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Relativistic Effects in Nuclear Forces*

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An attempt is made to reformulate in a relativistically invariant way a phenomenological theory proposed by Johnson and Teller. Such a generalized theory reproduces the earlier results, i.e., saturation of nuclear binding and approximately correct neutron-to-proton ratios. It also avoids the collapse of the nucleus occurring in the nonrelativistic theory for high kinetic energies. The theory predicts as a secondary result an extremely strong spin-orbit coupling which is of the order of magnitude of the phenomenological coupling introduced in the shell model. Furthermore, the theory predicts a strong attraction between nucleons and antinucleons. This could lead to a ^1S bound state as the ground state of the nucleon—antinucleon system having the properties of a pseudoscalar meson. The strong attraction between antinucleons and nuclear matter also suggests a high value for the collision cross section of antiprotons with nuclei, an effect which has recently been observed.

INTRODUCTION

IN a paper by Johnson and Teller[1] a velocity-dependent potential was introduced to explain certain striking nuclear properties. This phenomenological model was based on the following data on nuclear forces.

1. Nuclear forces lead to saturation of density and energy.
2. Nuclei show shell structure which can be explained by the introduction of a smooth classical potential as a first approximation in which the nucleons (at least the top nucleons) are thought to move more or less independently. To get the right shell structure a strong spin-orbit coupling has to be assumed.[2]
3. Nuclear forces are charge-independent.

The simplest way to treat charge independence would be to assume that differences between protons and neutrons result from their nonidentity (Fermi-statistics) and their different charge states. On the other hand the kinetic energy of the excess neutrons in nuclei do not balance completely the additional Coulomb energy of the protons.

In I, a nuclear model was proposed in which the nucleons move in a potential which transforms like a scalar in a nonrelativistic sense. For convenience, this potential was assumed to arise from a linear coupling with a neutral, scalar meson field. As an essential part of this theory an additional linear coupling was introduced which is proportional to the kinetic energy of the nucleon; this additional term is repulsive. According to Fermi statistics the total kinetic energy increases with density. Since the additional term in the potential also increases with density one can show that equilibrium can be obtained, and the saturation properties are correctly given. The kinetic-energy-dependent coupling has the effect of decreasing the mass of the nucleons within the nucleus. By assuming an effective mass of $\approx 0.4m$, the correct nuclear radii are reproduced. Because of the increased kinetic energy a smaller neutron excess can balance the repulsive Coulomb potential of the protons thus leading approximately to the correct neutron-proton ratios.

According to these assumptions the coupling between meson field and a nucleon is linear in the field amplitudes and decreases with increasing nuclear momentum. For a certain momentum (kinetic energy ≈ 60 Mev) the coupling vanishes and for higher momenta it becomes negative. The interaction between nucleons remains, however, negative and is proportional to the square of the coupling. Thus for very high kinetic energies extremely strong attractions are obtained and the nucleus should collapse. We shall show that in the relativistic treatment this collapse is avoided.

CHOICE OF COUPLING

We will try in this section to find a relativistic formulation of an interaction which in the nonrelativistic limit will lead to a velocity dependence as proposed in I. We restrict ourselves to the consideration of classical potential functions which have a smooth behavior inside the nucleus. All ∇ terms appearing in an interaction may therefore be discarded as surface terms. They do not give any contribution to the volume energy which is our primary interest. However, we do not restrict ourselves to fields transforming like ordinary potential functions, but investigate potentials which are linearly coupled to the nucleon field like scalars, vectors, tensors, pseudovectors or pseudoscalars. We write the Dirac equation (setting $\hbar = c = 1$):

$$i\gamma_\nu p_\nu + m = O_j, \tag{1}$$

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[1] M. H. Johnson and E. Teller, Phys. Rev. 98, 783 (1955), hereafter referred to as I.
[2] M. Goeppert-Mayer, Phys. Rev. 75, 1969 (1949); Haxel, Jensen, and Suess, Phys. Rev. 75, 1766 (1949).

with the possible interaction terms

$$\begin{array}{lll} O_1=-V_s, & & \text{scalar,} \\ O_2=i\gamma_\nu A_\nu, & A_\nu=(\mathbf{A},iA_0), & \text{vector,} \\ O_3=\frac{1}{2}i\gamma_\nu\gamma_\mu F_{\nu\mu}, & F_{\nu\mu}=-F_{\mu\nu}, & \text{tensor,} \\ O_4=i\gamma_5\gamma_\nu B_\nu, & B_\nu=(\mathbf{B},iB_0), & \text{pseudovector,} \\ O_5=i\gamma_5 V_{ps}, & & \text{pseudoscalar,} \end{array} \tag{2}$$

or linear combinations of these fields.

In calculating the nonrelativistic limit of the volume energy we can simplify our problem in two ways:

1. We can discard all commutators of the momentum $\mathbf{p}$ with the interaction potentials since they will lead to ∇ terms.

2. We can drop all spin-dependent interactions and interactions which are linear in the momentum $\mathbf{p}$ since they will average to zero for a closed-shell nucleus, and in other cases correspond to surface effects in phase space.

In the following we restrict ourselves to interactions which have a nonvanishing linear average. Terms which arise from averaging higher powers of the interaction will be disregarded. On the other hand, higher powers of linear averages will be retained. In time-independent problems this amounts to a self-consistent field treatment. On the basis of the transformation properties of the fields we observe that only the scalar interaction and the fourth component of the vector field interaction will contribute expressions linear in the field amplitudes to the nonrelativistic Hamiltonian since only these interactions transform like scalars in a nonrelativistic sense. Linear expression in the other interactions will either contain the ∇, the spin $\boldsymbol{\sigma}$, or the momentum $\mathbf{p}$, and they all can be dropped according to the approximations stated above.

Therefore we only consider the interaction with a scalar field V and the fourth component of a vector meson field A_0 and we will also keep any functions of these fields which may appear in the nonrelativistic limit. We set up our Dirac Hamiltonian in the form

$$H=\boldsymbol{\alpha}\cdot\mathbf{p}+\beta m-\beta V+A_0. \tag{3}$$

In this interaction a velocity dependence appears which can be seen by the following observation: all the operators involved are diagonal operators (i.e., do not mix large and small components) except for $\boldsymbol{\alpha}=\rho_1\boldsymbol{\sigma}$ which multiplies $\mathbf{p}$. The momentum therefore has the effect of mixing large and small components. Since

$$\beta=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix} \tag{4a}$$

and

$$I=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}, \tag{4b}$$

this has the consequence that the expectation value of β tends to zero with increasing $\mathbf{p}$, the expectation value of I, however, remains unity. If we consider both A_0 and V positive and $V>A_0$, the net potential $-\langle\beta\rangle V+\langle I\rangle A_0$ will be attractive for small momenta and will become repulsive for higher momenta.

This behavior is readily seen in the nonrelativistic approximation. If we introduce for later convenience the notation

$$V=am\phi, \tag{5}$$

$$A_0=bm\phi_0, \tag{6}$$

where a and b are coupling constants, ϕ and ϕ_0 field amplitudes, then we are led in the limit $p\ll m$ to the Schrödinger Hamiltonian with $H'=H-m$:

$$\begin{aligned} H' &= \frac{1}{2m(1-a\phi)}p^2-m(a\phi-b\phi_0) \\ &= \frac{1}{2m}p^2+V_{\text{eff}}, \end{aligned} \tag{7}$$

with

$$V_{\text{eff}}=-am\phi\left[1-\frac{1}{1-a\phi}\frac{p^2}{2m^2}\right]+bm\phi_0. \tag{8}$$

This obviously shows the decreasing contribution of the scalar interaction with increasing momentum. We have given here only that part of the Hamiltonian which is free of ∇ terms. The exact nonrelativistic limit of our Dirac equation, which can be obtained by a Foldy-Wouthuysen transformation, will be given later.

