α-Particle Model of O^{16}†*

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Lifetimes of the four lowest excited states of O^{16} were calculated on the α-particle model and are here compared with experimentally observed values. In addition, the computation of energy levels on each of the two possible level identification schemes has been extended to include all predicted energy levels up to 16 Mev, and compared with the current experimental data. Lastly, the α-particle model was used to provide the core wave function of a partially-excited-core shell model of O^{17}, and the lifetime of the $\frac{1}{2}^+$ 870-kev state of O^{17} and the electric quadrupole and magnetic moments of the ground state were computed and are compared with experimental values.

The energy level predictions on one identification scheme are in excellent agreement with experimental values in the 0–12.95 Mev range, in that of nineteen observed excited levels, fifteen can be matched within 1.1 Mev and three others tentatively matched (depending upon experimental determination of spins and parities); while two levels are predicted but not observed. The lifetime calculations agree to within factors of 15 with experimental values, except for an electric dipole transition which is forbidden on the model. The O^{17} calculations are useful only in evaluating one of the lifetime calculations.

THE lifetimes of the lowest excited states of the O^{16} nucleus have recently been determined by Devons[1] and found not to agree with those calculated on the single-particle model. Dennison[2,3] found previously that the energies of the low-lying levels of the O^{16} nucleus appear to agree better with the predictions of an α-particle model than with those of a single-particle or liquid-drop model. It therefore appeared somewhat promising to compute the lifetime of the lower excited states on the α-particle model.

In the course of the work the energy levels themselves were computed, paralleling the computations of Dennison. Several additional levels were found, and the energies of a few of the levels corrected.[4] In addition, the present work attempts to assign those energy levels which have been reported between October, 1954 and December, 1955 by Bittner and Moffat[5] (reviewed by Ajzenberg and Lauritsen[6]), and by Hornyak and Sherr[7] and Wilkinson, Toppel, and Alburger.[8]

Lastly, the wave function for the 2^+ 7.12-Mev state of O^{16} was used to describe the excited core state of a partially excited core shell model of O^{17}, in which Thirion and Telegdi[9] had found the lifetime of the $\frac{1}{2}^+$ 870-kev state shorter than that predicted on a strict shell model, but compatible with a model in which the O^{16} core was regarded as partially excited to a 2^+ state.

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INTRODUCTION

In the α-particle model, the O^{16} nucleus is regarded as a semirigid harmonically bound structure of four α particles whose equilibrium configuration is a tetrahedron. Excited energy levels are then ascribed to rotations and/or vibrations of the structure, the wave functions of which must, in accordance with the Bose statistics of α particles, be symmetric in the four particles. Once the parameters of the structure are fixed, the energy levels and rates of transitions between these are completely determined.

Although in principle, especially if only two-body forces were important, the parameterization of the structure could be effected from a knowledge of α-α forces; a simpler procedure is to deduce the parameters from a set of known energy levels. The latter procedure, which was used by Dennison,[2,3] and will be adopted here, is also safer in that three- and four-body forces may be included implicitly.

Thus the α-particle model is in effect a mathematical structure which relates certain observed quantities (lifetimes and energies) to other observed quantities (a specific set of energies), and the validity of the model may be judged according to the accuracy of its predictions.

I. THE HAMILTONIAN

The Hamiltonian which we employ is the zero-order approximation to the semirigid rotator Hamiltonian[2]:

$$H=\tfrac{1}{2}w_0(\mathbf{J}-\zeta\mathbf{L})^2+\tfrac{1}{2}w_1\left(\frac{p_1^2}{\hbar^2}+q_1^2\right)$$
$$+\tfrac{1}{2}w_2\sum_{i=2}^{3}\left(\frac{p_i^2}{\hbar^2}+q_i^2\right)+\tfrac{1}{2}w_3\sum_{i=4}^{6}\left(\frac{p_i^2}{\hbar^2}+q_i^2\right),$$

in which we use the following notation (which differs from that of Dennison[2,3]): $w_0=\hbar^2/I_0$; I_0=moment of inertia=(8/3) Ma^2, where M=α-particle mass and a=effective radius of tetrahedron; q_i=dimensionless normal coordinates of vibration; $p_i=-i\hbar\partial/\partial q_i$=conjugate momenta; q_1 refers to a nondegenerate dilatational mode of excitation energy w_1; q_2, q_3 refer to a doubly degenerate pair-twisting mode of excitation energy w_2; q_4, q_5, q_6 refer to a triply degenerate mode (in which one side of the tetrahedron expands while the opposite side contracts) of excitation energy w_3; $\mathbf{J}$=operators for (total angular momentum)/$\hbar$; $\zeta\mathbf{L}$=operators for (internal angular momentum)/$\hbar$, where $\zeta=-\frac{1}{2}$, $\mathbf{L}=L_x\mathbf{i}'+L_y\mathbf{j}'+L_z\mathbf{k}'$, where $\mathbf{i}'$, $\mathbf{j}'$, $\mathbf{k}'$ are a set of body axes for the tetrahedron, and L_x, L_y, $L_z=(p_5q_6-p_6q_5)/\hbar$, $(p_6q_4-p_4q_6)/\hbar$, $(p_4q_5-p_5q_4)/\hbar$.

