Remarks on Recent Measurements of the Paramagnetic Effect in Tin*

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It is shown that the measurements of Shibuya and Tanuma substantiate rather than invalidate the statement that the maximum permeability in the paramagnetic effect occurs for given values of the current I and the external field H at a temperature where the total magnetic field at the surface of the sample is equal to the critical field. It is further shown that their new equations, $I_0=\xi\gamma^*d(T_c-T)/4$ and $H_0=\xi(T_c-T)-4I_g/\gamma^*d$, follow as a first approximation from the statement that the maximum permeability depends only on $\gamma=4(I-I_g)/Hd$.

SHIBUYA and Tanuma have recently reported some measurements of the paramagnetic effect in tin.[1,2] The authors claim that their measurements are inconsistent with statements brought forth earlier.[3] These statements were:

1. The maximum apparent permeability μ^* (denoted by $\tilde{K}_m$ in reference 3) occurs for a given current, I and external field, H, at a temperature where the total magnetic field H_t at the surface of the sample equals the critical field H_c: $H_t=[H^2+(4I/d)^2]^{\frac{1}{2}}=H_c(T)$, where H is measured in oersteds, I in amperes, and the diameter d of the sample in mm.
2. The maximum apparent permeability is a function of $\gamma=4(I-I_g)/Hd$.

Our γ is dimensionless and considered as a variable. I_g is a constant characteristic of the metal. The exact dependence of μ^* on γ may vary to some extent from sample to sample. For values of $1\leq\mu^*\leq 2$ this dependence can be approximated by $\mu^*-1=\alpha(\gamma-\gamma^*)$, where γ^* is another constant, which is probably also characteristic for each metal. It has a value which is numerically four times greater than that of the constant γ (later denoted by $\gamma_{S\text{-}T}$) which Shibuya and Tanuma use.

Shibuya and Tanuma claim that the maximum permeability occurred in all their measurements at values of $H_t<H_c$. They try to prove this in Fig. 18 of reference 1, which shows a plot of I *versus* H with μ as parameter for a temperature of $T=3.590$°K. Instead of basing the argument on the measurements at one temperature only, we have calculated H_t for all their measurements of μ^* on sample 1 and plotted H_t *versus* T in Fig. 1. It is true that most of their measurements lie below the curve $H_t=138(T_c-T)$ which the authors claim to be the correct H_c-T relationship near T_c. But the value of $(dH_c/dT)_{T_c}$ varies to some extent from sample to sample. Lock, Pippard, and Shoenberg[4] report that $(dH_c/dT)_{T_c}=-152$ oersteds per degree. We find that most of the measurements of Shibuya and Tanuma are on the curve $H_c=130(T_c-T)$. Some measurements, however, are made with T and I fixed such that already $(4I/d)>H_c$. For all values of $H<H_c$ the sample consists in these cases of an intermediate core surrounded by a normal conducting sheath. (This does not prevent μ from going through a pseudomaximum if H is varied.) It has been shown[5] that one can calculate the over-all μ for these cases quite accurately. We will, however, refrain here from going into these calculations and disregard those points. The slight scattering which remains in the H_c-T values, even after adjustment of the slope and subtraction of the points for which already $(4I/d)>H_c$, is probably due to slight variations in the temperature. It is to be noted that a devia-

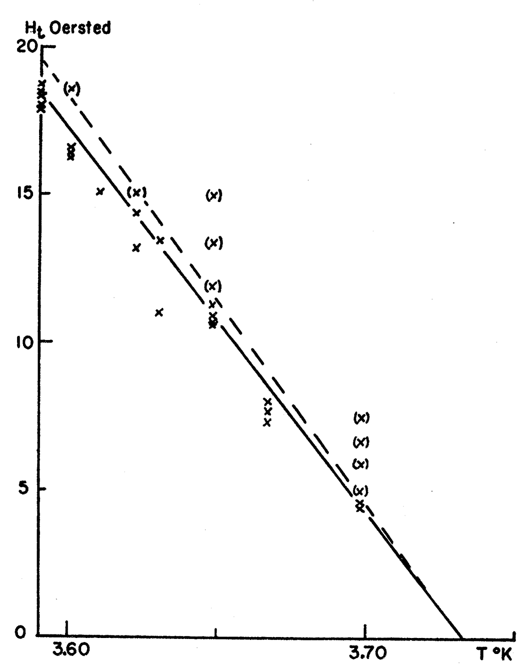

FIG. 1. Total magnetic field H_t at the surface of the sample as a function of the temperature T as calculated from Shibuya and Tanuma's measurements of the maximum permeability μ^* on their sample No. 1. The points enclosed in parentheses (×) are taken at values where $(4I/d)>H_c$. Broken curve: $H_c=138(T_c-T)$; solid curve: $H_c=130(T_c-T)$.

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[3] Hans Meissner, Phys. Rev. **97**, 1627 (1955).
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[5] Hans Meissner, Phys. Rev. **101**, 31 (1956).