We have constructed our interaction in a completely phenomenological way. However, the result is interesting from the view point of field theory. Assuming linear interactions with uncharged (isotopic singlet) meson fields, we may hope that already the adiabatic approximation (which neglects any changes in the nucleon state during emission and absorption of mesons) will give a fair result. In this approximation the scalar field and the fourth component of the vector meson field will give the main contribution, namely a classical instantaneous potential exactly of the type we introduced above. Meson theory also unambiguously determines the sign of the interaction. It can be shown quite generally that an interaction with an uncharged scalar field always lowers the energy and therefore leads to attraction. The interaction with the fourth component of an uncharged vector field always gives repulsion between like particles (e.g., nucleons-nucleons). We have here the same situation as for the electromagnetic field which leads to repulsion between particles of the same charge.

In our phenomenological approach, we have heavily relied on the assumption that the potential inside the nucleus is smooth. In field theory, this assumption corresponds to the introduction of a strong cutoff for meson momenta inside the nucleus. This assumption of low momentum cutoff would justify our replacement of the δ-function lattice of nuclear sources by the smooth

ρ distribution. Also the adiabatic approximation of our interaction would be quite reasonable. By disregarding the average of nonlinear terms we have excluded the processes in which two or more mesons are created and annihilated simultaneously. The justification of this restriction does not appear to be straightforward. In connection with the pseudoscalar theory arguments were given[3] which seem to imply that the contributions of the "pair term" to the interaction is small, and which depend on a weak momentum cutoff. In the case of a postulated strong momentum cutoff this conclusion may not hold.[4]

LAGRANGIAN AND HAMILTONIAN

We will set up a Lagrangian for the interaction of nucleons both with isotopic singlet scalar and vector mesons of mass μ_1 and μ_2, respectively. As stated, the interaction shall be of the type

$$L_{\text{int}} = -g_1\Psi^*\beta\Psi\varphi + g_2\Psi^*\Psi\varphi_0. \tag{9}$$

Ψ has here the meaning of the total nuclear field operators, while φ and $(\boldsymbol{\varphi}, i\varphi_0)$ are the field operators of the scalar field and the vector field, respectively. In the following discussion, we simplify our problem by treating the sources of the meson field in a nonrelativistic approximation. The nucleons obey Fermi statistics and can be effectively described by a Fermi gas of density ρ. The average kinetic energy of the nucleons can be written as

$$E_K = C\rho^{\frac{2}{3}}, \tag{10}$$

where

$$C = (3/10m)(\tfrac{3}{2}\pi^2)^{\frac{2}{3}}. \tag{11}$$

With these simplifications, the interaction Lagrangian assumes the form

$$L_{\text{int}} = -\left[1 - \frac{1}{1-a\varphi}\frac{E_K}{m}\right] am\varphi\rho + bm\varphi_0\rho, \tag{12}$$

if we introduce the new coupling constants

$$a = g_1/m, \tag{13a}$$

$$b = g_2/m, \tag{13b}$$

and define $(1-a\varphi)^{-1}$ by its series expansion.

[3] K. A. Brueckner and K. M. Watson, Phys. Rev. **92**, 1023 (1953); S. Drell and E. M. Henley, Phys. Rev. **88**, 1053 (1952); G. Wentzel, Phys. Rev. **86**, 802 (1952).

[4] One may argue in the following way. The nucleons are fermions occupying volumes of approximately $1/R^3$ in momentum space if R measures the nuclear spatial dimension. Emission and absorption of mesons will lead to recoil effects which can be accepted if the meson momenta $k \leq 1/R = (1/r_0)A^{-\frac{1}{3}}$ and the nucleon remains in the same state. Exchange of mesons with higher momentum would necessitate transitions to other momentum states which are already occupied. Transition to unoccupied momentum states above the Fermi level may be assumed to be less probable except for nucleons near the top of the Fermi distribution. These contributions, however, will then be proportional to $A^{\frac{2}{3}}$. This argument therefore introduces a strong momentum cutoff for only single meson events which are based on the exclusion principle rather than an assumption concerning the interaction. This line of reasoning is related to arguments given in the self-consistent field treatment by Brueckner, Levinson, and Mahmoud, Phys. Rev. **95**, 217 (1954).

Following the usual procedure of establishing a Hamiltonian form,[5] we arrive at the total Hamiltonian:

$$H' = H_{\text{nucl}} + H_{\text{int}}^s + H_{\text{int}}^v + H_m^s + H_m^v, \tag{14}$$

with

$$H_{\text{nucl}} = \int E_K\rho d\tau, \tag{15}$$

$$H_{\text{int}}^s = -\int\left[1 - \frac{1}{1-a\varphi}\frac{E_K}{m}\right] am\varphi\rho d\tau, \tag{16}$$

$$H_{\text{int}}^v = \int bm\varphi_0\rho d\tau, \tag{17}$$

$$H_m^s = \tfrac{1}{2}\int [\pi^2 + |\nabla\varphi|^2 + \mu_1^2\varphi^2] d\tau, \tag{18}$$

$$H_m^v = \tfrac{1}{2}\int\left[\boldsymbol{\pi}^2 + |\nabla\times\boldsymbol{\varphi}|^2 + \mu_2^2\boldsymbol{\varphi}^2 + \mu_2^2\varphi_0\left(\frac{2}{\mu_2^2}\nabla\cdot\boldsymbol{\pi} - \varphi_0\right)\right] d\tau. \tag{19}$$

In addition, an equation must be satisfied:

$$\mu_2^2\varphi_0 = bm\rho + \nabla\cdot\boldsymbol{\pi}. \tag{20}$$

Here φ and π are the canonically conjugated field operators of the scalar field satisfying the relation

$$\pi = \dot{\varphi}. \tag{21a}$$

For the analogous operators $(\boldsymbol{\varphi}, i\varphi_0)$ and $(\boldsymbol{\pi}, i\pi_0)$ of the vector meson field, we have

$$\boldsymbol{\pi} = \partial\boldsymbol{\varphi}/\partial t + \nabla\varphi_0, \quad \pi_0 \equiv 0. \tag{21b}$$

The condition (20) for φ_0 is connected with the fact that only three of the four components of the vector meson field are independent variables thus corresponding to an interaction with a meson of spin 1. It is easy to see that the Hamiltonian is positive definite as it should be.

We now want to separate the classical part of the interaction. Therefore we write

$$\varphi = \phi + \varphi', \tag{22}$$

and

$$\varphi_0 = \phi_0 + \varphi_0', \tag{23a}$$

$$\boldsymbol{\varphi} = \boldsymbol{\varphi}', \tag{23b}$$

where we now assume ϕ, ϕ_0 to be ordinary functions (i.e., not operators) describing a time-independent instantaneous classical field.

[5] E.g., G. Wentzel, *Quantum Theory of Fields* (Interscience Publishers, Inc., New York, 1949).

Substituting (23a) and (23b) into our Hamiltonian (14), the vector field part (17) and (19) can be written

$$H_{\text{int}}^{v}+H_{m}^{v}=\int\{bm\rho\phi_0-\tfrac{1}{2}[|\nabla\phi_0|^2+\mu_2^2\phi_0^2]$$

$$+\tfrac{1}{2}[\pi'^2+|\nabla\times\varphi'|^2+\mu_2^2\varphi'^2+\mu_2^2\varphi_0'^2]\}d\tau, \quad (24)$$

with the additional conditions,

$$\mu_2^2\varphi_0'=\nabla\cdot\pi' \quad (25)$$

and

$$-\nabla^2\phi_0+\mu_2^2\phi_0=bm\rho. \quad (26)$$

We recognize that the effect of the interaction of vector mesons with fixed nuclear sources can be described by a classical potential, ϕ_0 (particular solution), which obeys the field equation (26). The primed operators $(\varphi', i\varphi_0')$ correspond to free (uncoupled) mesons (solution of the homogeneous equation).