In the Hamiltonian, the terms $\frac{1}{2}w_0\mathbf{J}^2$, $\frac{1}{2}w_0\zeta^2\mathbf{L}^2$, $-w_0\zeta(\mathbf{J}\cdot\mathbf{L})$ refer respectively to the total angular momentum, the internal angular momentum associated with the vibration of excitation energy w_3, and Coriolis interaction between the two angular momenta; while the remainder of the Hamiltonian, H_{vib}, belongs to the vibrational motion. In the Hamiltonian no account is taken of a possible tunnel motion in the structure, but it is thought that perturbations made to the energy by tunnel motion should be in the kev range.[3]

II. ENERGY LEVELS

A wave function which is an eigenfunction of H is simultaneously an eigenfunction of H_{vib}, $\mathbf{J}^2$, $\mathbf{L}^2$, and $\mathbf{J}\cdot\mathbf{L}$, and has a definite parity p. We shall designate

$$\mathbf{I}=\mathbf{J}-\mathbf{L}.$$

Then the wave function is also an eigenfunction of $\mathbf{I}^2$, and

$$\mathbf{J}\cdot\mathbf{L}=\tfrac{1}{2}(\mathbf{J}^2+\mathbf{L}^2-\mathbf{I}^2).$$

The excitation energy of a state is given by

$$E=n_1w_1+n_2w_2+n_3w_3+w_0[\tfrac{1}{2}J(J+1)+\tfrac{1}{8}L(L+1)+\tfrac{1}{2}\mathbf{J}\cdot\mathbf{L}]$$

or

$$E=n_1w_1+n_2w_2+n_3w_3+w_0[\tfrac{3}{4}J(J+1)+\tfrac{3}{8}L(L+1)-\tfrac{1}{4}I(I+1)].$$

A state may be designated by specifying the value of the numbers

$$n_1,\quad n_2,\quad n_3,\quad J,\quad p,\quad L,\quad I;$$

and the energies are determined once the four constants w_0, w_1, w_2, w_3 are specified.

The symmetry requirement determines allowed combinations of the state numbers, as discussed in the Appendix.

The energy expressions for allowed states are listed in Table I together with the vibration numbers n_1, n_2, n_3, total angular momentum quantum numbers J, parities p, internal angular momentum quantum numbers L associated with the w_3 wave functions, parities p_L of the w_3 wave function under the permutation group, angular momentum quantum numbers I and parities p_I of the (angular plus w_3) wave functions under the permutation group. Where both parities are permitted, the parity value p is given as $\pm$ or $\mp$ to indicate whether the energy of the + or − state, respectively, would be expected to be enhanced by the tunnel energy.[10]

TABLE I. Energy levels of O^{16} on α-particle model.

n_1	n_2	n_3	J, p	L, p_L	I, p_I	Energy expression	E calculated Ident. (a)	E calculated Ident. (b)
0	0	0	0^+			0	0	0
0	0	0	3^-			$6w_0$	(6.14)	(6.14)
0	0	0	4^+			$10w_0$	10.23	10.23
1	0	0	0^+			w_1	(6.06)	(6.06)
1	0	0	3^-			w_1+6w_0	12.20	12.20
0	1	0	$2^\mp$			w_2+3w_0	(9.84)	(6.91)
0	1	0	$4^\mp$			w_2+10w_0	17.00	14.07
0	0	1	1^-	1^-	0^+	$w_3+9w_0/4$	(7.01)	(7.12)
0	0	1	2^+	1^-	3^-	$w_3+9w_0/4$	(7.01)	7.12
0	0	1	3^-	1^-	4^+	$w_3+19w_0/4$	9.57	9.68
0	0	1	3^+	1^-	3^-	$w_3+27w_0/4$	11.61	11.72
0	0	1	4^-	1^-	4^+	$w_3+43w_0/4$	15.71	15.82
0	0	1	4^+	1^-	3^-	$w_3+51w_0/4$	17.75	17.86
2	0	0	0^+			$2w_1$	12.12	12.12
1	1	0	$2^\mp$			$w_1+w_2+3w_0$	15.90	12.97
1	0	1	1^-	1^-	0^+	$w_1+w_3+9w_0/4$	13.07	13.18
1	0	1	2^+	1^-	3^-	$w_1+w_3+9w_0/4$	13.07	13.18
1	0	1	3^-	1^-	4^+	$w_1+w_3+19w_0/4$	15.63	15.74
0	2	0	0^+			$2w_2$	13.54	7.68
0	2	0	$2^\mp$			$2w_2+3w_0$	16.61	10.75
0	2	0	3^-			$2w_2+6w_0$	19.68	13.82
0	1	1	$1^\pm$	1^-	$2^\mp$	$w_2+w_3+3w_0/4$	12.24	9.43
0	1	1	$2^\pm$	1^-	$2^\mp$	$w_2+w_3+15w_0/4$	15.32	12.50
0	1	1	$3^\pm$	1^-	$4^\mp$	$w_2+w_3+19w_0/4$	16.34	13.52
0	0	2	0^+	0^+	0^+	$2w_3$	9.42	9.64
0	0	2	3^-	0^+	3^-	$2w_3+6w_0$	15.56	15.78
0	0	2	1^-	2^+	3^-	$2w_3+3w_0/4$	10.19	10.41
0	0	2	2^+	2^+	4^+	$2w_3+7w_0/4$	11.21	11.43
0	0	2	2^-	2^+	3^-	$2w_3+15w_0/4$	13.26	13.48
0	0	2	3^+	2^+	4^+	$2w_3+25w_0/4$	15.82	16.04
1	2	0	0^+			w_1+2w_2	19.60	13.74
1	1	1	$1^\pm$	1^-	$2^\mp$	$w_1+w_2+w_3+3w_0/4$	18.31	15.49
0	3	0	$0^\mp$			$3w_2$	20.31	11.52
0	3	0	$2^\mp$			$3w_2+3w_0$	23.38	14.55
0	2	1	$1^\pm$	1^-	$2^\mp$	$2w_2+w_3+3w_0/4$	19.02	13.27
0	2	1	1^-	1^-	0^+	$2w_2+w_3+9w_0/4$	20.55	14.80
0	2	1	2^+	1^-	3^-	$2w_2+w_3+9w_0/4$	20.55	14.80
0	1	2	$0^\mp$	2^+	$2^\mp$	$w_2+2w_3+3w_0/4$	16.96	14.25
0	1	2	$2^\mp$	2^+	$4^\mp$	$w_2+2w_3+7w_0/4$	17.98	15.27
0	0	3	1^-	3^-	4^+	$3w_3+w_0$	15.15	15.48
0	0	3	0^+	3^-	3^-	$3w_3+6w_0/4$	15.66	15.99
0	4	0	0^+			$4w_2$	27.08	15.36