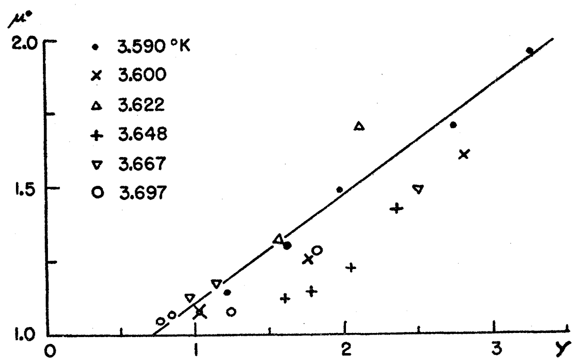

FIG. 2. Maximum permeability μ^* as a function of $\gamma = 4(I - I_g)/Hd$, as calculated from Shibuya and Tanuma's measurements on their sample No. 1. All the points for which $(4I/d) > H_c$ have been omitted. Solid curve: $\mu^* - 1 = \alpha(\gamma - \gamma^*)$, with $\gamma^* = 0.7$.

tion of 1 oersted corresponds to only 0.0072°K. It is therefore believed that the measurements of Shibuya and Tanuma substantiate rather than invalidate our first statement.

We have further calculated γ from their measurements and plotted μ^* as a function of γ in Fig. 2, omitting all measurements where $(4I/d) > H_c$. There is a considerable scattering around the curve $\mu^* - 1 = \alpha(\gamma - \gamma^*)$. This scattering, however, is typical for all measurements of the paramagnetic effect and increases with a decrease in sample size. We believe, therefore, that $\mu^* - 1 = \alpha(\gamma - \gamma^*)$ is a valid approximation in spite of the scattering.

We want now to demonstrate that the functional relationships shown in Figs. 8, 10, 12, and 13 of reference 1 and expressed in Eqs. (2) and (3) of reference 1 can be explained by using our two statements, the equation $\mu^* - 1 = \alpha(\gamma - \gamma^*)$ and the relation between H_c and T.

In their Fig. 8 Shibuya and Tanuma plot μ^* as function of I with T as parameter. $T = \text{const}$ means $H_c = \text{const}$ and we find that the equation $\mu^* - 1 = \alpha(\gamma - \gamma^*)$ has for this case the form

$$\mu^* - 1 = \alpha\left(\frac{4(I - I_g)}{d[H_c^2 - (4I/d)^2]^{\frac{1}{2}}} - \gamma^*\right). \qquad (1)$$

This equation represents the general trend of the measurements fairly well as long as $(4I/d) < H_c$. μ^* is larger than one as long as $I > I_0$, where I_0 is the value of I which makes the right side of Eq. (1) vanish. I_0 is the solution of a quadratic equation containing H_c, I_g, d, and γ^*. Since most of the measurements of I_0 are made in a region where $(H_c d/4I_g) \gg 1$ (see Fig. 12 of reference 1) we can write in first approximation

$$I_0 = \frac{\gamma^* H_c d}{4(1+\gamma^{*2})^{\frac{1}{2}}} = -\frac{dH_c/dT}{(1+\gamma^{*2})^{\frac{1}{2}}}\frac{\gamma^* d}{4}(T_c - T). \qquad (2)$$

Observing that our $\gamma^* = 4\gamma_{S\text{-}T}$, we find by comparison of Eq. (2) with Eq. (2) of reference 1 that the constant ξ introduced by Shibuya and Tanuma is given by $\xi = -(1+\gamma^{*2})^{-\frac{1}{2}}(dH_c/dT)$. This gives with $dH_c/dT = -130$ and $\gamma^* = 0.7$ a value of $\xi = 107$ oersted per degree as compared to their value of $\xi = 112$ oersted per degree.

We can, on the other hand, also express μ^* as a function of H^* with T as parameter (fixed H_c). H^* is the value of H which, for a given $I < H_c d/4$, makes $H_t = H_c$. We find, as in Eq. (1), that

$$\mu^* - 1 = \alpha\left(\frac{[H_c^2 - H^{*2}]^{\frac{1}{2}} - 4I_g/d}{H^*} - \gamma^*\right). \qquad (3)$$

This represents again the general trend of the curves in Fig. 10 of reference 1 fairly well as long as $I < H_c d/4$. μ^* is larger than one if $H^* < H_0$, where H_0 is the value of H^* which makes the right side of Eq. (3) vanish. H_0 is readily found as the solution of a quadratic equation containing H_c, I_g, d, and γ^*. For values of $H_c \gg 4I_g/d$, this equation can be greatly simplified. Observing that $\gamma^* = 4\gamma_{S\text{-}T}$ and $\xi = -(1+\gamma^{*2})^{-\frac{1}{2}}(dH_c/dT)$ we find finally

$$H_0 = \xi(T - T_c) - \frac{I_g}{\gamma_{S\text{-}T} d}\frac{\gamma^{*2}}{1+\gamma^{*2}}. \qquad (4)$$

Equation (4) differs from Eq. (3) of reference 1 by a factor $\gamma^{*2}/(1+\gamma^{*2}) = 0.3$ in the second (constant) term. Figure 13 of reference 1 shows, moreover, that the measured points can be fitted much better by a quadratic equation and that the simplification which was used in the derivation of Eq. (4) actually should not be applied. One finds readily from the correct solution for H_0 that the constant T_g which Tanuma and Shibuya use is the temperature at which $H_c = 4I_g/d$, i.e., $T_g = T_c + (4I_g/d)(dH_c/dT)^{-1}$.

It should further be noted that the authors use $\gamma_{S\text{-}T}$ in Eqs. (2) and (3) of reference 1 as a dimensionless quantity, but in Eq. (4) of reference 1 they use it as a constant of the dimension ampere/millimeter oersted. The numerical value which they use is that of the constant with dimension, while the dimensionless quantity is four times larger.

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