The reduction of the scalar meson interaction is similar. However, the calculation is a little more complicated by the nonlinear coupling which appeared in the nonrelativistic limit. Upon substituting (22) into (14) the Hamiltonian for the scalar meson interaction (16) and (18) can be written

$$H_{\text{int}}^{s}+H_{m}^{s}=\int\left\{-\left[1-\frac{1}{1-a(\phi+\varphi')}\frac{E_K}{m}\right]am\rho(\phi+\varphi')\right.$$

$$\left.+\tfrac{1}{2}[\pi^2+|\nabla(\phi+\varphi')|^2+\mu_1^2(\phi+\varphi')^2]\right\}d\tau, \quad (27)$$

where the operator in the denominator is defined by the expansion

$$[1-a(\phi+\varphi')]^{-1}=\frac{1}{\gamma}\left[1+\frac{a}{\gamma}\varphi'+\frac{a^2}{\gamma^2}\varphi'^2+\cdots\right], \quad (28)$$

and we use the abbreviation

$$\gamma=1-a\phi. \quad (29)$$

Ordering the terms and using partial integration, we finally get

$$H_{\text{int}}^{s}+H_{m}^{s}=\int\left\{-\left[1-\frac{1}{\gamma}\frac{E_K}{m}\right]am\phi\rho+\tfrac{1}{2}[|\nabla\phi|^2+\mu_1^2\phi^2]\right.$$

$$-\varphi'\left[\nabla^2\phi-\mu_1^2\phi+\left(1-\frac{1}{\gamma^2}\frac{E_K}{m}\right)am\rho\right]$$

$$+\frac{a^2\varphi'^2}{\gamma^3}E_K\rho\left[1+\frac{a}{\gamma}\varphi'+\cdots\right]$$

$$\left.+\tfrac{1}{2}[\pi'^2+|\nabla\varphi'|^2+\mu_1^2\varphi'^2]\right\}d\tau. \quad (30)$$

We shall define ϕ in such a way as to eliminate the linear coupling between φ' and the nucleons. This gives us a field equation for ϕ:

$$-\nabla^2\phi+\mu_1^2\phi=\left[1-\frac{1}{\gamma^2}\frac{E_K}{m}\right]am\rho. \quad (31)$$

In this case, however, the classical potential does not fully describe the interaction of the nucleons with the meson field. The mesons corresponding to the field φ' are still coupled by nonlinear terms. We will again assume that these terms can be neglected.

The essential part of the total Hamiltonian (14) now has the form

$$H'=\int\left\{E_K\rho-\left(1-\frac{1}{\gamma}\frac{E_K}{m}\right)am\phi\rho+bm\phi_0\rho\right.$$

$$\left.+\tfrac{1}{2}[|\nabla\phi|^2+\mu_1^2\phi^2]-\tfrac{1}{2}[|\nabla\phi_0|^2+\mu_2^2\phi_0^2]\right\}d\tau, \quad (32)$$

with the abbreviation (29) and the secondary conditions,

$$-\nabla^2\phi+\mu_1^2\phi=\left(1-\frac{1}{\gamma^2}\frac{E_K}{m}\right)am\rho, \quad (33)$$

$$-\nabla^2\phi_0+\mu_2^2\phi_0=bm\rho. \quad (34)$$

Obviously these field equations lead to Yukawa type potentials. Since ρ will always be a positive quantity, both potential functions ϕ and ϕ_0 will be positive if a and b are chosen positive. One therefore sees that the scalar field interaction, which gives rise to a negative term in (32), is attractive, while the vector meson field, is repulsive, since it gives rise to a positive term. The formulas hold only for small kinetic energies and hence the source function for the scalar field never becomes negative as may appear in formula (33) but tends to zero for high velocities (see Sec. 2).

SATURATION CONDITION

We wish now to minimize the total Hamiltonian (32) with respect to the nuclear density ρ. This is the only free variable assuming fixed interaction constants a and b and meson masses μ_1 and μ_2, since ϕ and ϕ_0 are expressed in terms of ρ by the field equations. We are only interested in the volume energy and consequently will drop all ∇ terms in these equations. We furthermore want to assume a constant nuclear density $\rho=\rho_0$, which leads also to the constant field amplitudes,

$$\mu_1^2\phi=\left(1-\frac{1}{\gamma^2}\frac{E_K^0}{m}\right)am\rho_0, \quad (35)$$

$$\mu_2^2\phi_0=bm\rho_0. \quad (36)$$

Observing the normalization condition,

$$\int\rho d\tau=\rho_0\int d\tau=A, \quad (37)$$

and defining the radius of the nucleus by

$$R=r_0A^{\frac{1}{3}}, \tag{38}$$

where A is the number of nucleons, we get

$$\rho_0=3/4\pi r_0^3. \tag{39}$$

The energy per particle (neglecting surface terms) will be designated by E_V. We find

$$E_V=\frac{H'}{A}=\frac{1}{\gamma}E_K^0-V_1+V_2+R_1-R_2, \tag{40}$$

with the abbreviations

$$E_K^0=C\rho_0^{\frac{2}{3}}, \tag{41a}$$

$$V_1=am\phi, \tag{41b}$$

$$V_2=bm\phi_0, \tag{41c}$$

$$R_1=\tfrac{1}{2}\mu_1^2\phi^2/\rho_0, \tag{41d}$$

$$R_2=\tfrac{1}{2}\mu_2^2\phi_0^2/\rho_0. \tag{41e}$$

It is interesting to note that in (40) one may consider ϕ and ϕ_0 as independent parameters by disregarding for the moment the field equations (35) and (36). Setting the variation with respect to ϕ and ϕ_0 equal to zero, one obtains

$$\phi\partial E_V/\partial\phi=(a\phi/\gamma^2)E_K^0-V_1+2R_1=0, \tag{42}$$

$$\phi_0\partial E_V/\partial\phi_0=V_2-2R_2=0, \tag{43}$$

which gives us back the field equations (35) and (36). That the derivatives of H vanish with respect to the classical fields ϕ and ϕ_0 is, of course, a consequence of the assumed time independence of these fields. Minimizing (40) with respect to ρ_0 will give us the saturation condition

$$\rho_0\partial E_V/\partial\rho_0=\tfrac{2}{3}(1/\gamma)E_K^0-R_1+R_2=0. \tag{44}$$

If we subtract (42) from (43), we obtain with (44)

$$V_{12}=V_1-V_2=\left(\frac{1}{3}+\frac{1}{\gamma}\right)\frac{1}{\gamma}E_K^0. \tag{45}$$

Inserting this into the energy equation (40), we get, with (44),

$$E_V=-\left[\frac{1}{\gamma}-\frac{4}{3}\right]\frac{1}{\gamma}E_K^0. \tag{46}$$

This expression is the volume part of the binding energy.

Up to now we have treated protons and neutrons equally. If we take into account the electrostatic repulsion experienced by the protons and treat neutrons and protons as separate Fermi gases of density ρ_n and ρ_p, respectively, we obtain the following total kinetic energy per nucleon:

$$E_K^{0\prime}=E_K^0[1+(5/9)\Delta^2], \tag{47}$$

with

$$\Delta=(N-Z)/(N+Z)=\rho_n-\rho_p/\rho_n+\rho_p=\delta\rho/\rho, \tag{48}$$

where N is the number of neutrons, Z the number of protons.

Minimizing with respect to the total density ρ leads to the binding energy formula

$$E_B=-\left(\frac{1}{\gamma}-\frac{4}{3}\right)\frac{1}{\gamma}E_K^0+\frac{5}{9}\left(\frac{10}{3}-\frac{1}{\gamma}\right)\frac{1}{\gamma}E_K^0\Delta^2 +E_{Cb}+\text{surface terms}. \tag{49}$$

The first term is the volume energy as determined by (46). The second term is the symmetry energy, and the third term is the Coulomb energy.