Also in Table I are listed the energies of the states on both schemes of correlation given by Dennison.[3] Those energy values used in the identification are set off in parentheses. The two schemes of identification differ principally in the assignment of the excitation energy

[10] According to Dennison,[3] the tunnel energy is to be ascribed to the pair twisting mode of excitation energy w_2. In a $\pm$ doublet in which state w_2 is excited, which of the two parity states is of enhanced energy will thus depend upon the parity of the wave function for the w_2 vibration plus rotation. Since the wave function for the triply degenerate vibration of excitation energy w_3 has intrinsic parity $p_L=(-1)^{n_3}$, then for total parity p, the wave function for w_2 plus rotation will have parity $p_I=p_Lp$. Hence we expect that the energy of the − or + state of a doublet will be enhanced by the tunnel energy according as n_3 is even or odd.

TABLE II. Assignment of energy levels.

Calculated-ident. (a) E	J, p	Observed E	J, p	Calculated-ident. (b) E	J, p	Observed E	J, p
0	0^+	0	0^+	0	0^+	0	0^+
(6.06)	0^+	6.06	0^+	(6.06)	0^+	6.06	0^+
(6.14)	3^-	6.14	3^-	(6.14)	3^-	6.14	3^-
(7.01)	2^+	6.91	2^+	(6.91)	2^+	6.91	2^+
(7.01)	1^-	7.12	1^-	6.91	2^-		
9.42	0^+			7.12	2^+		
9.57	3^-			(7.12)	1^-	7.12	1^-
(9.84)	2^+	9.84	2^+	7.68	0^+		
9.84	2^-	[8.87	2^-3^+][a]	9.43	1^-	9.58	1^-
10.19	1^-	9.58	1^-	9.43	1^+		
10.23	4^+	10.36	4^+	9.64	0^+		
11.21	2^+	11.51	2^+	9.68	3^-		
11.61	3^+	[11.08	2^-3^+][a]	10.23	4^+	10.36	4^+
12.12	0^+	11.25	0^+	10.41	1^-	11.10	(1^-)
12.20	3^-	11.62	3^-	10.75	2^+	9.84	2^+
12.24	1^-	[12.43	1^-]	10.75	2^-	[11.08	2^-3^+][a]
12.24	1^+	[12.02	(1^+)][a]	11.43	2^+	11.51	2^+
13.07	2^+	12.5+	2^+	11.52	0^+	11.25	0^+
13.07	1^-	13.09	1^-	11.52	0^-	[12.02	(0^-)][a]
13.26	2^-	12.51	2^-	11.72	3^-	11.62	3^-
13.54	0^+	[12.43	0^+]	12.12	0^+	[12.43	0^+]
15.15	1^-			12.20	3^-		
15.32	2^-			12.50	2^+	[12.5+	2^+]
15.32	2^+			12.50	2^-	12.51	2^-
15.56	3^-			12.97	2^-	12.95	2^-
15.63	3^-			12.97	2^+		
15.66	0^+			13.18	2^+		
15.71	4^-			13.18	1^-	[12.43	1^-]
15.82	3^+			13.27	1^-	13.09	1^-]
15.90	2^+			13.27	1^+	[13.65	1^+
15.90	2^-			13.48	2^-	[13.65	2^-]
				13.52	3^-		
				13.52	3^+		
				13.74	0^+		
				13.82	3^-		
				14.07	4^+	13.24	4^+
				14.07	4^-		
				14.25	0^+		
				14.25	0^-		
				14.55	2^+		
				14.55	2^-		
				14.80	2^+		
				14.80	1^-		
				15.27	2^+		
				15.27	2^-		
				15.36	0^+		
				15.48	1^-		
				15.49	1^-		
				15.49	1^+		
				15.74	3^-		
				15.78	3^-		
				15.82	4^-		
				15.99	0^+		

