells increases. Moreover, the argument of the logarithm is proportional to  $\nu_p^{-1}$  which decreases with increasing  $Z$ . Thus with reasonable values of the constants entering into Eq. (17), one finds<sup>16</sup>  $W_b(\infty)=0.017$  Mev/g cm<sup>-2</sup> for O<sub>2</sub> and 0.006 Mev/g cm<sup>-2</sup> for Xe. These results show that the Čerenkov loss is probably negligible for the heavier gases ( $Z \gtrsim 10$ ). However, it should be noted that the values of  $w_j$  and  $b$  which appear in (17) have large uncertainties. Nevertheless, since these quantities enter only behind the logarithm, the order of magnitude of  $W_b$  is probably given correctly by the present estimates.

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<sup>16</sup> R. M. Sternheimer, Phys. Rev. **93**, 1434 (1954). The value 0.058 for Xe should be 0.0058 Mev/g cm<sup>-2</sup>.

<sup>15</sup> See also P. Budini, Nuovo cimento **10**, 236 (1953).