This formula has to be compared with the empirical Weizsäcker mass formula, which we take from Green's[6] "best fit" values ($r_0=1.22\times10^{-13}$ cm):

$$E_B=-15.75+23.42\Delta^2+E_{Cb}+\text{surface terms}. \tag{50}$$

(Numbers are given in Mev.) With this value of

$$r_0=1.22\times10^{-13}\text{ cm}, \tag{51}$$

we get for the Fermi energy, using (41a), (39), and (11),

$$E_K^0=19.25\text{ Mev}. \tag{52}$$

We adjust γ in such a way that the volume terms in (49) and (50) should become equal. This gives

$$\gamma=0.559\approx0.56, \tag{53a}$$

or

$$a\phi\approx0.44. \tag{53b}$$

This would mean that the nucleons act inside the nucleus if they had an apparent mass

$$m_{\text{eff}}=\gamma m=0.56m. \tag{54}$$

Using (49), one finds

$$E_B=-15.75+29.6\Delta^2+\cdots. \tag{55}$$

The symmetry energy is comparable with the value obtained empirically by Green (50) and therefore will lead to approximately the proper neutron-proton ratios.

We point out, however, that in case neutrons and protons occupy the different volumes

$$\text{V}_n=\text{V}[1+\tfrac{1}{2}\delta\text{V}/\text{V}] \quad\text{and}\quad \text{V}_p=\text{V}[1-\tfrac{1}{2}\delta\text{V}/\text{V}],$$

respectively, then $\delta\rho/\rho\neq\Delta=N-Z/N+Z$ but

$$\frac{\delta\rho}{\rho}=\left[\Delta-\frac{\text{V}_n-\text{V}_p}{\text{V}_n+\text{V}_p}\right]=\Delta\left[1-\frac{1}{2\Delta}\frac{\delta\text{V}}{\text{V}}\right]. \tag{56}$$

The symmetry energy then will be only

$$\text{E}_{\text{sym}}=29.6\left[1-\frac{1}{2\Delta}\frac{\delta\text{V}}{\text{V}}\right]<29.6, \tag{57}$$

[6] A. E. S. Green, Phys. Rev. **95**, 1006 (1954).

because of the expected smaller volume of the proton distribution.

Using Eq. (45), we get, for the potential energy,

$$V_{12}=m(a\phi-b\phi_0)=73.2 \text{ Mev}, \quad (58)$$

and therefore, for the vector field coupling,

$$b\phi_0=0.363\approx 0.36, \quad (59)$$

We may point out that the saturation condition does not fix the coupling constants a and b but rather the dimensionless products $a\phi$ and $b\phi_0$. The meson field variables are determined completely only if we know the masses of the corresponding mesons, i.e., the ranges of the fields. It may be helpful to express our coupling constants a and b in a form analogous to the fine structure constant $\alpha=1/137$ in electrodynamics.

Using the nucleon Compton wavelength, $\lambda\!\!\!^{-}=m^{-1}$, we get

$$\alpha_1=\tfrac{1}{3}(r_0^3/\lambda\!\!\!^{-3})(a\phi)(1-E_K{}^0/\gamma^2 m)^{-1}(\mu_1^2/m^2), \quad (60)$$

$$\alpha_2=\tfrac{1}{3}(r_0^3/\lambda\!\!\!^{-3})(b\phi_0)(\mu_2^2/m^2). \quad (61)$$

With the values (51), (53), and (59), and $\lambda\!\!\!^{-}=0.210\times 10^{-13}$ cm, therefore,

$$\alpha_1=30.8\mu_1^2/m^2, \quad (62)$$

$$\alpha_2=23.6\mu_2^2/m^2. \quad (63)$$

SURFACE ENERGY

Any statement about the surface energy is quite doubtful since ∇ terms were neglected in the section on Lagrangian and Hamiltonian. Some of these terms give strong negative contributions to the surface energy. Nevertheless let us investigate the contribution to the surface energy arising from the static potential in a special case assuming a constant ρ value up to a radius $R=r_0A^{\frac{1}{3}}$ and $\rho=0$ outside this radius. We can solve the field equations (33) and (34) for ϕ and ϕ_0. This will lead to a potential function which decreases rapidly within a meson Compton wavelength. In addition to the volume term an energy term proportional to $A^{\frac{2}{3}}$ is obtained as a first approximation which is identified as a surface energy. The surface energy per particle is approximately

$$E_s\approx\tfrac{3}{4}m\left[\frac{1}{\mu_1 r_1}a\phi-\frac{1}{\mu_2 r_0}b\phi_0\right]A^{-\frac{1}{3}}. \quad (64)$$

If we assume both meson masses to be of the order of the π-meson mass ($\mu_1=\mu_2=\mu\approx 1/r_0$), we would expect

$$E_s=\tfrac{3}{4}(V_{12}/\mu r_0)A^{-\frac{1}{3}}\approx 55A^{-\frac{1}{3}}(\text{Mev}). \quad (65)$$

As in I, we get quite a big positive contribution from the potential to the surface energy. To account for the relatively small observed surface energy of $E_s\approx 15A^{-\frac{1}{3}}$ Mev, we should assume meson masses higher than the π meson mass. However, such considerations are only meaningful if we introduce all the other possible contributions. As has been pointed out, some contributions are negative and we therefore cannot conclude that the mesons used here are heavier than the π mesons.

SINGLE PARTICLE MOTION

We will concentrate now on the motion of a single particle in the nuclear field which is produced by the common action of all nucleons. This over-all field will have all the features of a classical scalar field in a nonrelativistic sense. However, we have not yet shown that the particles will move in first approximation independently of each other in such a field as is suggested by the success of the shell model. We do not want to prove this now, but simply will assume here that a self-consistent treatment of our theory will approximately lead to such a behavior, at least for the particles of highest energies. We shall give an argument for the weaker statement that actual fluctuations of the potential will be smaller than would follow from additive pair interactions.

In our theory, the nucleons are sources of two kinds of mesons: scalar mesons and vector mesons. The exchange of the first kind leads to attraction, the exchange of the second kind to repulsion. However, the source strength of the scalar mesons is not a constant, but has the form

$$a_{\text{eff}}=a\left[1-\frac{1}{1-a\phi}\frac{p^2}{2m^2}\right], \quad (66)$$

i.e., it decreases with increasing momentum p of the nucleons (measured relatively to the static field) and it decreases for $p\neq 0$ with increasing field amplitude. The latter can be interpreted as a saturation of the scalar meson interaction, an effect which is the stronger the higher the momentum of the nucleon. If two nucleons approach, ϕ, ϕ_0 and the kinetic energy are expected to increase. The greater values of ϕ and the kinetic energy will reduce the attraction, but the repulsive term ϕ_0 is not reduced. Thus one must not expect as great fluctuations in the attractive potential as would result if short-range attractions of particle pairs would be summed.