Levels observed but not predicted

Ident. (a)		Ident. (b)	
[11.10	$J(-1)^J$][a]	[8.87	2^-3^+][a]
12.95	2^-		
13.24	4^+		
13.65	2^-		

[a] See reference 7.

w_2 of the doubly degenerate vibration which in identification (a) first appears in the 2^+ state at 9.83 Mev, while in identification (b) occurs in the 2^+ state at 6.91 Mev. It is believed that all states whose predicted energy on either identification is less than 16 Mev have been included.

In Table II the energy levels predicted on the two identification schemes are listed in order of increasing energy and compared with observed energies. Predicted energies used in the identification are placed in parentheses, while observed energies whose angular momenta are uncertain are set off in brackets. Observed energies are those given by Ajzenberg and Lauritsen,[6] with some additions from Hornyak and Sherr.[7]

III. MEAN LIVES OF LOWER EXCITED STATES

Devons[1] has made experimental determinations of the lifetime of the 0^+, 3^-, 2^+, and 1^- states at 6.06, 6.14, 6.91, and 7.12 Mev, respectively. In order to compute the lifetimes of these states on the α-particle model, we chose a particular set of standard coordinates $q_1 \cdots q_6$ of vibration and Euler angle coordinates of rotation. After expressing in these coordinates the wave function for the four excited states, the ground state, and the operators which provide the transition to the ground state, we could then compute the lifetimes. The four lifetimes are discussed separately below and the results tabulated in Table III.

We use the following notation: W_0, W_3, W_2, W_1 = excitation energies for first four excited states, $0^+, 3^-, 2^+, 1^-$, on identification (b); W_1' = excitation energy for 2^+, or 1^- state on identification (a); Mc^2 = rest energy of α-particle; $\alpha = e^2/\hbar c = 1/137.03$, the fine structure constant; $w_2 = W_2 - \frac{1}{2}W_3$ = vibrational excitation energy for doubly degenerate vibrational mode; $w_3 = W_1 - \frac{3}{8}W_3$ = vibrational excitation energy for triply degenerate vibrational mode on identification (b); $w_3' = W_1' - \frac{3}{8}W_3$ = vibrational excitation energy for triply degenerate vibrational mode on identification (a);

$$0^+(E = W_0 = 6.06 \text{ Mev}) \rightarrow 0^+(E = 0). \qquad \text{(a)}$$

This transition goes by pair production. The transition rate was computed for the direct first order process involving pair production by the time-varying electric field inside the nucleus. Computation of higher order corrections to the pair production process did not appear to be justified here.

The calculated mean life,

$$\tau_{00} = \frac{15\pi}{8\alpha^2} \frac{W_3(Mc^2)^2\hbar}{(w_1)^4} = 4.6 \times 10^{-12} \text{ sec},$$

TABLE III. Mean lives of low-lying states of O^{16}.

State	Energy (Mev)	Mean life in seconds: Experimental[a]	Theoretical[b]	Remarks
0^+	6.06	$(7 \pm 1) \times 10^{-11}$	4.6×10^{-12}	
3^-	6.14	$4.3 \times 10^{-12} \leq \tau \leq 1.2 \times 10^{-11}$	3.2×10^{-11}	
2^+	6.91	$\leq 1.7 \times 10^{-14}$	1.95×10^{-15}	Ident. (b)
			2.4×10^{-14}	Ident. (a)
1^-	7.12	$\leq 1.2 \times 10^{-15}$	$4 \times 10^{-16} < \tau < 5 \times 10^{-13}$	Ident. (b)
			$9 \times 10^{-15} < \tau < 5 \times 10^{-13}$	Ident. (a)

[a] The experimental mean life for the 0^+ state is taken from reference 9; the others from reference 10.

[b] The theoretical limits for the 1^- state are values given by the indirect approach of paragraphs (1) and (2) of Sec. III(d) above. On the strict α-particle model, $\tau = 1.6 \times 10^{-11}$ sec.

is shorter than the experimental value by a factor of 15.

(b) $3^-(E=W_3=6.14\text{ Mev})\rightarrow 0^+(E=0)$.