SPIN-ORBIT COUPLING

Let us now investigate the motion of a single particle in a central nuclear field which is composed of the self-consistent classical meson fields ϕ and ϕ_0. The Dirac Hamiltonian will have the form

$$H=\boldsymbol{\alpha}\cdot\mathbf{p}+\beta m-\beta a m\phi+b m\phi_0. \quad (67)$$

We are only interested in the nonrelativistic limit of this expression. We know that in this limiting process a spin-orbit coupling term ($E_{\text{s.o.}}$) will appear. The contribution of the scalar field to this coupling can be obtained from the well-known Thomas precession,[7]

$$\boldsymbol{\omega}_T = -\tfrac{1}{2}[\dot{\mathbf{v}}\times\mathbf{v}]. \tag{68}$$

This leads to the spin-orbit energy,

$$E_{\text{s.o.}}(\text{scalar}) = +\tfrac{1}{4}\boldsymbol{\sigma}\cdot[\dot{\mathbf{v}}\times\mathbf{v}]. \tag{69}$$

Replacing the acceleration by the force, assuming a central potential $V_s(r)$, introducing the momentum $\mathbf{p}=m\mathbf{v}$ and orbital angular momentum $\mathbf{l}=\mathbf{r}\times\mathbf{p}$, we get the usual form

$$E_{\text{s.o.}}(\text{scalar}) = -\frac{1}{4m^2}\left(\frac{1}{r}\frac{\partial V_s}{\partial r}\right)(\boldsymbol{\sigma}\cdot\mathbf{l}). \tag{70}$$

From the fourth component of a vector field (which behaves like an electrostatic field), we will get the same Thomas precession term since there is no difference in the nature of a scalar field and the fourth component of a vector field in the nonrelativistic limit. However, the particle in its orbital motion will not only experience the fourth component of the field but also an admixture of the other three components. In fact, a particle moving in a pure electric field will experience a magnetic field contribution,

$$\mathbf{H} = -(\mathbf{v}\times\mathbf{E}),$$

which, with

$$e\mathbf{E} = m\dot{\mathbf{v}}$$

and

$$\boldsymbol{\omega}_L = (e/m)\mathbf{H},$$

leads to the Larmor precession

$$\boldsymbol{\omega}_L = \dot{\mathbf{v}}\times\mathbf{v}. \tag{71}$$

The total precession resulting from the vector meson field therefore yields

$$\boldsymbol{\omega}_\Sigma = \boldsymbol{\omega}_T + \boldsymbol{\omega}_L = +\tfrac{1}{2}[\dot{\mathbf{v}}\times\mathbf{v}]. \tag{72}$$

The corresponding spin-orbit energy,

$$E_{\text{s.o.}}(\text{vector}) = +\frac{1}{4m^2}\left(\frac{1}{r}\frac{\partial V_v}{\partial r}\right)(\boldsymbol{\sigma}\cdot\mathbf{l}), \tag{73}$$

is similar to $E_{\text{s.o.}}(\text{scalar})$ except that it carries the opposite sign.

In our theory, we have a superposition of a strong attractive scalar and a strong repulsive vector potential (V_s is negative but V_v positive). Therefore in this case the vector field precession will add to the scalar field precession constructively and so will lead to an extremely strong spin-orbit coupling. This coupling is in addition enhanced by the fact that the effective mass in the interior of the nucleus is reduced. The spin-orbit coupling will also have the correct sign required by the shell model.

The stated results can be obtained by finding the nonrelativistic limit of our Dirac Hamiltonian (67) by using the Foldy-Wouthuysen[8] transformation,

$$\Psi' = e^{iS}\Psi. \tag{74}$$

Here S is some Hermitian operator which will be chosen in such a way that the transformed Hamiltonian,

$$\bar{H}' = e^{iS}He^{-iS}, \tag{75}$$

has only diagonal operators (i.e., operators which do not mix large and small components). This can be accomplished in approximate steps for small momenta ($p\ll m$) and potentials which are sufficiently smooth (their variation within a nucleon Compton wavelength should be small). Then S will be a small operator of order p/m and we can use the expansion

$$\bar{H}' = H + i[S,H] + \tfrac{1}{2}i^2[S,[S,H]] + \cdots. \tag{76}$$

We choose, for our S in first approximation,

$$S = -\frac{i}{4m}\beta\boldsymbol{\alpha}\cdot\left[\frac{1}{1-a\phi}\mathbf{p} + \mathbf{p}\frac{1}{1-a\phi}\right] - \frac{1}{4m(1-a\phi)^2}\boldsymbol{\alpha}\cdot\nabla(b\phi_0), \tag{77}$$

and get

$$H' = \beta m + \beta\frac{1}{8m}\left[\mathbf{p}\frac{2}{1-a\phi}\mathbf{p} + \mathbf{p}^2\frac{1}{1-a\phi} + \frac{1}{1-a\phi}\mathbf{p}^2\right] - \beta am\phi + bm\phi_0 + \frac{1}{8m}\nabla\cdot\left[\frac{\nabla b\phi_0}{(1-a\phi)^2}\right] + \frac{1}{4m(1-a\phi)^2}\boldsymbol{\sigma}\cdot[\nabla(\beta a\phi + b\phi_0)\times\mathbf{p}]. \tag{78}$$

This Hamiltonian is Hermitian. To see this for the last term, one uses the assumption that ϕ and ϕ_0 are central fields. Positive and negative energy states are clearly separated up to order of p^2 (higher orders were dropped). For the nucleons we have to take the upper rows of the Dirac operators, i.e., $\beta\to 1$. We at once recognize the very large spin-orbit coupling term,

$$H_{\text{s.o.}} = \frac{1}{4m(1-a\phi)^2}\boldsymbol{\sigma}\cdot[\nabla(a\phi+b\phi_0)\times\mathbf{p}] = \frac{1}{4m(1-a\phi)^2}\frac{1}{r}\frac{\partial}{\partial r}(a\phi+b\phi_0)(\boldsymbol{\sigma}\cdot\mathbf{l}). \tag{79}$$

In calculations with the shell model, it is customary to introduce the proposed strong spin-orbit coupling term in the form of the Thomas precession term multiplied by a large phenomenological factor λ, i.e.,

$$H_{\text{s.o.}}{}^T = \lambda\frac{1}{4m^2}\left(\frac{1}{r}\frac{\partial V}{\partial r}\right)(\boldsymbol{\sigma}\cdot\mathbf{l}). \tag{80}$$

[7] L. H. Thomas, Phil. Mag. **3**, 1 (1927). D. R. Inglis, Phys. Rev. **50**, 783 (1936); Phys. Rev. **56**, 1175 (1939).

[8] L. L. Foldy and S. A. Wouthuysen, Phys. Rev. **78**, 29 (1950).

The potential usually chosen for heavy nuclei, if we use $r_0=1.22\times10^{-13}$ cm, is a square well of depth

$$V_0=+40\text{ Mev}=v_0m=0.043m. \qquad (81)$$

Let us assume for a first crude estimate that the eigenfunction for a particle in our velocity dependent potential is similar to the eigenfunction of a particle in an ordinary potential of 40 Mev with the same binding energy. Then we can compare the spin-orbit energies,

$$E_{\text{s.o.}}{}^{T}=\lambda\frac{\boldsymbol{\sigma}\cdot\mathbf{l}}{4m^2}\int_0^\infty\psi^{*T}(r)\frac{1}{r}\frac{\partial V}{\partial r}\psi^T(r)r^2dr$$
$$=\frac{R(\boldsymbol{\sigma}\cdot\mathbf{l})}{4m}\lambda v_0\,|\psi^T(R)|^2 \qquad (82)$$

for the phenomenological expression, and

$$E_{\text{s.o.}}=\frac{\boldsymbol{\sigma}\cdot\mathbf{l}}{4m}\int_0^\infty\psi^*(r)\frac{1}{(1-a\phi)^2}\frac{1}{r}\frac{\partial}{\partial r}(a\phi+b\phi_0)\psi(r)r^2dr$$
$$=\frac{R(\boldsymbol{\sigma}\cdot\mathbf{l})}{4m}\frac{a_0+b_0}{1-a_0}\,|\psi(R)|^2 \qquad (83)$$

from our theory, if we define inside the well the constants $a_0=a\phi$, $b_0=b\phi_0$. With the values a_0 and b_0 of Eqs. (53) and (59),

$$\lambda=\frac{a_0+b_0}{(1-a_0)v_0}\frac{|\psi(R)|^2}{|\psi^T(R)|^2}\approx33\frac{|\psi(R)|^2}{|\psi^T(R)|^2}. \qquad (84)$$

The value $\lambda=33$ which one obtains for $\psi(R)=\psi^T(R)$ is in reasonable agreement with the estimated phenomenological coupling.[9,10] A more careful consideration of the level ordering in nuclei with our velocity dependent potential may require higher values of λ. At the same time the change in effective mass near the surface leads to

$$|\psi(R)|^2>|\psi^T(R)|^2. \qquad (85)$$

A detailed discussion of this question will be given elsewhere. We note here that the serious discrepancy between the ordinary Thomas term and the phenomenological spin-orbit coupling is removed.

INTERACTIONS OF NUCLEONS WITH NUCLEI

We calculate the effective potential which acts on a nucleon of kinetic energy $E'=E-m$ impinging on a heavy nucleus at rest. We assume that the potential of the nucleus will not change appreciably by the presence of the bombarding nucleon. This approximation should be quite good for heavy nuclei since here the fluctuation in nuclear density caused by the addition of the incoming particle to the nucleus will be small and the nucleus itself is in a state of minimum energy which does not change with small changes of ρ.

For constant values of ϕ and ϕ_0, the Hamiltonian operator (67) for a nucleon reduces inside the nucleus to the form

$$H=[m^2(1-a\phi)^2+p^2]^{\frac{1}{2}}+bm\phi_0=E, \qquad (86)$$

where a, b, ϕ, and ϕ_0 have the same meaning as in the last sections. We will consider this energy as resulting from an effective potential V_{eff}, which we define by the equation

$$H=[m^2+p^2]^{\frac{1}{2}}+V_{\text{eff}}=E. \qquad (87)$$

By elimination of p^2 from Eqs. (86) and (87) V_{eff} can be expressed in terms of the total energy E of the bombarding nucleon. For convenience, we introduce the dimensionless quantities,

$$\epsilon=E/m, \qquad (88)$$

$$v_{\text{eff}}=V_{\text{eff}}/m, \qquad (89)$$

$$a_0=a\phi=0.44, \qquad (90)$$

$$b_0=b\phi_0=0.36, \qquad (91)$$

and get

$$v_{\text{eff}}=\epsilon-[(\epsilon-b_0)^2+a_0(2-a_0)]^{\frac{1}{2}}. \qquad (92)$$

In Fig. 1, v_{eff} is plotted against ϵ. We observe that the effective potential is negative up to energies $\epsilon=1.13$ ($E'=121$ Mev) and then becomes repulsive for higher particle energies.

The effective potential can be compared approximately with the real (dispersive) part of the potential introduced in the optical model. For very small kinetic energies ($E'\rightarrow0$), we find

$$V_{\text{eff}}=-0.047m=-43.8\text{ Mev}. \qquad (93)$$

From the position of the giant S-wave neutron reso-

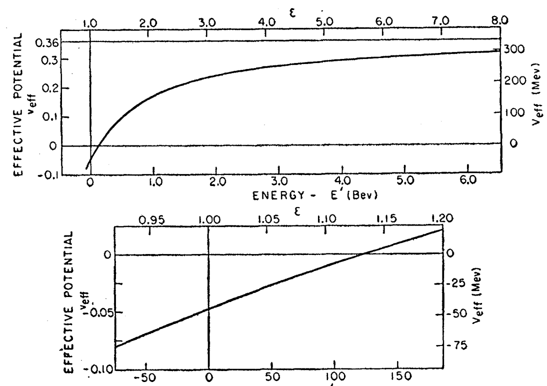

FIG. 1. The effective potential, v_{eff}, for nucleons is plotted against nucleon energy. The upper panel shows the behavior between 0 and 6 Bev. The lower panel shows the same curve on a larger scale for low energies including negative energies which correspond to bound states in the nuclear well. The starting point of the curve is determined by the depth of the nuclear potential for zero kinetic energy.

[9] W. Heisenberg, *Theorie des Atomkerns* (Max Planck Institute, Göttingen, 1951).
[10] Ross, Mark, and Lawson, Phys. Rev. **102**, 1613 (1956).

nances at low energies Adair[11] and Feshbach, Porter, and Weisskopf[12] estimated the depth of the ordinary square well potential with the radius $R=1.45\times10^{-13}A^{\frac{1}{3}}$ cm to be $V_0=42$ Mev. Recent calculations by Lawson[13] and Ross, Mark, and Lawson[10] with a diffuse well,

$$V=V_0[1+e^{\alpha'(r-R)}]^{-1},$$

indicate that a potential depth $V_0=42.8$ Mev with a radius $R=1.3\times10^{-13}A^{\frac{1}{3}}$ cm and $\alpha'=1.45\times10^{13}$ cm^{-1} also reproduces the experimental data. Comparing the values V_0R^2 (which roughly takes into account the different definitions of R) with our $V_{\text{eff}}R^2$ as given by (93) and (51) leads to a fair agreement. We will point out, however, that this comparison can be made only in an approximate way since the behavior of the wave functions on the surface of a velocity-dependent well will be a little different from the behavior on the surface of an ordinary well. The correct resonance positions will therefore be reproduced at a slightly different (smaller) value for the effective depth of the potential than given by these authors.

The analysis of proton elastic scattering at 5, 17, and 31 Mev on the basis of a diffuse well[14] indicates an energy dependence of the real part of the potential in agreement with our theory.

Calculations by Taylor[15] based on total neutron cross sections between 30 and 400 Mev[16] indicate a strong decrease of the real part of the potential around 120 Mev. On the basis of our theory such an effect should be expected. Calculations of cross sections in collisions of nucleons with heavy nuclei will be given in a later paper.

INTERACTIONS OF ANTINUCLEONS WITH NUCLEI

Up to now we have discussed the interactions of nucleons with nuclei. It is possible, however, to draw some conclusions on the negative energy states, i.e., the antinucleons. Our Dirac Hamiltonian (67) and its nonrelativistic limit (78) clearly exhibits a striking difference in the behavior of nucleons and antinucleons. In case of nucleon-nucleus interaction the difference of the scalar meson and vector meson potential constitutes the effective potential. In case of antinucleon-nucleus interactions the vector meson potential also becomes attractive since it is similar to the electrostatic field in the case of opposite charges. Therefore it will add to the scalar meson potential which is always attractive. On the other hand the spin-orbit coupling will be only of the order of the ordinary Thomas term.

We can describe the interaction of heavy nuclei with the bombarding antinucleons of total energy E by an

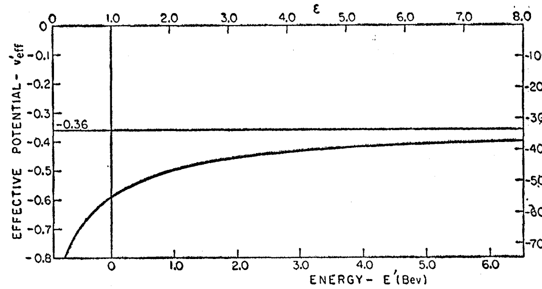

FIG. 2. The effective potential, v'_{eff}, for antinucleons is plotted against antinucleon energy. In contrast to the nucleon potential, this potential is always negative.

effective potential,

$$V'_{\text{eff}}=v'_{\text{eff}}m. \tag{94}$$

We only have to give b_0 the opposite sign in (86) and (92). We get

$$v'_{\text{eff}}=\epsilon-[(\epsilon+b_0)^2+a_0(2-a_0)]^{\frac{1}{2}}. \tag{95}$$

In Fig. 2, this effective potential is plotted against the total energy $E=\epsilon m$, of the bombarding antinucleon. We observe that this potential is always strongly attractive. The decreasing attraction with increasing energy is again due to the decreasing contribution of the scalar interaction.

The strong attraction between nucleons and antinucleons may give rise to a bound state of the nucleon-antinucleon system[17] analogous to the electron-positron system. Perhaps this could lead, as in the case of the positronium, to a ^1S ground state which would exhibit the properties of a pseudoscalar meson. Since our theory is not based on the π-meson field it may be quite satisfactory to explain the π meson as a consequence of these nuclear interactions.