An electric octupole transition is involved. The calculation is straightforward and yields a mean life

$$\tau_{30}=\frac{245}{24\alpha}\frac{(Mc^2)^3\hbar}{(W_3)^4}=3.2\times10^{-11}\text{ sec},$$

which is two or three times longer than the experimental value.

(c) $2^+(6.91\text{ Mev})\rightarrow 0^+$ (ground state)

An electric quadrupole transition is involved. The calculation is straightforward except that for the 2^+ state, the two identification schemes, (a) and (b), assign different vibration numbers, 001 and 010, respectively. The wave functions thus differ and the lifetime computed on identification (a) is ten times longer than that computed on identification (b).

According to identification (b) with $E=W_2=6.91$ Mev, $w_2=3.84$ Mev and the mean life is

$$\tau_{20}=\frac{25}{24\alpha}\frac{(W_3Mc^2)^2\hbar}{w_2(W_2)^4}=1.95\times10^{-15}\text{ sec},$$

which is well within the limits set by experimental data.

On identification (a), with $E=W_1'=7.01$ Mev, $w_3'=4.71$ Mev and the mean life is

$$\tau_{2'0}=\frac{45}{4\alpha}\frac{(W_3Mc^2)^2\hbar}{w_3'(W_1')^4}=2.4\times10^{-14}\text{ sec},$$

which is just outside of the experimental limits.

(d) $1^-(E=W_1=7.12\text{ Mev})\rightarrow 0^+(E=0)$.

This is an electric dipole transition. On the α-particle model however, electric dipole transitions are forbidden, since all particles have the same charge/mass ratio. Thus the center of charge coincides with the center of mass, and the electric dipole moment vanishes so that the dominant contribution to the transition rate is given by the third-order electric dipole moment operator. The mean life calculated on the strict α-particle model on identification (b) with $W_1=7.21$, $w_3=4.91$ Mev is

$$\tau_{10}^{(3)}=\frac{225}{12\alpha}\frac{w_3(W_3)^2(Mc^2)^3\hbar}{(w_1)^7}=1.6\times10^{-11}\text{ sec},$$

and identification (a) would produce a similar result. The value is larger by a factor $>10^4$ than the limit set by experimental data. It is not expected that this value for the lifetime has any significance. (A single-particle model would give a lifetime of the order of 10^{-18} sec for this transition, so that a very small admixture of single-particle wave function with a predominantly α-particle wave function in the 1^- state would account for the observed lifetime.)

It is possible to get rough but perhaps more meaningful estimates for the lifetime of the 1^- state by following some arguments proposed by Wilkinson.[11]

(1) We obtain an upper limit for the lifetime of the 1^- state by considering that the $(1^-\rightarrow 3^-)/(1^-\rightarrow 0^+$ (ground)) branching ratio is observed to be less than 0.008. Since the $1^-\rightarrow 3^-$ transition goes by electric quadrupole, we can presumably calculate the $1^-\rightarrow 3^-$ lifetime correctly on the α-particle model and we thus obtain the upper limit 5×10^{-13} sec for the 1^- lifetime.

(2) To get a very rough lower limit for the lifetime, we consider that the $(2^+\rightarrow 3^-)/(2^+\rightarrow 0^+$ (ground)) branching ratio is observed to be less than 0.005. We assume the same single particle admixture to be present in the 2^+ state as in the 1^-, so that the rate for the $2^+\rightarrow 3^-$ transition differs from that for the $1^-\rightarrow 0^+$ transition by a factor of the cube of the ratio of energy difference. Lastly, we use the $2^+\rightarrow 0^+$ lifetime calculation given by the α-particle model and combine the factors. The lower limit is dependent upon which of the two identifications used for the 2^+ state: the lower limit for the 1^- lifetime under identification (a) is 9×10^{-15} sec, while under identification (b) it is 4×10^{-16} sec.

The value given by the identification (b) is within experimental limits, while that given by identification (a) falls outside.

IV. APPLICATION TO O^{17}

Thirion and Telegdi[9] found that in O^{17} the $\frac{1}{2}^+$ 870-kev state decays to the $(5/2)^+$ ground state with a mean life of $(2.5\pm1)\times10^{-10}$ sec. They computed that on a strict shell model of a neutron outside an O^{16} core, the lifetime would be $\sim10^{-7}$ sec; and suggested that an admixture of partially excited $(2^+)O^{16}$ core states could account for the observed lifetime, provided that the lifetime of the 6.91 Mev, 2^+ state in O^{16} were short enough.