As a consequence of this strong attraction a cross section higher than the geometric cross section was predicted[18] for collisions of high energy antinucleons with nuclei. Recent measurements on the attenuation of the intensity of the antiproton "beam" by Cu and Be at the Berkeley Bevatron, indeed, seem to indicate such a result.[19,20]

We will try to give an estimate of this cross section. As in the section on surface energy we describe the nucleus by a meson source distribution of constant density ρ_0 up to a radius $R=r_0A^{\frac{1}{3}}$ and $\rho=0$ outside this region. This will give rise to an effective potential for antinucleons which decreases exponentially for $r>R$ (Fig. 3)

$$V'=\tfrac{1}{2}V'_{\text{eff}}(R/r)e^{-\alpha(r-R)}. \tag{96}$$

Here α is a measure for the slope of the potential which

[11] R. K. Adair, Phys. Rev. **94**, 737 (1954).
[12] Feshbach, Porter, and Weisskopf, Phys. Rev. **96**, 448 (1954).
[13] R. D. Lawson, Phys. Rev. **101**, 311 (1956).
[14] Melkanoff, Moszkowski, Nodvik, and Saxon, Phys. Rev. **101**, 507 (1956).
[15] T. B. Taylor, Phys. Rev. **92**, 831 (1953).
[16] J. de Juren and B. Moyer, Phys. Rev. **81**, 919 (1951); A. E. Taylor and E. Wood, Phil. Mag. **44**, 95 (1953).
[17] E. Fermi and C. N. Yang, Phys. Rev. **76**, 1739 (1949).
[18] H.-P. Duerr and E. Teller, Phys. Rev. **101**, 494 (1956).
[19] Brabant, Cork, Horowitz, Moyer, Murray, Wallace, and Wenzel, Phys. Rev. **101**, 498 (1956).
[20] Chamberlain, Keller, Segrè, Steiner, Wiegand, and Ypsilantis, Phys. Rev. **102**, 1637 (1956).

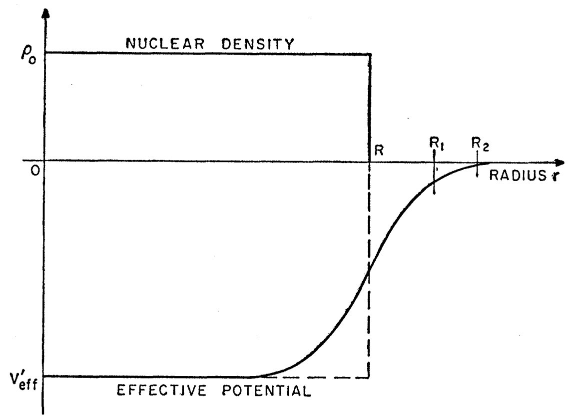

FIG. 3. The nucleon density and effective potential assumed in the calculation of the antinucleon cross section is shown in this figure.

in general will not only depend on the meson masses of the vector and the scalar field, but perhaps can have contributions from other fields which were dropped in the calculation of the volume energy. It also will depend on the slope of the ρ distribution which has been assumed here to be infinitely steep. We therefore will take for α some semiempirical value as suggested by calculations on energy levels in nuclei using diffuse wells.[10]

For simplicity we will treat the high-energy antinucleons as classical particles which have a certain impact parameter d. This approximation will be valid for momenta $p \gg (\hbar/4E')(\partial V/\partial r)$. In this classical picture we then have to distinguish four cases:

(1) Particles with impact parameter $d \leq R$ will interact with the nucleons. We will assume that they are all annihilated.

(2) Particles with impact parameter $R < d \leq R_1$ will be strongly deflected by the attractive field and as a consequence fall into the nucleus, thus increasing the annihilation cross section from πR^2 to πR_1^2.

(3) Particles with impact parameter $R_1 < d \leq R_2$ will be elastically scattered and will contribute to the scattering cross section.

(4) Particles with impact parameter $d > R_2$ will be scattered by angles smaller than the angle ϑ_0 given by the geometry of the experiment and will not contribute to the cross section.

In case of gradually decreasing ρ-distribution and a finite "annihilation length," (3) would also include annihilation and inelastic scattering events.

In an attenuation experiment with poor geometry, where shadow scattering is not measured, the total cross section for high-energy particles will therefore be

$$\sigma = \pi R_2^2 = f\pi R^2, \tag{97}$$

where

$$f = R_2^2(E,\vartheta_0)/R^2 \tag{98}$$

gives the factor multiplying the geometrical cross section for a certain particle energy E and cutoff angle ϑ_0 of the experimental arrangement.

We consider an impinging antinucleon of classical energy

$$H = [m^2(1-a\phi)^2 + p^2]^{\frac{1}{2}} - bm\phi_0 = E, \tag{99}$$

and impact parameter d in its incident plane (Fig. 4). ϕ and ϕ_0 will be constant inside the nucleus but will have an exponential dependence on r outside the nucleus. In the Hamiltonian operator, we therefore have to retain the ∇ terms which appear from the commutators of p with the potentials and which are proportional to $\hbar$. In a classical calculation these terms can all be neglected ($\hbar \to 0$) and the Hamiltonian reduces to its classical form (99).

According to the first Hamilton equation, the change in momentum will be

$$d\mathbf{p}/dt = -(\nabla H)_p, \tag{100}$$

where

$$\frac{d\mathbf{p}}{dt} = \frac{dp}{dt}\mathbf{s}^0 + \left[\frac{d\chi}{dt} \times \mathbf{p}\right]. \tag{101}$$

Here $\mathbf{s}^0$ is the unit vector in the direction of the path of the antinucleon and $d\chi^0$ is the infinitesimal rotation of this direction. We are only interested in small deflections and consequently will replace in the calculation of the force, instantaneous velocity and momentum the actual path by the straight line

$$d = r\cos\delta, \tag{102}$$

where (r,δ) are the polar coordinates of the particle. However, the changes in velocity due to the presence of a velocity-dependent potential will be taken into account. The velocity of the particle $v(r) = ds/dt$ of certain impact parameter d along the path is given by

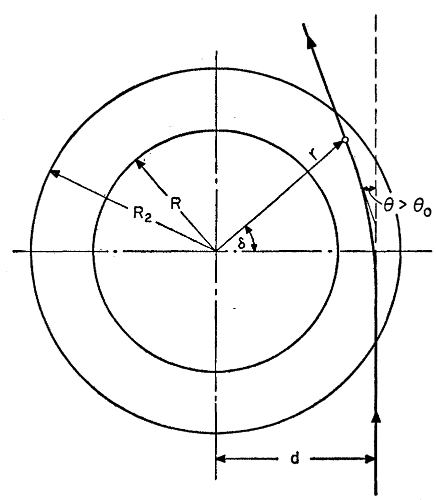

FIG. 4. The classical orbit of the antinucleon in the nuclear field.

the second Hamilton equation,

$$v(r)=\partial H/\partial p(r), \quad (103)$$

where $p(r)$ is the momentum along the assumed straight path (102) at r.

The radial velocity will be

$$dr/dt=v(r)\sin\delta, \quad (104)$$

and we get, from (100) and (101),

$$p(r)v(r)\partial\chi/\partial r=(\partial H/\partial r)\cot\delta. \quad (105)$$

With (102), this yields the total deflection,

$$\vartheta=2\int_0^\infty \frac{d}{[r^2-d^2]^{\frac{1}{2}}}\frac{1}{p(r)v(r)}\left(\frac{\partial H}{\partial r}\right)dr. \quad (106)$$

Assuming that ϕ and ϕ_0 have the same functional dependence on r as given by (96), we introduce the dimensionless quantities,

$$a\phi=\tfrac{1}{2}a_0 z, \quad (107a)$$

$$b\phi_0=\tfrac{1}{2}b_0 z, \quad (107b)$$

and

$$z=\frac{R}{r}e^{-\alpha(r-R)}$$
$$=\frac{R}{d}\exp[-\alpha(d-R)]\exp[-\bar{\alpha}(r-d)], \quad (108)$$

with approximately

$$\bar{\alpha}=\alpha[1+(1/\alpha d)]. \quad (109)$$

We can write

$$F(z)=\frac{1}{pv}\frac{\partial H}{\partial r}$$
$$=\tfrac{1}{2}\bar{\alpha}z\frac{(a_0+\epsilon b_0)-\frac{1}{2}(a_0^2-b_0^2)z}{(\epsilon^2-1)+(a_0+\epsilon b_0)z-\frac{1}{4}(a_0^2-b_0^2)z^2}. \quad (110)$$

For high energies E (small deflections), i.e.,

$$(\epsilon^2-1)\gg(a_0+\epsilon b_0)z$$
$$\gg\tfrac{1}{4}(a_0^2-b_0^2)z^2, \quad (111)$$

the denominator can be expanded and we are led to

$$F(z)=\tfrac{1}{2}\bar{\alpha}z\frac{a_0+\epsilon b_0}{\epsilon^2-1}[1-A_1z+A_2z^2-+\cdots], \quad (112)$$

with

$$A_1=\left[\frac{a_0+\epsilon b_0}{\epsilon^2-1}+\frac{1}{2}\frac{a_0^2-b_0^2}{a_0+\epsilon b_0}\right], \quad (113a)$$

$$A_2=\frac{1}{\epsilon^2-1}\left[\frac{(a_0+\epsilon b_0)^2}{\epsilon^2-1}+\tfrac{3}{4}(a_0^2-b_0^2)\right]. \quad (113b)$$

Since the function $F(z)$ is proportional to z and will consequently decrease very rapidly with r, we can approximate the square root in the integral (106) by

$$[r^2-d^2]^{-\frac{1}{2}}=[2d(r-d)]^{-\frac{1}{2}}\left[1-\frac{r-d}{4d}+\cdots\right]. \quad (114)$$

Introducing the notation

$$y=[\bar{\alpha}(r-d)]^{\frac{1}{2}}, \quad (115)$$

the integral (106) can be written

$$\vartheta=[2\bar{\alpha}d]^{\frac{1}{2}}\frac{a_0+\epsilon b_0}{\epsilon^2-1}\frac{R}{d}e^{-\alpha(d-R)}\int_0^\infty\left\{\left[1-\frac{y^2}{4\bar{\alpha}d}\right]\exp(-y^2)\right.$$
$$\times\left[1-A_1\frac{R}{d}e^{-\alpha(d-R)}\exp(-y^2)\right.$$
$$\left.\left.+A_2\frac{R^2}{d^2}e^{-2\alpha(d-R)}\exp(-2y^2)-+\cdots\right]\right\}dy. \quad (116)$$

This gives

$$\vartheta=\left[\frac{\pi\alpha d}{2}\right]^{\frac{1}{2}}\frac{a_0+\epsilon b_0}{\epsilon^2-1}\frac{R}{d}e^{-\alpha(d-R)}$$
$$\times[I_0-A_1I_1+A_2I_2-\cdots], \quad (117)$$

with

$$I_0=\left(1+\frac{3}{8\alpha d}\right), \quad (118a)$$

$$I_1=\frac{1}{\sqrt{2}}\left(1+\frac{7}{16\alpha d}\right)\frac{R}{d}e^{-\alpha(d-R)}, \quad (118b)$$

$$I_2=\frac{1}{\sqrt{3}}\left(1+\frac{11}{24\alpha d}\right)\frac{R^2}{d^2}e^{-2\alpha(d-R)}. \quad (118c)$$

The deflection angle depends exponentially on the impact parameter. According to (111) the expansion in brackets of (117) converges rapidly for $\vartheta\ll[\frac{1}{2}\pi\alpha R]^{\frac{1}{2}}$.

For a rough estimate of ϑ, we use the first term in the expansion (i.e., $A_1I_1=A_2I_2=0$ and $I_0=1$) and set $(R/d)^{\frac{1}{2}}\approx\exp[-(d-R)/2R]$. This gives

$$\vartheta=\left[\frac{\pi\alpha R}{2}\right]^{\frac{1}{2}}\frac{a_0+\epsilon b_0}{\epsilon^2-1}\exp\left[-\alpha\left(1+\frac{1}{2\alpha R}\right)(d-R)\right]. \quad (119)$$

For the calculation of the cross section, we have to express the impact parameter in terms of the deflection angle. An experimental arrangement with a cutoff angle ϑ_0 will measure a collision cross section equal to the geometric cross section times the factor (98):

$$f=\left[1+\frac{1}{0.5+\alpha R}\ln\left(\frac{(\pi\alpha R/2)^{\frac{1}{2}}}{\vartheta_0}\frac{a_0+\epsilon b_0}{\epsilon^2-1}\right)\right]^2. \quad (120)$$

Chamberlain *et al.*[20] have measured the cross section of antiprotons of average kinetic energy $E'=430$ Mev ($\epsilon=1.4$) in an attenuation experiment with Cu ($A=63$). The cutoff angle is $\vartheta_0=13°$ which excludes the shadow scattering in the cross section. With the choice of

$$\alpha=1.0\times10^{13}\ \text{cm}^{-1}, \tag{121}$$

and the values of a_0 and b_0 given in (53) and (59), we get

$$f_{\text{Cu}}=2.15. \tag{122}$$

Using (38) and (51), the cross section will be

$$\sigma_{\text{Cu}}=1.59\times10^{-24}\ \text{cm}^2. \tag{123}$$

The more exact calculation on the basis of (117) gives $f_{\text{Cu}}=2.17$ and $\sigma_{\text{Cu}}=1.60\times10^{-24}$ cm^2 which shows that our crude approximation (119) is already very good.

The experimental value

$$(\sigma_{\text{Cu}})_{\text{exp}}=(1.58\pm0.22)\times10^{-24}\ \text{cm}^2 \tag{124}$$

is in good agreement with our result.

The calculated value for σ, however, will depend strongly on the choice of α, which is only known within wide limits. Ross, Mark, and Lawson[10] have calculated the energy levels in nuclei with the Saxon potential

$$V=V_0[1+e^{\alpha'(r-R)}]^{-1},$$

and found that the choice $\alpha'=1.45\times10^{13}$ cm^{-1} led to an improved level ordering. This α' is not directly comparable with our α. However, we can show that a choice $\alpha=1.0\times10^{13}$ cm^{-1} will lead to a potential of similar slope to the Saxon potential with $\alpha'=1.45\times10^{13}$ cm in the region $d\approx R_2\approx1.5R$ which is of interest to us.

Similar calculations on Be^9 ($E'=455$ Mev, $\vartheta_0=19°$) gives $f_{\text{Be}}=2.51$, i.e., $\sigma_{\text{Be}}=0.508\times10^{-24}$ cm^2 which is to be compared with the experimental value of $(0.365\pm0.059)\times10^{-24}$ cm^2. The application of our theory to this very light nucleus is, however, quite doubtful.

On the basis of our classical model, we can also estimate the annihilation cross section, which we define by

$$\sigma_a=f_a\pi R^2, \tag{125}$$

with

$$f_a=R_1^2(E)/R^2. \tag{126}$$

The factor f_a can be determined in a classical manner and depends on the energy of the bombarding antinucleon. In this calculation we explicitly have to take into account the deviation of the particle orbit from a straight line in the field of the nucleus.

In case of the 430-Mev antiprotons of the Cu attenuation experiment, we get

$$(f_a)_{\text{Cu}}=1.81. \tag{127}$$

The comparison with (121) shows that approximately 84% of the collision cross section is due to annihilation.

The analogous calculation for Be^9 ($E'=455$ Mev) yields $(f_a)_{\text{Be}}=1.76$, which implies by comparison with the calculated value for the collision cross section that here only 70% of the collision cross section is due to annihilation.

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