We have accordingly used α-particle wave functions to describe the O^{16} core of a partially-excited-core shell model of O^{17}, and computed the $\frac{1}{2}^+\rightarrow(5/2)^+$ transition rate and the electric quadrupole and magnetic moment of the ground state, to see whether such a model could fit the three data. The wave functions were of the form

$$\Psi_{\frac{5}{2}+}{}^m(O^{17})=a_0\Phi_{d\frac{5}{2}}{}^m(n)\Psi_0(O^{16})+a_2\sum_{j=-\frac{1}{2}}^{\frac{1}{2}}C_{j,m-j,m}{}^{\frac{1}{2},2,5/2}\Phi_{s\frac{1}{2}}{}^j(n)\Psi_{2+}{}^{m-j}(O^{16})$$

$$\Psi_{\frac{1}{2}+}{}^m(O^{17})=a_0\Phi_{s\frac{1}{2}}{}^m(n)\Psi_0(O^{16})+b_2\sum_{j=-\frac{5}{2}}^{\frac{5}{2}}C_{j,m-j,m}{}^{5/2,2,\frac{1}{2}}\Phi_{d\frac{5}{2}}{}^j\Psi_{2+}{}^{m-j}(O^{16}),$$

[11] D. H. Wilkinson and G. A. Jones, Phil. Mag. 44, 542 (1953).

where the C's are Clebsch-Gordan coefficients, and a_0, a_2, b_0, b_2 are adjustable parameters. a_2 and b_2 are regarded as small; a_0 and b_0 are real; $a_0^2=1-|a_2|^2$; $b_0^2-1-|b_2|^2$.

The computed $\frac{1}{2}^+ \rightarrow (5/2)^+$ transition rate, disregarding the small neutron transition contribution, is given by

$$\omega=|a_0b_2+\sqrt{3}b_0a_2^*|^2\left(\frac{W}{W_2}\right)^5\frac{1}{\tau_{20}};$$

with $W=870$ kev and W_2, τ_{20}, being interpreted as either the primed or the unprimed quantities according to the identification scheme used to describe the 2^+ state. On identification (b),

$$\omega=|a_0b_2+\sqrt{3}b_0a_2^*|^2\times 1.6\times 10^{10}\ \text{sec}^{-1},$$

while on identification (a),

$$\omega=|a_0b_2+\sqrt{3}b_0a_2^*|^2\times 1.2\times 10^{9}\ \text{sec}^{-1}.$$

Comparing these expressions with the observed transition rate of 4×10^9 sec^{-1}, we see that on identification (a) no exact fit is possible, consistent with weak excitation of the core. On identification (b), however, the observed rate can be matched if $|a_0b_2+\sqrt{3}b_0a_2^*|^2=0.5$; for which, given the most favorable phase relationship $b_2\propto a_2^*$, a quite low amplitude of core excitation, $a_2=b_2\sim 0.2$ (4% probability of excitation) is possible.

In the latter case, a ground state quadrupole moment of $-2e\times 10^{-27}$ cm^2 and magnetic moment of -1.81 nuclear magnetons are obtained, in fair agreement with the observed values[12,13] of $(-5\pm 2)e\times 10^{-27}$ cm^2 and -1.893 nuclear magnetons, respectively.

V. INTERPRETATIONS

1. Energy Levels

On either scheme of identification it is possible to match almost all observed levels below 12.95 Mev, to within 1 Mev. (The only exceptions here are the levels at 8.87, 11.08, and 11.10 Mev, for none of which are the angular momentum and parity known exactly). The first two are reported by Hornyak and Sherr[7] to be most likely 2^- but possibly 3^+. On identification (a), levels at 8.87 and 11.08 Mev can be fitted only if their spins and parities are 2^-, 3^+, respectively; the 8.87-Mev level is regarded as the 2^- component of a $2^\mp$ doublet (the other component being the 9.84-Mev 2^+ state) which, in this identification must be inverted with respect to the predictions of the model. On identification (b), the 8.87-Mev level cannot be fitted at all and the 11.08 level can be matched only if its spin and parity is 2^-. The 11.10-Mev level is reported by Bittner and Moffat[5] to have appeared in the $C^{12}(\alpha,\alpha)$ scattering and thus must have parity $(-1)^J$. On identification (a), no such state can be matched; while on identification (b), a fit is possible only if $J=1$. The 12.02-Mev state is reported by Hornyak and Sherr[7] to be a gamma-emitting level and thus should have parity $(-1)^{J+1}$. On identification (a), assignment of this level is possible only for J, $p=1^-$. On identification (b), it is possible to identify the 12.02-Mev level either with the 0^- level at 11.52 Mev, or with the 2^- level predicted at 12.97 Mev, provided that the reported 12.95-Mev 2^- level is not ascribed to the α-particle model (see below).

One must note that in the 0–12.95 Mev region, the correlation between observed and predicted levels is not one-to-one on either identification scheme. In the most optimistic case on identification (a), regarding the 8.87, 11.08, and 12.02 Mev levels as matched (i.e., spins and parities 3^+, 2^-, 1^+, respectively), one finds that there are two levels (9.42 and 9.57 Mev) which are predicted but not observed, and one level (11.10 Mev) which is observed but not predicted. On identification (b), for the most optimistic case, regarding the 11.08- and 11.10-Mev states as matched (i.e., spins and parities 2^-, 1^-, respectively), one finds that there are at least seven levels [6.91(2^-), 7.12(2^+), 7.68, 9.43(1^+), 9.64, 9.68, 12.20, and perhaps 11.52 Mev (0^-)] which are predicted but not observed, and one level (8.87 Mev) which is observed but not predicted.

For energies above 12.95 Mev inclusive, there are three or four observed levels (depending upon whether there are one or two levels at 13.65 Mev) which can be fitted on identification (b) but which cannot be matched within 1 Mev on identification (a). However, the equivalence theory for neutrons and protons requires the correspondence of four levels in O^{16} to the four known levels (0 to 0.31 Mev) in N^{16}. Since N^{16} has isotopic spin $T=1$ ($T_z=1$) the corresponding O^{16} levels must have $T=1$ ($T_z=0$), but since α particles have intrinsic $T=0$, these levels could not appear on the α-particle model. Ajzenberg and Lauritsen[14] find that these levels (the lowest has J, $p=2^-$) should appear at energies 12.95 through 13.34 Mev in O^{16}. Thus the lack of predicted levels 12.95 Mev ($J=2^-$) 13.24($J=4^+$) and perhaps also 13.65 Mev ($J=1^+$ or 2^-) is to be considered to be evidence favoring identification (a).

Thus, as previously stated by Dennison,[3] it would appear that the energy level evidence favors identification (a) over identification (b).

2. Lifetimes

The results of the lifetime calculations are only in fair agreement with experiment. The 1^- lifetime corresponding to an electric dipole transition, is forbidden on the α-particle model and thus a strict calculation was expected to come out qualitatively incorrect. The 3^-

[12] Geschwind, Gunther-Mohr, and Townes, Phys. Rev. **83**, 209 (1951).

[13] F. Alder and F. C. Yu, Phys. Rev. **81**, 1067 (1951).

[14] F. Ajzenberg and T. Lauritsen, Quarterly Progress Report 4, Department of Physics, Boston University, September 30, 1954 (unpublished), Appendix B.

and 2^+ lifetimes should, on the other hand, be correctly predicted qualitatively and perhaps quantitatively. Thus for the 3^- lifetime, where experiment yields narrow limits, the model yields a value which is close, being a factor of 3 to 8 times too large. For the 2^+ state, the theoretical lifetime depends upon the identification scheme employed. Identification (b) yields a lifetime which is well within the limit set experimentally, while the lifetime on identification (a) falls outside by a factor of 1.5.

For the 0^+ pair emission lifetime, the α-particle model produces a lifetime which is shorter by a factor of 15 than the experimental value. This is in agreement with computations by Schiff[15] who points out that any pure collective model (in which the excitation of the 0^+ state at 6.06 Mev is attributed to a dilatational oscillation of the entire nucleus) must produce too large a matrix element for this transition. This and other evidence such as the short 1^- lifetime show that quantitatively correct model of the O^{16} nucleus cannot be achieved by a pure collective model but only by a model which considers other types of excitation, such as single-particle excitation, as well.

3. O^{17} Calculations

The calculations on O^{17} show only that if the partially excited core shell model of O^{17} is correct, then the half-life of the 2^+ state of O^{16} is probably equal to or less than that predicted on identification (b) and certainly less by a factor of 10 than that predicted on identification (a).

VI. SUMMARY AND CONCLUSIONS

As previously stated by Dennison,[3] the α-particle model on identification (a) shows a remarkable agreement with experiment in the prediction of energy levels of O^{16}. Of the first nineteen excited states which are experimentally observed (0–12.95 Mev), fifteen are definitely predicted and three others tentatively predicted (awaiting experimental determination of their spins and parities), all to within 1.1 Mev or less; two levels are predicted but not observed. On the basis of charge symmetry considerations relating states of N^{16} to those of O^{16}, it is felt that there is little use in attempting to carry level identification to higher energies.

Lifetimes of the first four excited states (0^-, 3^-, 2^+, 1^-) calculated on the α-particle model (except that of the $1^- \rightarrow 0^+$ decay which is forbidden on the model) are within about one and one-half orders of magnitude of the experimental values. These results probably represent what one should expect under the approximations which are implied in the calculations. The slightly greater success in lifetime predictions afforded by identification (b), which appears to give a better approximation to the 2^+ lifetime than does identification (a), probably does not represent anything significant.

[15] L. I. Schiff, Phys. Rev. **98**, 1281 (1955).

In conclusion, we can say that the α-particle model appears to give order-of-magnitude correct answers in lifetime calculations involving only first excited vibrational levels, so long as only those processes consistent with the model, are involved. When further experimental evidence on the O^{16} nucleus is obtained, it will be interesting to perform other dynamical calculations, to see whether similar order-of-magnitude agreement with experiment is obtained.

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APPENDIX. SYMMETRIZATION OF WAVE FUNCTIONS

A wave function of an assemblage of four α particles is required to be invariant under the group P of permutations of the four α particles. P is isomorphic to the group P_4 of permutation of four objects and hence possesses four irreducible representations (1111), (211), (22), (31), (4) of dimensions 1, 3, 2, 3, 1, respectively, in which the basis vector of the first is symmetric in all four objects, those of the second are antisymmetric in a pair of objects and symmetric between that pair and the others, the third is antisymmetric in each of two pairs and symmetric between the two pairs, etc.

The set of wave functions for first excited states of a particular degenerate vibrational level must form basis vectors of particular irreducible representations of P_4, and it is easily found that q_1 belongs to the (1111) representation; q_2, q_3 to the (22) representations; q_4, q_5, q_6 to the (211) representation.

The permutations P on the vertices of a tetrahedron form a subgroup of the complete rotation group, so that for any fixed J, p, and $m=J_z$ the permutations P induce on the $(2J+1)$ angular wave functions $Y_J^{\pm km}$ a group of unitary transformations which form a representation R of P_4, of order $2J+1$ and are, in general, reducible. (For a particular, fixed J, p, m the similarity transformation which completely reduces R is independent of m and produces linear combinations of functions of different k values; such that under P these combinations transform only within independent sets, each set according to an irreducible representation of P_4.)

The number of irreducible representations of P_4 induced in functions of angular momentum and parity $J^\pm$ are given in Table IV.

For the total wave function to be invariant under P, the vibrational and rotational parts must belong to the same irreducible representation. Thus

(1) The only pure rotational states (vibrational ground state belongs to (1111)) are those containing the (1111) representation, and have J, $p=0^+, 3^-, 4^+ \cdots$.

TABLE IV. Number of irreducible representations of P_4 induced in functions of angular momentum and parity $J^{\pm}$.

$J^{\pm}$	0^+	0^-	1^+	1^-	2^+	2^-	3^+	3^-	4^+	4^-	5^+	5^-
$n(1111)$	1	0	0	0	0	0	0	1	1	0	0	0
$n(211)$	0	0	0	1	1	0	1	1	1	1	1	2
$n(22)$	0	0	0	0	1	1	0	0	1	1	1	1
$n(31)$	0	0	1	0	0	1	1	1	1	1	2	1
$n(4)$	0	1	0	0	0	0	1	0	0	1	0	0

(2) The vibration number, n_1, never affects the symmetry; hence there is no restriction on the value of n_1.

(3) The vibrational state $n_2=1$ belongs to representation (22); the vibrational state $n_2=2$ belongs to representation (1111) or (22); the vibrational state $n_2=3$ belongs to representation (1111) or (22) or (4). Hence states with $n_2, n_3=1, 0$ have $J, p=2^{\pm}, 4^{\pm}, 5^{\pm}, \cdots$; states with $n_2, n_3=2, 0$ have $J, p=0^+, 2^{\pm}, 3^-, 4^{\pm}, 5^{\pm}, \cdots$; and states with $n_2, n_3=3, 0$ have $J, p=0^{\pm}, 2^{\pm}, 3^{\pm}, 4^{\pm}, 5^{\pm}, \cdots$.

(4) States with $n_3=0$ will have to be eigenfunctions of $\mathbf{L}^2$ and $\mathbf{J}\cdot\mathbf{L}$ in addition to being symmetric.

(a) For a given value of n_3, eigenvalues of $\mathbf{L}^2$ have $L=n_3, n_3-2, n_3-4, \cdots$ with $L\geq 0$.

(b) Call $V_L{}^i$ an eigenfunction of $L^2=L(L+1)$ and $L_z=j$. Then the operator $\mathbf{J}\cdot\mathbf{L}$ operating on $Y_J{}^{km}V_L{}^i$ can increase k and j, decrease k and j, or leave both unchanged. Hence an eigenfunction of $\mathbf{J}\cdot\mathbf{L}$ will be made of $Y_J{}^{km}V_L{}^i$ combinations in which $k-j=$constant, or in which functions of total angular momentum J and internal angular momentum L combine as $\mathbf{I}=\mathbf{J}-\mathbf{L}$.

Under the group P, the wave function for (angular momentum J)$+$(n_3 state with internal angular momentum L) will transform like an angular function with angular momentum $\mathbf{I}=\mathbf{J}-\mathbf{L}$. The parity we should associate with I is $p_I=p_Lp$; where $p_L=(-1)^L=(-1)^{n_3}$, since the vibration wave function for $n_3=1$ transforms under P exactly like the angular function for J, $p=1^-$.

(5) The rules governing the choice of L, p_L, I, p_I, J, p for given n_1, n_2, n_3 are therefore:

(i) $L=n_3, n_3-2, n_3-4\cdots; L\geq 0$;
(ii) $p_L=(-1)^{n_3}$;
(iii) I, p_I are given as a function of n_2 by the table:

n_2	I, p_I
0	$0^+, 3^-, 4^+, 6^{\pm}\cdots$
1	$2^+, 4^{\pm}, 5^{\pm}, 6^{\pm}\cdots$
2	$0^+, 2^{\pm}, 3^-, 4^{\pm}, 5^{\pm}, 6^{\pm}\cdots$
3	$0^+, 2^{\pm}, 3^{\pm}, 4^{\pm}, 5^{\pm}, 6^{\pm}\cdots$;

(iv) $|J-L|\leq I\leq|J+L|$;
(v) $p=p_Lp_I$.